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BUREAU OF EDUCATION

CIRCULAR OF INFORMATION NO. 3, 1890

THE TEACHING AND HISTORY OF MATHEMATICS IN THE UNITED STATES

BY

FLORIAN CAJORI, M. S. (University of Wisconsin)

FORMERLY PROFESSOR OF APPLIED MATHEMATICS IN THE TULANE UNIVERSITY OF LOUISIANA, NOW PROFESSOR OF PHYSICS IN COLORADO COLLEGE.

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DEPARTMENT OF THE INTERIOR,
BUREAU OF EDUCATION,
Washington, D. C., February 19, 1889.

SIR: I have the honor to transmit herewith the manuscript of a History of Mathematical Teaching in the United States, by Prof. Florian Cajori, a graduate of the University of Wisconsin, student at Johns Hopkins University, and recently professor of applied mathematics in Tulane University of Louisiana—a work prepared with your approval, under the direction of this Office.

The table of contents indicates the wide scope of the work and the variety of subjects treated, but scarcely more than suggests the painstaking labor involved in its preparation. Professor Cajori's researches have extended through several years, and have been pursued in the libraries of Baltimore, Philadelphia, and Washington. He has personally conducted a large correspondence with alumni, and past and present instructors in the higher educational institutions, and has been aided by 1,000 circulars of inquiry sent from this Office relating to the present condition of mathematical teaching in schools of all grades.

I am convinced that this monograph will prove of great value to all teachers and students of mathematics, and will be not without interest to any person engaged in the work of education. I therefore respectfully recommend its publication.

I have the honor to be, sir, very respectfully, your obedient servant,

N. H. R. DAWSON,
Commissioner.

Hon. WM. F. VILAS,
Secretary of the Interior, Washington, D. C.

DEPARTMENT OF THE INTERIOR,
Washington, D. C., April 11, 1889.

The COMMISSIONER OF EDUCATION:

SIR: I acknowledge the receipt of your letter of February 19, 1889, in which you recommend the publication of a monograph, a history of mathematical teaching in the United States.

Authority is hereby given for the publication of the monograph, provided there are funds in sufficient amount available for such purpose.

Very respectfully,

JOHN W. NOBLE,
Secretary.

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THE TEACHING AND HISTORY OF MATHEMATICS IN THE UNITED STATES.

I.

COLONIAL TIMES.

Elementary Schools.

On the study of mathematics in elementary schools of the American colonies but little can be said. In early colonial days schools did not exist except in towns and in the more densely settled districts; and even where schools were kept, the study of mathematics was often not pursued at all, or consisted simply in learning to count and to perform the fundamental operations with integral numbers. Thus, in Hampstead, N. H., in 1750, it was voted "to hire a school-master for six months in ye summer season to teach ye children to read and write." Arithmetic had not yet been introduced there. As late as the beginning of this century there were schools in country districts in which arithmetic was not taught at all. Bronson Alcott, the prominent educator, born in Massachusetts in 1799, in describing the schools of his boyhood, says: "Until within a few years no studies have been permitted in the day school but spelling, reading, and writing. Arithmetic was taught by a few instructors one or two evenings in a week. But in spite of the most determined opposition arithmetic is now permitted in the day school." This was in Massachusetts at the beginning of this century.

In secondary schools, "ciphering" was taught during colonial times, which consisted generally in drilling students in the manipulation of integral numbers. He was an exceptional teacher who possessed a fair knowledge of "fractions" and the "rule of three," and if some pupil of rare genius managed to master fractions, or even pass beyond the "rule of three," then he was judged a finished mathematician.

The best teachers of those days were college students or college graduates who engaged in teaching as a stepping-stone to something better. An example of this class of teachers was John Adams, afterwards President of the United States. Immediately after graduating at Harvard and before entering upon the study of law, he presided, for a few years, over the grammar school at Worcester. From a letter

written by him at Worcester, September 2, 1755, we clip the following description of the teacher's daily work:

As a haughty monarch ascends his throne, the pedagogue mounts his awful *great chair*, and dispenses right justice through his whole empire. His obsequious subjects execute the imperial mandates with cheerfulness, and think it their high happiness to be employed in the service of the emperor. Sometimes paper, sometimes his penknife, now birch, now arithmetic, now a ferule, then ABC, then scolding, then flattering, then thwacking, calls for the pedagogue's attention.

School appliances in those days were wholly wanting (excepting the ferule and birch rods). Slates were entirely unknown for school use until some years after the Revolution; blackboards were introduced much later. Paper was costly in colonial days, and we are told that birch bark was sometimes used in schools in teaching children to write and figure. Thirty-six years ago a writer in one of our magazines* wrote as follows:

"There are probably men now living who learned to write on birch and beech bark, with ink made out of maple bark and copperas." But more generally "ciphering" was done on paper. Dr. L. P. Brockett says that on account of its dearness and scarcity, "the backs of old letters, the blank leaves of ledgers and day-books, and even the primer books were eagerly made use of by the young arithmeticians."

Since few or none of the pupils had text-books it became necessary for the teacher to dictate the "sums." As in the colleges of that time, so in elementary schools, *manuscript books* were used whenever printed ones were not accessible. To advanced boys the teacher would give exercises from his manuscript or "ciphering-book," in which the problems and their solutions had been previously recorded. "With a book of his own the pupil solved the problems contained in it in their proper order, working hard or taking it easy as pleased him, showed the solutions to the master, and if found correct generally copied them in a blank-book provided for the purpose. * * * Some of these old manuscript ciphering-books, the best, one may suppose, having come down through several generations, are still preserved among old family records, bearing testimony to the fair writing and the careful copying, if not to the arithmetical knowledge, of those who prepared them. When a pupil was unable to solve a problem he had recourse to the master, who solved it for him. It sometimes happened that a dozen or twenty pupils stood at one time in a crowd around the master's desk waiting with * * * problems to be solved. There were no classes in arithmetic, no explanations of processes either by master or pupil, no demonstrations of principles either asked for or given. The problems were solved, the answers obtained, the solutions copied, and the work was considered complete. That some persons did obtain a good knowledge of arithmetic under such teaching must be admitted, but this result was clearly due rather to native talent or hard personal labor than to wise direction."† Those

*North Carolina University Magazine, Raleigh, 1853, Vol. II, p. 452.

†*History of Education* in Pennsylvania, by James Pyle Wickersham, p. 205.

teachers who were the fortunate possessors of a printed arithmetic used it as a guide in place of the old "ciphering-book."

In the early schools, arithmetic was hardly ever taught to girls. Rev. William Woodbridge says that in Connecticut, just before the Revolution, he has "known boys that could do something in the first four rules of arithmetic. Girls were never taught it."* In the two "charity schools" in Philadelphia, which before the Revolution were the most celebrated schools in Pennsylvania, boys were taught reading, writing, arithmetic; girls, reading, writing, sewing. Thus, sewing was made to take the place of arithmetic. Warren Burton, in his book entitled, "The District School as it was, by one who went to it," says that, among girls, arithmetic was neglected. The female portion "generally expected to obtain husbands to perform whatever arithmetical operations they might need beyond the counting of fingers." Occasionally women were employed in summer schools as teachers, but they did not teach arithmetic. A school-mistress "would as soon have expected to teach the Arabic language as the numerical science."

The early school-books in New England and in all other English settlements were much the same as those of Old England. John Locke, in his Thoughts concerning Education (1690), says that the method of teaching children to read in England has been to adhere to "the ordinary road of horn-book, primer, psalter, testament, and bible." This same road was followed in New England. We are told that books of this kind were sold to the people by John Pynchon, of Springfield, from 1656 to 1672 and after.† Regular arithmetics were a great rarity in this country in the seventeenth century. The horn-book has been raised by some to the dignified name of a "primer" for teaching reading and imparting religious instruction. If this be permissible, then why should we not also speak of it as an *arithmetical* primer? For, what was the horn-book? It consisted of one sheet of paper about the size of an ordinary primer, containing a cross (called "criss-cross"), the alphabet in large and small letters, followed by a small regiment of monosyllables; then came a form of exorcism and the Lord's Prayer, and, finally, the *Roman numerals*. The leaf was mounted on wood, and protected with transparent horn,

"To save from fingers wet the letters fair."

It is on the strength of the Roman numerals that we venture to propose the horn-book as a candidate for the honor of being the first mathematical primer used in this country. Horn-books were quite common in England and in the English colonies in America down to the time of George II. They disappeared entirely in this country before the Revolution. In early days the common remark expressive of ignorance was "he does not know his horn-book." This is equivalent to the more modern saying, "he does not know his letters."

* Reminiscenses of Female Education, in Barnard's Journal of Education, 1864, p. 137.

† *Barnard's Journal* of Education, Vol. XXVII.

George Fox, the founder of the Society of Friends, published in 1674, in England, a primer or spelling-book, which was republished at Philadelphia in 1701, at Boston in 1743, and at Newport, R. I., in 1769.* Wickersham describes this little book as containing the alphabet, lessons in spelling and reading, explanations of scripture names, *Roman numerals, lessons in the fundamental rules of arithmetic and weights and measures, a perpetual almanac,* and catechism with the doctrine of the Friends. It may be imagined that a mere primer, covering such a wide range of subjects, could contain only a very few of the simplest rudiments of a subject like arithmetic. Fox's book was used little outside of the Society of Friends.

Wickersham (p. 201) speaks of another book which is of interest as illustrating the book-making of those old times. It is entitled, "The American Instructor, or Young Man's Best Companion, containing Spelling, Reading, Writing, *Arithmetic, in an Easier Way than any yet Published,* and how to Qualify any Person for Business without the Help of a Master." It was written by George Fisher, and printed in Philadelphia, in 1748, by Franklin and Hall. This work never attained any popularity.

Dr. Brockett says that in New Jersey and, perhaps, also in Virginia, a book resembling the "New England Primer," but as intensely Royalist and High Church in religion as the New England Primer was Puritan and Independent, was in use in schools. It was called "A Guide for the Child and Youth, in two parts; the First for Children, * * * the second for Youth: Teaching to write, *cast accounts* and read more perfectly; with several other varieties, both pleasant and profitable. By T. H., M. A., Teacher of a Private School, London, 1762." It does not appear that this book was reprinted here.

Wickersham gives another book of similar stamp but of much later date. "Ludwig Höcker's Rechenbüchlein was published at Ephrata [Pennsylvania] in 1786. The Ephrata publication is an exceedingly curious compound of religious exercises and exercises in arithmetic. The creed, the Lord's Prayer, hymns, and texts of scripture, are strangely intermixed with problems and calculations in the simplest parts of arithmetic."†

One of the earliest purely arithmetical books used in this country was the arithmetic of James Hodder. It may possibly have fallen into the hands of as early a teacher as Ezekiel Cheever, "the father of Connecticut school-masters, the pioneer and patriarch of elementary classical culture in New England."‡ In a history of schools at Salem, Mass., we are told that "among our earliest arithmetics was James Hodder's."

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* History of Education in Pennsylvania, by James Pyle Wickersham, p. 194.

† *Ibid.*, p. 200.

‡ After having been a faithful school-master for seventy years, he died in 1708, at the age of ninety-four, having "held his abilities in an unusual degree to the very last."

Hodder was a famous English teacher of the seventeenth century. Later writers have borrowed largely from his arithmetic of which the first edition, entitled "Hodder's Arithmetick, or that necessary art made most easy," appeared in London in 1661. An American edition from the twenty-fifth English edition was published in Boston in 1719. This is the first purely arithmetical book known to have been printed in this country.

In New York the Dutch teachers of the seventeenth century imported from Holland an arithmetic called the "Coffer Konst," written by Pieter Venema, a Dutch school-master, who died about 1612. So popular was the book that an English translation of it was published in New York in 1730. Venema's appeared to be the second oldest arithmetic printed in America.

An English work almost as old as Hodder's, which met with a limited circulation in this country, is Cocker's Arithmetic. The first edition appeared in England after the death of Cocker, in 1677. According to its title page it was "perused and published by John Hawkins, * * * by the author's correct copy." De Morgan is perfectly satisfied that "Cocker's Arithmetic was a forgery of Hawkins's, with some assistance, it may be, from Cocker's papers." Regarding the book itself, De Morgan says:* "Cocker's Arithmetic was the first which entirely excluded all demonstration and reasoning, and confined itself to commercial questions only. This was the secret of its extensive circulation. There is no need of describing it; for so closely have nine out of ten of the subsequent school treatises been modelled upon it, that a large proportion of our readers would be able immediately to turn to any rule in Cocker, and to guess pretty nearly what they would find there. Every method since his time has been "according to Cocker." This book was found here and there in the colonies at an early date. Thus we read in Benjamin Franklin's Autobiography that (at about the age of sixteen; *i. e.*, about 1722) "having one day been put to the blush for my ignorance in the art of calculation which I had twice failed to learn while at school, I took Cocker's treatise on arithmetic and went through it by myself with the utmost ease." An American edition of the work appeared in Philadelphia in 1779. It contains the rude portrait of the author, "which might be taken for a caricature," and also the following poetical recommendation:

> Ingenious Cocker, now to Rest thou 'rt gone,
> No Art can show thee fully, but thine own;
> Thy rare Arithmetick alone can show
> Th' vast Thanks we for thy labours owe.

Wickersham† mentions Daniel Fenning's *Der Geschwinde Rechner* as having been published by Sower in 1774.

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* Article, "Cocker," Penny Cyclopædia.

† History of Education in Pennsylvania, p. 200.

The first arithmetic written by an American author and printed here was that of Prof. Isaac Greenwood of Harvard College, in 1729. The book was probably used by the author in his classes at Harvard. We have nowhere seen it mentioned except in a biographical sketch of its author.* So far as we know, there are only three copies of Greenwood's Arithmetic in existence, two in the Harvard library and one in the Congressional Library. Prof. J. M. Greenwood, superintendent of schools in Kansas City, sends the writer the following description of it:

The book is a small duodecimo volume of 158 pages, exclusive of an advertisement (4 pages) prefixed, and the table of contents (4 pages) put at the end. The following is a transcript of the title-page: "Arithmetick, Vulgar and Decimal: with the Application thereof to a variety of Cases in Trade and Commerce. (Vignette.) Boston: N. E., Printed by S. Kneeland and T. Green, for T. Hancock at the Sign of the Bible and Three Crowns, in Ann Street, MDCCXXIX."

The headings of chapters are as follows: The introduction; chapter 1, Numeration; chapter 2, Addition; chapter 3, Subtraction; chapter 4, Multiplication; chapter 5, Division; chapter 6, Reduction; chapter 7, Vulgar Fractions; chapter 8, Decimal Fractions; chapter 9, Roots and Powers; chapter 10, Continued Proportion; chapter 11, Disjunct Proportion; chapter 12, Practice; chapter 13, Rules relating to Trade and Commerce.

From the preface: "The Author's Design in the following Treatise is to give a very concise Account of such Rules, as are of the easiest practice in all the Parts of Vulgar and Decimal Arithmetick and to illustrate each with such examples, as may be sufficient to lead the Learner to the full Use thereof in all other Instances."

"The Reader will observe that the Author has inserted under all those Rules, where it was proper, Examples with Blanks for his Practice. This was a Principal End to the Undertaking; that such persons as were desirous thereof might have a comprehensive Collection of all the best Rules in the Art of Numbering, with Examples wrought by themselves. And that nothing might be wanting to favour this Design, the Impression is made upon several of the best sorts of Paper. This method is entirely new, * * *."

The paper used in the book is thick, the type large. Words and phrases to which the author desires to call special attention are printed in italic characters, and as more than half the book is, in the author's eyes, important, more than half the book is printed in italics.

In 1788, when Nicholas Pike published his arithmetic, Greenwood's book was entirely unknown, and Pike's was believed to be the first arithmetic written and printed in America.

The first arithmetic which enjoyed general popularity and reached an extended circulation in the colonies was the School-master's Assistant, by Thomas Dilworth. The first edition of this was published in London in 1744 or '45. According to Wickersham, there appeared a reprint of this in Philadelphia in 1769. Other American editions were brought out at Hartford in 1786, New York in 1793 and 1806, Brooklyn in 1807,

* Appleton's Dictionary of American Biography.

New London 1797, and Albany 1824. At the beginning of the Revolution this was the most popular arithmetic, and it continued in use long after.

We have now enumerated all the arithmetics which were used to our knowledge in the American colonies. It may be instructive to give the last book which we have mentioned a closer examination; for Dilworth's School-master's Assistant was the most noted arithmetic of its time. As an arithmetician Dilworth belonged to the school founded by Cocker, which scrupulously excluded all demonstration and reasoning. The School-master's Assistant gives all rules and definitions in the form of questions and answers. Let us turn to page 44 of the twenty-second London edition, 1784, and examine his mode of explaining proportion, or, as the subject was then called, the "Rule of Three."

OF THE SINGLE RULE OF THREE.

Q. How many Parts are there in the Rule of Three?
A. Two: Single or Simple, and Double or Compound.
Q. By what is the single Rule of Three known?
A. By three Terms, which are always given in the Question, to find a fourth.
Q. Are any of the terms given to be reduced from one Denomination to another?
A. If any of the given terms be of several denominations, they must be reduced into the lowest Denomination mentioned.
Q. What do you observe concerning the first and third Terms?
A. They must be of the same Name and Kind.
Q. What do you observe concerning the fourth Term?
A. It must be of the same Name and Kind as the second.
Q. What do you observe of the three given Terms taken together?
A. That the two first are a Supposition, the last is a Demand.
Q. How is the third Term known?
A. It is known by these, or the like Words, *What cost? How many? How much?*
Q. How many Sorts of Proportion are there?
A. Two: Direct and Inverse.

And so on. We have quoted enough to give an idea of the book. It is not easy to see how a pupil beginning the subject of proportion could get clear notions from reading the above. Nor can we see how a boy who had never before heard of fractions could get any idea whatever of a fraction from Dilworth's definition, which is (p. 111): A fraction "is a broken number and signifies the part or parts of a whole number."

A closer examination of this arithmetic discloses many other strange things. It consists really of three parts, more or less complete in themselves, namely: Part I, on whole numbers; Part II, on vulgar fractions; Part III, on decimal fractions. In Part I, the student is carried through the elementary rules, and through interest, fellowship, exchange, double rule of three, alligation, single and double position, geometrical progression, and permutations. He is carried through all these without having as yet even heard of fractions. The advanced and comparatively unimportant subjects, such as alligation and progressions, are made

to precede so important and fundamental a subject as fractions. The teaching of decimal fractions *after* interest is illogical, to say the least. In Part II, after fractions have been explained, the rule of three is taken up a second time; and in Part III, under decimal fractions, it is resumed a third time. Thus this rule is explained three times; the first time with whole numbers, the second time with common fractions, the third time with decimal fractions—thus leaving the impression that the rule is different in each one of the three cases.

The whole book is nothing but a Pandora's box of disconnected rules. It appeals to memory exclusively and completely ignores the existence of reasoning powers in the mind of the learner. Noticeable is the fact that in the treatment of common fractions, the process of "cancellation," which may be made to shorten operations so much, is not even mentioned. The book abounds in unnecessary and perplexing technical terms, such as "practice," "conjoined proportion," "alligation medial," "alligation alternate," "comparative arithmetic," "biquadrate roots," "sursolids," "square cubes," "second sursolids," "biquadrates squared," "third sursolids," and "square cubes squared." Under the head of duodecimals are given rules like these: "Feet multiplied by feet give feet;" "feet multiplied by inches give inches," etc. These rules, taken literally, are absurd. We can no more multiply feet by feet than we can multiply umbrellas by umbrellas. These rules are in opposition to the fundamental ideas of multiplication in arithmetic. A concrete number can not be multiplied by a concrete number. It seems strange that so gross an error should not have been corrected in later editions of the book; but still more strange is the fact that nearly all arithmetics down to the present day should have persisted in making this mistake.

As an instance of the confusion of ideas to which it gives rise, I quote the following from an article "Early School Days" in Indiana, contributed by Barnabas C. Hobbs.* A law had just been passed requiring that teachers' examinations should be conducted by three county examiners instead of the township trustees, as had been the practice before. "I shall not forget," says Hobbs, "my first experience under the new system. The only question asked me at my first examination was, 'What is the product of 25 cents by 25 cents?' * * * We were not as exact then as people are now. We had only Pike's Arithmetic, which gave the sums and the rules. These were considered enough at that day. How could I tell the product of 25 cents by 25 cents, when such a problem could not be found in the book? The examiner thought it was $6\frac{1}{4}$ cents, but was not sure. I thought just as he did, but this looked too small to both of us. We discussed its merits for an hour or more, when he decided that he was sure I was qualified to teach school, and a first-class certificate was given me."

* The Indiana Schools, by James H. Smart, 1876.

We have spoken of Dilworth's School-master's Assistant at some length, because from it we can see what sort of arithmetics we inherited from the English. All arithmetics of that time were much alike. The criticisms upon one will therefore apply to all.

Before proceeding to another subject we shall examine briefly the "Short Collection of Pleasant and Diverting Questions" in Dilworth. We shall meet there with a company of familiar friends. Who has not heard of the farmer, who, having a fox, a goose, and a peck of corn, and wishing to cross a river, but being able to carry but one at a time, was confounded as to how he should carry them across so that the fox should not devour the goose, nor the goose the corn? Who has not heard of the perplexing problem of how three jealous husbands with their wives may cross a river in a boat holding only two, so that none of the three wives shall be found in company of one or two men, unless her husband be present? Many of us, no doubt, have also been asked to place the nine digits in a quadrangular form in such a way that any three figures in a line may make just 15? When these pleasing problems were first proposed to us, they came like the morning breeze, with exhilarating freshness. We little suspected that these apparently new-born creatures of fancy were in reality of considerable antiquity; that they were found in an arithmetic used in this country one hundred years ago. Still greater is our surprise when we learn that at the time they were published in Dilworth's School-master's Assistant some of these questions for amusement had already seen as many as one thousand birthdays. The oldest record bearing upon this subject is found in a manuscript entitled *Propositiones ad acuendos juvenes*. The authorship of this paper has been generally attributed to Alcuin, whose years of greatest activity were spent in France, in the court of the great Charlemagne, and who was one of the most learned scholars and celebrated teachers of the eighth century. The MS. attributed to him contains the puzzle about the wolf, goat, and cabbage, which in the modern version is known as the "fox, goose, and peck of corn" puzzle.

In a MS. coming from the thirteenth century, two learned German youths, named Firri and Tyrri, are made to propose to each other problems and puzzles. Firri takes among others the hard nut of Alcuin about the wolf, goat, and cabbage-head, and lays it before Tyrri in the modified and improved version of the three wives and the three jealous husbands. This same document contains also the following: "Firri says: There were three brothers in Cologne, having nine vessels of wine. The first vessel contained 1 quart (amam), the second 2, the third 3, the fourth 4, the fifth 5, the sixth 6, the seventh 7, the eighth 8, the ninth 9. Divide the wine equally among the three brothers, without mixing the contents of the vessels."

This problem admits of more than one solution, and is closely related to the last problem we quoted from Dilworth's collection. It is of spe-

cial interest, since it gives rise to the following magic square, in which any three figures in a straight line have 15 for their sum.

2	7	6
9	5	1
4	3	8

The history of magic squares is a rich field for investigation. The Germans were by no means the originators of them. This honor must be given to the Brahmins in India. Later on the study of these curious problems was zealously pursued by the Arabs, who transmitted the fruits of their study to the Europeans.

Had we the time, we would attempt to trace the history of some other familiar puzzles. But enough has been said to show that many of them possess great antiquity. Nevertheless, when they were first proposed to us, they betrayed no signs of old age. May they continue perpetually in their youth, and may they delight the minds of men for numberless centuries to come!

COLLEGES.

HARVARD COLLEGE.

As early as 1636 the people of Massachusetts stamped their approval upon the cause of higher education by the founding of Harvard College. The nature of the early instruction given at this oldest of American colleges is of special interest to us. The earliest record bearing on the history of the rise of mathematical studies at Harvard is a tract entitled, "New England's First Fruits." It was originally published in 1643, or five years after the college had opened, and contained the curriculum of studies then pursued. Whoever expects to find in it an extended course of mathematical studies resembling that in our colleges of to-day will be much disappointed.

In the first place, a student applying for admission to Harvard in 1643 was not confronted and embarrassed by any entrance examinations in mathematics. The main requirement for admission was Latin. Contrary to the practice of to-day, Latin was then taught as a spoken language. "So much Latin as was sufficient to understand Tully, or any like classical author, and to make and speak true Latin in prose or verse, and so much Greek as was included in declining perfectly the paradigms of the Greek nouns and verbs," were the necessary requisites for admission; but in mathematics applicants were *required* to know not even the multiplication table.

When we come to examine the college course, which extended originally through only three years, we meet with other surprises. Boys did

not receive that thorough "grinding" in the elements during the first years of college that they do now; on the contrary, no mathematics at all was taught except during the last year. The mathematical course began in the Senior year, and consisted of arithmetic and geometry during the first three-quarters of the year, and astronomy during the last quarter. Algebra was then an unknown science in the New World. It is interesting to notice that, in this original curriculum, the attention of each class was concentrated for a whole day upon only one or two subjects. Thus, Mondays and Tuesdays were devoted by the third year students exclusively to mathematics or astronomy, Wednesdays to Greek, Thursdays to "Eastern tongues," and so on. The importance attached to mathematical studies, as compared with other branches of discipline, may be inferred from the fact that ten hours per week were devoted to philosophy, seven to Greek, six to Rhetoric, four to Oriental languages, but only two to mathematics. According to these figures, Oriental languages were considered twice as important as mathematics. But we must remember that this course was laid out for students who were supposed to choose the clerical profession. For that reason, philosophical, linguistic, and theological studies were allowed to monopolize nearly the whole time, while mathematics was excluded almost entirely.

In what precedes we have measured the college work done in 1643 by the standards of 1889. Let us now compare it with the contemporaneous work in English universities. We may here premise that in the middle of the seventeenth century rapid progress was made in the mathematical sciences. In 1643, Galileo had just passed away; Cavalieri, Torricelli, Pascal, Fermat, Roberval, and Descartes were at the zenith of their scientific activity; John Wallis was a young man of twenty-seven, Isaac Barrow a youth of thirteen, while Isaac Newton was an infant feeding from his mother's breast. Though much original work was being done, especially by French and Italian mathematicians, the enthusiasm for mathematical study had hardly reached the universities. Some idea of the state of mathematics at Cambridge, England, previous to the appearance of Newton, may be gathered from a discourse by Isaac Barrow, delivered in Latin, probably in 1654, or eighteen years after the founding of Harvard College. In it occurs the following passage: "The once horrid names of Euclid, Archimedes, Ptolemy, and Diophantus, many of us no longer hear with trembling ears. Why should I mention the fact that by the aid of arithmetic, we have now learned, with easy and instantaneous work, to compute accurately the number of the very sands (themselves). * * * And indeed that horrible monster that men call algebra many of us brave men (that we are) have overcome, put to flight, and (fairly) triumphed over; (while) very many (of us) have dared, with straight-along glance, to look into optics; and others (still), with intellectual rays unbroken, have dared to pierce (their way) into the still subtler and highly useful doctrine of dioptrics."

From this it would seem that mathematical studies had been introduced into old Cambridge only a short time before Barrow delivered his speech. It thus appears that about 1636, when new Cambridge was founded in the wilds of the west, old Cambridge was not mathematical at all. In further support of this view we quote from the Penny Cyclopædia, article "Wallis," the following statement: "There were no mathematical studies at that time [when Wallis entered Emmanuel College in 1632] at Cambridge, and none to give even so much as advice what books to read. The best mathematicians were in London, and the science was esteemed no better than mechanical. This account is confirmed by his [Wallis's] contemporary, Horrocks, who was also at Emmanuel and whose works Wallis afterwards edited." In a biography of Seth Ward, an English divine and astronomer, we meet with similar testimony.* He entered Sidney Sussex College, Cambridge, in 1632. "In the college library he found, by chance, some books that treated of the mathematics, and they being wholly new to him, he inquired all the college over for a guide to instruct him that way, but all his search was in vain; these books were Greek, I mean unintelligible, to all the fellows in the college."

If so little was done at old Cambridge, then we need not wonder at the fact that new Cambridge failed to be mathematical from the start. The fountain could not rise higher than its source. It was not until the latter half of the seventeenth century that mathematical studies at old Cambridge rose into prominence. Impelled by the genius of Sir Isaac Newton, old Cambridge advanced with such rapid strides that the youthful college in the west became almost invisible in the distant rear.

The mathematical course at Harvard remained apparently the same till the beginning of the eighteenth century. Arithmetic and a little geometry and astronomy constituted the sum total of the college instruction in the exact sciences. Applicants for the master's degree had only to go over the same ground more thoroughly. Says Cotton Mather: "Every scholar that giveth up in writing a system or synopsis or sum of logic, natural and moral philosophy, arithmetic, geometry, and astronomy, and is ready to defend his theses or positions, withal skilled in the originals, as above said, and of godly life * * * is fit to be dignified with the second degree."†

These few unsatisfactory data are the only fragments of information that we could find on the mathematical course at Harvard during the seventeenth century. The following note on the nature of the instruction given in physics is not without interest: Mr. Abraham Pierson, jr. (first rector of Yale College), graduated at Harvard in 1668. The college (Yale) possesses several of his MSS., "containing notes made by

* Life of Right Reverend Seth, Lord Bishop of Salisbury, by Walter Pope. London, 1697, p. 9.

† Magnalia, Book IV, 128th ed., 1702.

him during his student life at Harvard on logic, theology, and physics, and so throwing light on the probable compass of the manuscript text-book on physics compiled by him, which was handed down from one college generation to another for some twenty-five years, until superseded by Clarke's Latin translation of Rohault's *Traité de Physique*. The Harvard notes on physics seem (from an inscription attached) to to have been derived in like manner from the teachings of the Rev. Jonathan Mitchel (Harvard College, 1647); they are rather metaphysical than mathematical in form, and it is even difficult to determine what theories of physical astronomy the writer held. Suffice it to say that he ranged himself somewhere in the wide interval between the Ptolemaic theory (generally abandoned one hundred years earlier) and the Newtonian theory (hardly known to any one in this part of the world until the eighteenth century). In other words, while recognizing that the earth is round, and that there is such a force as gravity, there is no proof that he had got beyond Copernicus to Kepler and Galileo." *

In this extract our attention is also called to the common practice among successive generations of students at that time of copying manuscript text-books. As another instance of this we mention the manuscript works, a System of Logic and a Compendium Physical, by Rev. Charles Morton, which (about 1692) were received as text-books at Harvard, "the students being required to copy them."†

We shall frequently have occasion to observe that astronomical pursuits have always been followed with zeal and held in high estimation by the American people. As early as 1651 a New England writer, in naming the "first fruits of the college," speaks of the "godly Mr. Sam Danforth, who hath not only studied divinity, but also astronomy; he put forth many almanacs," and "was one of the fellows of the college." Another fellow of Harvard was John Sherman. He was a popular preacher, an "eminent mathematician," and delivered lectures at the college for many years. He published several almanacs, to which he appended pious reflections. The ability of making almanacs was then considered proof of profound erudition. A somewhat stronger evidence of the interest taken in astronomy was the publication at Cambridge of a set of astronomical calculations by Uriah Oakes. Oakes, at that time a young man, had graduated at Harvard in 1649, and in 1680 became president *pro tem.* and afterwards president of Harvard College. In allusion to his size, he attached to his calculations the motto, "*Parvum parva decent, sed inest sua gratia parvis.*" (Small things beût the small, yet have a charm their own.)

The preceding is an account of the mathematical and physical studies at Harvard during the seventeenth century. We now proceed to the eighteenth century. It appears that in 1700 algebra had not yet be-

* Yale Biographies and Annals, 1701–1745, by Franklin Bowditch Dexter, p. 61.
† Quincy's History of Harvard University, Vol. 1, p. 70.

come a college study. The Autobiography of Rev. John Barnard* throws some light on this subject. Barnard took his first degree at Harvard in 1700, then returned to his father's house, where he betook himself to studying. "While I continued at my father's I prosecuted my studies and looked something into the mathematics, though I gained but little, our advantages therefor being noways equal to what they have who now have the great Sir Isaac Newton and Dr. Halley and some other mathematicians for their guides. About this time I made a visit to the college, as I generally did once or twice a year, where I remember the conversation turning upon the mathematics, one of the company, who was a considerable proficient in them, observing my ignorance, said to me he would give me a question, which if I answered in a month's close application he should account me an apt scholar. He gave me the question. I, who was ashamed of the reproach cast upon me, set myself hard to work, and in a fortnight's time returned him a solution of the question, both by trigonometry and geometry, with a canon by which to resolve all questions of the like nature. When I showed it to him he was surprised, said it was right, and owned he knew no other way of resolving it but by algebra, which I was an utter stranger to." Though a graduate of Harvard, he was an utter stranger to algebra. From this we may safely conclude that in 1700 algebra was not yet a part of the college curriculum.

What, then, constituted the mathematical instruction at that time? Was it any different from the course given in 1643? Until about 1655, the entire college course extended through only three years; at this time it was lengthened to four years. We might have supposed that the mathematics formerly taught in the third year would have been retained as a study for the third or Junior year, but this was not the case. In the four-years' course, mathematics was taught during the last, or Senior year. Quincy, in his history of Harvard University (Vol. I, p. 441), quotes from Wadworth's Diary the list of studies for the year 1726. The Freshmen recited in Tully, Virgil, Greek testament, rhetoric, Greek catechism; the Sophomores in logic, natural philosophy, classic authors, Heerebord's Meletemata, Wollebins's Divinity; the "Junior sophisters" in Heerebord's Meletemata, physics, ethics, geography, metaphysics; while the "Senior sophisters, besides arithmetic, recite Alsted's Geometry, Gassendi's Astronomy in the morning; go over the arts towards the latter end of the year, Ames's Medulla on Saturdays, and dispute once a week." This quotation establishes the fact that ninety years after the founding of Harvard, the mathematical course was essentially the same as at the beginning. Arithmetic, geometry, and astronomy still constituted the entire course. Mathematics continued to be considered the crowning pinnacle instead of a corner-stone of college education; natural philosophy and physics were

* Collections of the Mass. Hist. Soc., Third Series, Vol. V, pp. 177–243.

still taught before arithmetic and geometry. But we must observe that, in 1726, *printed* treatises were used as text-books in geometry and astronomy. We are not informed at what time these printed books were introduced. They may have been used as text-books much earlier than the above date. The authors of these books were in their day scholars of wide reputation. Johann Heinrich Alsted (1558–1638), the author of the Geometry, was a German Protestant divine, a professor of philosophy and divinity at Herborn in Nassau, and afterwards in Carlsburg in Transylvania. In one of his books he maintained that the millenium was to come in 1694.

Pierre Gassendi (1592–1655), whose little astronomy of one hundred and fifty pages was used as a class-book at Harvard, was a contemporary of Descartes and one of the most distinguished naturalists, mathematicians, and philosophers of France. He was for a time professor of mathematics at the Collège Royal of Paris. What seems very strange to us is that nearly a century after the first publication of these books they should have been still in use and apparently looked upon as the best of their kind. Forty years after the publication of Newton's Principia an astronomy was being studied at Harvard whose author died before the name of Newton had become known to science. The wide chasm between the theories of Newton and those of Gassendi is brought to full view by the following quotation from Whewell's History of the Inductive Sciences (Third edition, Vol. I, p. 392): "Gassendi's own views of the causes of the motions of the heavenly bodies are not very clear. * * * In a chapter headed 'Quæ sit motrix siderum causa,' he reviews several opinions; but the one which he seems to adopt is that which ascribes the motion of the celestial globes to certain fibers, of which the action is similar to that of the muscles of animals. It does not appear therefore that he had distinctly apprehended, either the continuation of the movements of the planets by the first law of motion, or their deflection by the second law."

The year 1726 is memorable in the annals of Harvard for the establishing of the Hollis professorship of mathematics. Thomas Hollis, a kind-hearted friend of the college, transmitted to the treasurer of the college the then munificent sum of twelve hundred pounds sterling, and directed that the funds should be applied to "the instituting and settling a professor of mathematics and experimental philosophy in Harvard College." To the same benefactor Harvard was indebted for the establishment of the professorship of divinity. Down to the commencement of the nineteenth century only one additional professor was appointed in the undergraduate department, namely, the Hancock professor of Hebrew, in 1765. Hence, it follows that almost all regular instruction was given by tutors. Previous to the establishment of the Hollis professorship the mathematical instruction was entirely in the hands of tutors. Since almost any minister was considered competent to teach mathematics, and since tutors held their place sometimes for

only one year, we may imagine that the teaching was not of a very high order.

The first appointment to the Hollis professorship of mathematics and natural philosophy was that of Isaac Greenwood. He was the first to occupy a collegiate chair of mathematics in New England, but not the first in America, as is sometimes stated. This honor belongs to a professor at William and Mary College. Greenwood graduated at Harvard in 1721, then engaged in the study of divinity, visited England, and began to preach in London with some approbation.*. He also attended lectures delivered in that metropolis on experimental philosophy and mathematics. In 1727 he entered upon his duties at Harvard. "In scientific attainments Greenwood seems to have been well qualified for his professorship." He made astronomical contributions to the Philosophical Transactions of 1728, and published in 1729 an arithmetic. That seems to have been the earliest arithmetic from the pen of an American author. This is all we know of Greenwood as a mathematician and teacher. Unfortunately he did not prove himself worthy of his place. We regret to say that the earliest professor of mathematics in the oldest American college was "guilty of many acts of gross intemperance, to the dishonor of God and the great hurt and reproach of the society." His intemperance brought about his removal from his chair in 1738.

On the dismissal of Greenwood, Nathaniel Prince, who had been tutor for thirteen years, aspired to the professorship. He was, says Elliot, superior "to any man in New England in mathematics and natural philosophy." But his habits being notoriously irregular, John Winthrop of Boston, was appointed in his stead. Winthrop graduated at Harvard in 1732, and was only twenty-six years old when he was chosen professor of mathematics and natural philosophy. He filled this chair for over forty years (until 1779) with marked ability. In mathematical science he came to be regarded by many the first in America.

If we could turn the wheel of time backward through one hundred and twenty revolutions, and then enter the lecture-room of Professor Winthrop and listen to his instruction, what a chapter in the history of mathematical teaching would be uncovered! But as it is, this history is hidden from us. We know only that during the early part of his career as professor, "and probably many years before," the text-books were the following: Ward's Mathematics, Gravesande's Philosophy, and Euclid's Geometry; besides this, lectures were delivered by the professors of divinity and mathematics.†

From this we see that some time between the years 1726 and 1738, Ward's Mathematics had been introduced, and Alsted's old Geometry had given place to the still older but ever standard work of Euclid. This is the first mention of Euclid as a text-book at Harvard. The in-

* Quincy's History of Harvard University, Vol. II, p. 14.

† Peirce's History of Harvard, p. 237.

troduction of Gravesande's Philosophy is another indication of progress. Gravesande was for a time professor of mathematics and astronomy at the University of Leyden. He was the first who on the continent of Europe publicly taught the philosophy of Newton, and he thus contributed to bring about a revolution in the physical sciences. By the adoption of his philosophy as a text-book at Harvard we see that the teachings of Newton had at last secured a firm footing there. Ward's Mathematics continued for a long time to be a favorite text-book.*

It is probable that with the introduction of Ward's Mathematics, algebra began to be studied at Harvard. The second part of the Young Mathematician's Guide consists of a rudimentary treatise on this subject. It is possible, then, that the teaching of algebra at Cambridge may have begun some time between 1726 and 1738. But I have found no direct evidence to show that algebra actually was in the college curriculum previous to 1786.

Since Ward's Mathematics were used, to our knowledge, not only at Harvard, but also at Yale, Brown, and Dartmouth, and as a book of reference at the University of Pennsylvania, a description of the Young Mathematician's Guide may not be out of place.†

The first part treats of arithmetic (143 pages). Though very deficient according to modern notions, the presentation of this subject is superior to that in Dilworth's School-master's Assistant. It is less obscure.

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* According to ex-President D. Woolsey, the author of this book was the Ward who had been "president of Trinity College, Cambridge, and bishop of Exeter." (Yale College; A Sketch of its History, William L. Kingsley, Vol. II, p. 499.) Now, the only individual answering to this description is Seth Ward, the astronomer, whose time of activity preceded the epoch of Newton. We shall show that the book in question was not written by Seth Ward, but by John Ward, who flourished half a century later than Seth Ward and whose Young Mathematician's Guide was for a long time a popular elementary text-book in England. Wherever we have seen Ward's book mentioned in the curricula of American colleges it was always called "Ward's Mathematics." The baptismal name of the author was never given. This shows that there was only one Ward (either Seth or John) whose mathematical books were known and used in our colleges. Now, Benjamin West, professor of mathematics in Brown University from 1786 to 1799, published in the first volume of the American Academy of Arts and Sciences a paper "On the extraction of roots," in which he offers improvements on "Ward's" method. Now, I have seen a copy of Seth Ward's Astronomia Geometrica, but have found nothing in it on root extraction. One would hardly expect to find anything on it in Seth's "Trigonometry" or "Proportion." John Ward, on the other hand, treats of roots in his "Guide," and gives a "general method of extracting roots of all single powers." West takes two examples (two numbers, one of 14, the other of 18 digits) from "Ward," and shows how the required roots can be extracted by his method. But both these examples are given in John Ward's Young Mathematician's Guide. This evidence in favor of John Ward's book may be considered conclusive. Further information on "Ward's Mathematics" will be found in an article by the writer in the Papers of the Colorado College Scientific Society, Vol. I.

† The copy which the writer has before him (Twelfth edition, London, 1771), was kindly lent him by Dr. Artemas Martin, of the U. S. Coast Survey, who has for years been making a collection of old and rare books on mathematics.

Like all books of that time, it contains rules, but no reasoning. What seems strange to us is the fact that subjects of no value to the beginner, such as arithmetical and geometrical proportion (*i. e.*, progression), alligation, square root, cube root, biquadrate root, sursolid root, etc., are given almost as much space and attention as common and decimal fractions.

The second part (140 pages) is devoted to algebra. Ward had published a small book on algebra in 1698, but that, he says, was only "a compendium of that which is here fully handled at large." Like Harriot, he speaks of his algebra as "Arithmetick in species." This name is appropriate, inasmuch as he does not (at least at the beginning) recognize the existence of negative quantities, but speaks of the *minus sign* always as meaning only subtraction, as in arithmetic. A little further on, however, he brings in, by stealth, "affirmative" and "negative" quantities. The knowledge of algebra to be gotten from this book is exceedingly meagre. Factoring is not touched upon. The rule of signs in multiplication is proved, but further on all rules are given without proof. The author develops a rule showing how binomials can be raised "to what height you please without the trouble of continued involution." He then says: "I proposed this method of raising powers in my Compendium of Algebra, p. 57, as wholly new (viz, as much of it as was then useful), having then (I profess) neither seen the way of doing it, nor so much as heard of its being done. But since the writing of that tract, I find in Doctor Wallis's History of Algebra, pp. 319 and 331, that the learned Sir Isaac Newton had discovered it long before." The subject of "interest" is taught in the book algebraically, by the use of equations.

Part III (78 pages) treats of geometry. In point of precision and scientific rigor, this is quite inferior. After the definitions follow twenty problems, intended for the excellent purpose of exercising the "young practitioner," and bringing "his hand to the right management of a ruler and compass, wherein I would advise him to be very ready and exact." Then follows a collection of twenty-four "most useful theorems in plane geometry demonstrated." This part is semi-empirical and semi-demonstrative. A few theorems are assumed and the rest proved by means of these. The theorem, "If a right line cut two parallel lines, it will make the opposite (*i. e.*, alternate interior) angles equal to each other," is proved by aid of the theorem, that "If two lines intersect each other, the opposite angles will be equal." The proof is based on the idea that "parallel lines are, as it were, but one broad line," and that by moving one parallel toward the other, the figure for the former theorem reduces to that of the latter. The next chapter contains the algebraical solution of twenty geometrical problems.

Part IV, on conic sections (36 pages), gives a semi-empirical treatment of the subject. Starting with the definition of a cone, it shows how the three sections are obtained from it, and then gives some of their principal properties.

Part V (36 pages) is on the arithmetic of infinites. Judging from this part of the book, its author knew nothing of fluxions. The first edition appeared in 1707, after Newton had published the first edition of his Principia, in 1687, but his Method of Fluxions was not published till 1736, though written in 1671. Ward employs the method of integration by series of Cavalieri, Roberval, and John Wallis, and, thereby, finds the superficial and solid contents of solid figures. It does not appear that this part of the book was ever studied in American colleges.

Ward's book met with favor in England. In the preface to the twelfth edition he says: "I believe I may truly say (without vanity) this treatise hath proved a very helpful guide to near five thousand persons, * * * and not only so, but it hath been very well received amongst the learned, and (I have been often told) so well approved on at the universities, in England, Scotland, and Ireland, that it is ordered to be publicly read to their pupils."

In former times all professors of mathematics in American colleges gave instruction, not merely in pure mathematics, but also in natural philosophy and astronomy; and it appears that as a general rule these professors took more real interest and made more frequent attempts at original research in the fields of astronomy and natural philosophy than in pure mathematics. The main reason for this lies probably in the fact that the study of pure mathematics met with no appreciation and encouragement. Original work in abstract mathematics would have been looked upon as useless speculations of idle dreamers. The scientific activity of John Winthrop was directed principally to astronomy. His reputation abroad as a scientist was due to his work in that line. In 1740 he made observations on the transit of Mercury, which were printed in the Transactions of the Royal Society. In 1761 there was a transit of Venus over the sun's disk, and as Newfoundland was the most western part of the earth where the end of the transit could be observed, the "province" sloop was fitted out *at the public expense* to convey Winthrop and party to the place of observation.* He took with him two pupils who had made progress in mathematical studies. One of these, Samuel Williams, became later his successor at Harvard. In 1769 Winthrop had another chance for observing the transit of Venus, at Cambridge. "As it was the last opportunity that generation could be favored with, he was desirous to arrest the attention of the people. He read two lectures upon the subject in the college chapel, which the students requested him to publish. The professor put this motto upon the title page: *Agite, mortales! et oculos in spectaculum vertite, quod hucusque spectaverunt perpaucissimi; spectaturi iterum sunt nulli.*" (Come, mortals! and turn your eyes upon a sight which, to this day, but few have seen, and which not one of us will ever see again.) The transit of 1769 was also observed in Philadelphia by David Rittenhouse, and in Providence by Benjamin West. These observations

* "John Winthrop," in the Biographical Dictionary by John Eliot, 1809.

were an important aid in determining the sun's parallax. Most gratifying to us is the interest in astronomical pursuits manifested in those early times. Expeditions fitted out at public expense, and private munificence in the purchase of suitable instruments, bear honorable testimony to the enlightened zeal which animated the friends of science.

In 1767 John Winthrop wrote his *Cogita de Cometis*, which he dedicated to the Royal Society, of which he had been elected a member. This was reprinted in London the next year, and gave him an extensive literary reputation.

In 1764 a calamity befell Harvard College. The library and philosophical apparatus—the collections of over a century—were destroyed by fire. Among the books recorded as having been lost are the following: "The Transactions of the Royal Society, Academy of Sciences in France, Acta Eruditorum, Miscellanea Curiosa, the works of Boyle and Newton, with a great variety of other mathematical and philosophical treatises."* It is seen from this that, before the fire, books of reference in higher mathematics had not been entirely wanting.

John Winthrop died in 1779, and the robe of the departing prophet fell upon his former disciple, the Rev. Samuel Williams. Williams filled the mathematical chair for eight years. Having inherited from his master a love of astronomy, he frequently published observations and notices of extraordinary natural phenomena in the memoirs of the American Academy of Arts and Sciences. He occupied the mathematical chair at Harvard until 1788. Then he lectured at the University of Vermont on astronomy and natural philosophy for two years, and was subsequently minister at Rutland and Burlington, Vermont.

YALE COLLEGE.

Yale, the second oldest New England college, was founded in 1701, or sixty-three years after the opening of Harvard. During the first fifteen years it maintained a sort of nomadic existence. Previous to 1816 instruction seems to have been given partly at Saybrook and partly at Killingworth and Milford. Its course of instruction was then very limited. The mathematical teaching during the first years of its existence was even more scanty than in the early years at Harvard. Benjamin Lord, a Yale graduate of 1714, wrote in 1779 as follows in reply to inquiries by President Stiles: "As for mathematics, we recited and studied but little more than the rudiments of it, some of the plainest things in it. Our advantages in that way were too low for any to rise high in any branch of literature."† Doctor Johnson, of the same class, says: "Common Arithmetick and a little surveying were the *ne plus ultra* of mathematical acquirements." It appears from this that *surveying* was taken at Yale, instead of the *geometry* which formed part

* *Vide* Quincy's History of Harvard University, Vol. II, p. 481.

† Yale Biographies and Annals, 1701–45, by Franklin Bowditch Dexter, pp. 115 and 116.

of the course at Harvard. In a new and only partially settled country some knowledge of surveying was a great desideratum. But the study of surveying without a preliminary course in geometry and trigonometry is truly characteristic of the purely practical tendencies of the times. Men took eager interest in the applications of science, but cared nothing for science itself. The little mathematics studied was evidently not pursued for its own sake, nor for the mental discipline which it afforded, but simply for the pecuniary profit which it would afterwards bring.

As at Harvard, so at Yale, the mathematics were studied, at that time, during the last year of the college course and after the study of physics had been completed.* During the next six or seven years, the course at Yale was extended somewhat. In 1720 it was identical with the Harvard course of 1726. In 1719, when Jonathan Edwards was a member of the Junior class at New Haven, he wrote as follows to his father: "I have enquired of Mr. Cutter, what books we shall have need of the next year. He answered he would have me get against that time, Alsteds' Geometry and Gassendi's Astronomy."†

At this time progress was also made in the teaching of physics. The earliest guide in this study were the manuscript lectures by Rector Pierson, which were a repetition of lectures he had heard while a student at Harvard College. They were metaphysical rather than mathematical, "recognizing the Copernican theory, but knowing nothing of Kepler and Galileo, and much less of Newton."‡

During the first seventeen years at Yale the doctrines of the schoolmen in logic, metaphysics, and ethics still held sway. Descartes, Boyle, Locke, Bacon, and Newton were regarded as innovators from whom no good could be expected. It is pleasing to think that the introduction of Newtonian ideas and the rise of mathematical studies at Yale was partly due to an act of charity by the great Sir Isaac Newton himself. In the year 1715 a collection of books made in England by Mr. Drummer, the agent of the colony, amounting to about eight hundred volumes, was sent over to the college. The collection consisted of donations by well-spirited gentlemen in Britain. "Sir Isaac Newton gives the second edition of his Principia (which appeared in 1713)"; "Doctor Halley sends his edition of Apollonius."§ But these and many other donations would have been barren of results had there not been young men of talent and energy to master the contents of these precious volumes. Such a man was Samuel Johnson. He graduated in 1714 and was appointed tutor a few years later. Drummer's collection furnished him with a "feast of fat things." To use his own words: "He seemed to himself like a person suddenly emerging out of the glimmer of twilight into the full sunshine of open day." He and

* Yale College; a sketch of its history, William L. Kingsley, Vol. II, p. 496.
† Edwards' Works, Vol. I, p. 30.
‡ Ex-President D. Woolsey, in Yale Book, Vol. II, p. 499.
§ Yale Biographies and Annals, 1701-45, by Franklin Bowditch Dexter, p. 141.

Mr. Brown, another young tutor, exerted themselves to the utmost for the improvement of the students under their charge. Imbued with the grand ideas of Newton, they extended the mathematical course for the understanding of the Newtonian system, and then taught this system in place of the older. There was at that time much contention as to the place where the college should be permanently located. This was a fortunate circumstance for the young tutors, since these troubles without withdrew public attention from the innovations within.* In 1722 Johnson and Brown resigned their tutorships and sailed for England to receive ordination from an English bishop. Johnson became later president of King's (now Columbia) College in New York.

Soon after this the Physics of Rohault was introduced at Yale as a text-book. Rohault (1620–75) was a French philosopher and an implicit follower of the Cartesian theory. The edition used was that by the celebrated Samuel Clarke, who had taken the rugged Latin version of the treatise of Rohault (then used as a text-book at the University of Cambridge, England), and published it in better Latin, together with numerous critical notes, which he had added with a view of bringing the Cartesian system into disrepute by exposing its fallacies. This disguised Newtonian treatise maintained its place at Yale until 1743, when it was superseded by the work of Gravesande.

During President Clap's time, Martin's Philosophy, in three volumes, was the text-book in this science; when this work came to be out of print, President Stiles procured Enfield's Philosophy, which was the first introduction into American colleges of that now obsolete work.

It is worthy of remark that, in 1749, Benjamin Franklin presented to the college an electric machine, and that, a few years later, Ezra Stiles, then tutor at the college, began to make experiments with it. These are supposed to have been the earliest of the kind made in New England.

It appears that in 1733, Euclid was being used as a text-book in geometry. The earliest mention of Euclid at Harvard is in 1737. In 1733, Dr. John Hubbard of New Haven, who had received the honorary degree of master of arts three years previously, testified his gratitude by writing a panegyric, "The Benefactors of Yale College."

He introduced a recent gift of mathematical books by Joseph Thompson, of London, with the following stanza:

> "The Mathematicks too our tho'ts employ,
> Which nobly elevate the Student's joy:
> The little Euclids round the table set
> And at their rigid demonstrations sweat." †

This same Joseph Thompson donated to the college also "a complete set of surveying instruments, valued at £21." "A reflecting telescope, a microscope, a barometer, and other mathematical instruments—valued at £37, were bought by a subscription from the trustees and others."‡

* Barnard's Journal, Vol. XXVII, 1877; Article: "Samuel Johnson."
† Yale Biographies and Annals, 1701–45, by F. B. Dexter, p. 473.
‡ *Ibid.*, p. 521.

In 1742, elementary mathematics came to be removed from its august position in the curriculum as a senior study, and to be assigned an humbler but more befitting place nearer the beginning of the course. In 1742 the rector of the college advised the students to pursue a regular course of academic studies in the following order: "In the first year to study principally the tongues, arithmetic, and algebra; the second, logic, rhetoric, and geometry; the third, mathematics, and natural philosophy; and the fourth, ethics and divinity."*

That these changes were not made earlier than 1742 is evident from a passage in the memoir of Samuel Hopkins, who graduated in 1741, stating that then "metaphysics and mathematics found their place in in the fourth year, being in their turn the subject of study and recitation for the first four days of every week."†

At what time this dethronement of elementary mathematics as a senior study took place at Harvard, we are not able to state. It will be noticed that, at Yale, mathematics and natural philosophy had at this period exchanged places, the former now preceding the latter. From the above it is also evident that algebra was studied at Yale in 1742. The earliest mention of algebra at Cambridge is in 1786, though it doubtless began to be taught there much earlier. What branch of mathematics constituted the study for the third or Junior year remains a matter of conjecture. The "mathematics" spoken of in the extract probably referred to trigonometry, possibly together with some other branches.

A strong impetus to the study of mathematics at Yale was given during President Clap's administration. Thomas Clap graduated at Harvard in 1722. Doctor Stiles, his successor in the presidency at Yale, says that Clap studied the higher branches of mathematics, and was one of the first philosophers America has produced, "that he was equalled by no man, except the most learned Professor Winthrop." In his history of Yale, written in 1766, the year of his resignation, President Clap gives the following account of the studies pursued by students at the college:

"In the first year they learn Hebrew, and principally pursue the study of the languages, and make a beginning in logic and some parts of the mathematics. In the second year they study the languages, but principally recite logic, rhetoric, oratory, geography, and natural philosophy; and some of them make good proficiency in trigonometry and algebra. In the third year they will pursue the study of natural philosophy and most branches of mathematics. Many of them well understand surveying, navigation, and the calculation of eclipses; and some of them are considerable proficients in *conic sections* and *fluxions*. In the fourth year they principally study and recite metaphysics, ethics, and divinity."‡

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*Yale Biographies and Annals, 1701–45, p. 724.

†New Englander, August, 1852, p. 452: Professor Park's Memoir of Hopkins.

‡ Yale College; a Sketch of its History, by Wm. L. Kingsley, Vol. II, pp. 497 and 498.

The mathematical course in the above curriculum is indeed one that Yale had reason to be proud of. It shows that not only algebra and geometry, but also trigonometry, and even conic sections and fluxions, were studied at Yale previous to the year 1766. This is the earliest distinct mention of conic sections and fluxions as college studies in America.

Mathematics seem to have come to occupy some of the time which was given at first to logic. President Clap does not enumerate the text-books employed, but his successor, Doctor Stiles, in his diary for November 9, 1779, mentions a list of books recited in the several classes at his accession to the presidency, in 1777. The mathematical books are, for the Freshman class, Ward's Arithmetic; Sophomore class, Hammond's Algebra, Ward's Geometry (Saturday), Ward's Mathematics; Junior class, Ward's Trigonometry, Atkinson and Wilson's Trigonometry.

On comparing this mathematical course with that given by President Clap eleven years previous we observe some changes. The study of conic sections and fluxions had been apparently discontinued. This waning of mathematical enthusiasm was probably due to the departure of President Clap, and also to the political disturbances and confusions of the times. It would seem that during Clap's administration not all the students took higher mathematics, but only those who were particularly fond of them. Clap says, "*Many of them* well understand surveying, navigation, and the calculation of eclipses; and *some of them* are considerable proficients in the conic sections and fluxions."

That optional studies were then pursued occasionally is evident from a statement by President Stiles that he began instructing a class in Hebrew and Oriental languages, which he "selected out of all other classes, as they voluntarily offered themselves." The extent to which each of these branches was studied may probably be correctly inferred from the contents of Ward's Young Mathematician's Guide. This consists of five parts: arithmetic, algebra, geometry, conic sections, and arithmetic of infinites. Students that were mathematically inclined went through the entire work it would seem, excepting the algebra, which was studied from Hammond's book.

The year 1770 is memorable for the creation of the chair of "mathematics and natural philosophy" at Yale. This was done apparently to fill the gap caused by the departure of President Clap, who was uncommonly skilled in those sciences. The first occupant of this chair was Nehemiah Strong, who kept it eleven years. He belonged to the class of 1755 at Yale, and was tutor there from 1757 to '60. Before entering upon the duties of his chair, he had been pastor. After his resignation of his chair, he entered upon the study and practice of law. He published an "Astronomy Improved" (New Haven, 1784). President T. Dwight speaks of him as "a man of vigorous understanding."

WILLIAM AND MARY COLLEGE.

William and Mary is next to Harvard the oldest of American colleges. From 1688, the year of its organization at Williamsburg, Va., until the inauguration of the University of Virginia, it was the leading educational institution in the South. Owing to the repeated destruction by fire of the college buildings and records, not even the succession of the professors has been preserved. The early courses were in all probability much the same as the contemporaneous courses at Harvard. According to Campbell, 5 professorships were provided for by the charter, namely, those of Greek and Latin, mathematics, moral philosophy, and two of divinity. In speaking of the early course of study, Howison says that it embraced also a "natural philosophy which was just beginning to believe that the earth revolved round the sun, rather than the sun round the earth."

The earliest mathematical professor at William and Mary whose name has come down to us, was Rev. Hugh Jones. The college had a *professorship* of mathematics from its very beginning, and at a date when mathematical teaching at Harvard was still in the hands only of tutors. The names of the predecessor or predecessors of Hugh Jones are not known. He is the earliest *professor* of mathematics in America whose name has been handed down to us. He was an Englishman of university education; came to Maryland in 1696; was for a time pastor of a church; and then was appointed to the chair of mathematics at William and Mary. He was a man of broad, scholarly attainments, and endeared himself to the student of history quite as much as to the mathematician, by writing his invaluable book on The Present State of Virginia (1724). Says Dr. Herbert B. Adams: "His monograph is acknowledged to be one of the best sources of information respecting Virginia in the early part of the eighteenth century." The following quotations from it (p. 44) may be of interest: "They (the Virginians) are more inclinable to read men by business and conversation than to dive into books, and are for the most part only desirous of learning what is absolutely necessary in the shortest and best method."

"Having this knowledge of their capacities and inclination from sufficient experience, I have composed on purpose some short treatises adapted with my best judgment to a course of education for the gentlemen of the plantations, consisting in a short *English Grammar*, an *Accidence of Christianity*, an *Accidence to the Mathematick* in all its parts and applications, *Algebra, Geometry, Surveying of Land*, and *Navigation*."

"These are the most useful branches of learning for *them*, and such as they willingly and readily master, if taught in a plain and short method, truly applicable to their *genius*; which I have endeavored to do, for the use of *them* and *all others* of their temper and parts."

We are not to understand by the above that his "Accidence to the

Mathematick" and the other books mentioned were actually printed; they existed only in manuscript copies. From the above it appears that about 1724 the mathematical course at William and Mary was quite equal to that in either of the two New England colleges. We must, of course, guard ourselves against the impression that full and exhaustive courses were given in algebra, geometry, surveying, and navigation. As is pointed out by the author himself, the merest rudiments only were imparted.

Reverend Jones was succeeded by Alexander Irvine, and he in turn by Joshua Fry. Fry was educated at Oxford, and, after coming to this country, was made master of the grammar school connected with William and Mary, and later, professor of mathematics in the college. In company with Peter Jefferson, the father of Thomas Jefferson, he made a map of Virginia. He also served on a commission appointed to determine the Virginia and North Carolina boundary line. He was succeeded in 1758 by William Small.

A few years before the outbreak of the Revolutionary War William and Mary College had among her students several who afterwards rose to prominence; she had four who became signers of the Declaration of Independence, and also the illustrious Thomas Jefferson, who became the author of this great document. At William and Mary, Jefferson was a passionate student of mathematics. The college long exercised the duties of the office of surveyor-general of the Colony of Virginia. Thomas Jefferson's father was a practical surveyor, who had been chosen in 1747 with Joshua Fry, then professor of mathematics at William and Mary, to continue the boundary line between Virginia and North Carolina.

When Thomas Jefferson, at the age of seventeen, entered the Junior class, he came into intimate relation with Dr. William Small, of Scotland, who was then the professor of mathematics. As an instructor he had the happy gift of making the road of knowledge both easy and profitable. In his Autobiography Jefferson says: "It was my great good fortune, and what probably fixed the destinies of my life, that Dr. William Small, of Scotland, was then professor of mathematics, a man profound in most of the useful branches of science, with a happy talent of communication, correct and gentlemanly manners, and an enlarged and liberal mind. He, most happily for me, became soon attached to me, and made me his daily companion when not engaged in the school; and from his conversation I got my first views of the expansion of science, and of the system of things in which we are placed."

In 1773 Thomas Jefferson was appointed surveyor of the county of Albemarle. But the college of Williamsburg left its stamp upon Jefferson, not merely as a qualified surveyor, but also as a statesman, philosopher, economist, and educator. We dwell with special interest upon his association at college with Dr. Small, because in later years, when filling the office of President of the United States, we shall marvel at the rich fruits his early association with a lover of exact science brought

forth. It was during Jefferson's administration that a systematic plan of conducting the Government surveys of the great North-West Territory was initiated; it was during his administration that the great work of the U. S. Coast Survey was first inaugurated. He took also great interest in the enlargement of the U. S. Military Academy. In these great movements the personal interest and enlightened zeal of Jefferson himself were the primary motive power. His biographers tell us that he was the first discoverer of the formula for constructing the mould-board of a plow on mathematical principles. He wrote to Jonathan Williams on this subject, July 3, 1796: "I have a little matter to communicate, and will do it ere long. It is the form of a mould board *of least resistance.* I had some years ago conceived the principles of it, and I explained them to Mr. Rittenhouse." We quote the following to show that even in his old age he still loved the favorite study of his youth. Said he in a letter to Col. William Duane, dated October, 1812, "When I was young, mathematics was the passion of my life. The same passion has returned upon me, but with unequal powers. Processes which I then read off with the facility of common discourse, now cost me labor and time, and slow investigation." Of interest are also certain passages in a course of legal study which he drew up for a young friend: "Before you enter on the study of law a sufficient groundwork must be laid. * * * Mathematics and natural philosophy are so useful in the most familiar occurrences of life and are so peculiarly engaging and delightful as would induce every one to wish an acquaintance with them. Besides this, the faculties of the mind, like the members of the body, are strengthened and improved by exercise. Mathematical reasoning and deductions are, therefore, a fine preparation for investigating the abstruse speculations of the law." Among the books in mathematics recommended by Jefferson to his young friend are, Bezout's *Cours de Mathématique*—the best for a student ever published; Montucla, or Bossut, *Histoire des Mathématiques;* Astronomy—Ferguson, and Le Monnier or De Lalande.

It should not be left unmentioned here that George Washington once applied to the College of William and Mary for an elective course in land surveying, and that he received his first commission as county surveyor from the faculty of the college. In this connection we can not refrain quoting a passage from the excellent monograph by Dr. Herbert B. Adams on the College of William and Mary.* "It is interesting," says he, "to trace the evolution of men as well as of institutions. It is generally known that Washington began his public life as a county surveyor, but, in all probability, few persons have ever thought of his service in that office as the historical and economic germ of his political greatness. Most people regard this early work as a passing incident in his career, and not as a determining cause, and yet it is possible to show that Washington's entire public life was but the natural out-

* Circular of Information of the Bureau of Education, No. 1, 1887, p. 30.

growth of that original appointment given him in 1749, at the age of seventeen, by the College of William and Mary. That appointment, in the colonial days of Virginia, was the equivalent of a degree in civil engineering, and it is interesting to observe what a peculiar bias it gave to Washington's life before and after the Revolution."

Professor Small's successors in the mathematical chair at William and Mary were Rev. Thomas Gwatkin, George Blackburn, Ferdinand S. Campbell, Robert Saunders, Benjamin S. Ewell, and Thomas T. L. Snead.

UNIVERSITY OF PENNSYLVANIA.

The University of Pennsylvania was chartered in 1755, and was known before the Revolution as the College, Academy, and Charitable School of Philadelphia. The celebrated Dr. William Smith, D. D., was the first provost. He was a man of great learning and superior executive ability. Under his administration, previous to the outbreak of the Revolution, the college made marvellous progress. The teachers were men of well-established reputation throughout the colonies. Dr. Smith, who was very fond of mathematical studies, gave lectures on mathematics, natural philosophy, astronomy, and rhetoric. In 1769 he appears as one of the founders of the American Philosophical Society. The first volume of the transactions of that society contains accurate observations by Rittenhouse and himself of the transits of Venus and Mercury. Associated with him at the college as professor of mathematics, from 1760 to 1763, was Hugh Williams. He was a graduate of the institution, and a minister. Afterward he studied medicine abroad and then practiced in Philadelphia. He took great interest in astronomy, and observed the transit of Venus and Mercury for the Philosophical Society.

Theophilus Grew is also mentioned as a mathematical instructor. Rev. Ebenezer Kinnersley, Franklin's assistant in his electrical experiments, gave instruction in physics. "In this institution," says Dr. Smith, "there is a good apparatus for experiments in natural philosophy, done in England by the best hands and brought over from thence in different parcels. There is also in the experiment room an electrical apparatus, chiefly the invention of one of the professors, Mr. Kinnersley, and perhaps the completest of the kind now in the world." The courses of study mapped out by Dr. Smith are preserved in his works.* According to this, the mathematical and physical instruction during the three years at college was as follows (in 1758):

First year.—Common and decimal arithmetic reviewed, including fractions and the extraction of roots; algebra through simple and quadratic equations, and logarithmical arithmetic; first six books of Euclid.

Second year.—Plane and spherical trigonometry; surveying, dialing, navigation; eleventh and twelfth books of Euclid; conic sections; fluxions; architecture, with fortification; physics.

Third year.—Light and color, optics, perspective, astronomy.

* William Smith's Works, 1803, p. 238.

There is given, in addition to this, the following list of "books recommended for improving the youth in the various branches."

First year.—Barrow's Lectures, Pardie's Geometry, Maclaurin's Algebra, Ward's Mathematics, Keil's Trigonometry.

Second year.—Patoun's Navigation, Gregory's Geometry and Fortification; Simson's Conic Sections; Maclaurin's and Emerson's Fluxions.

Third year.—Helsham's Lectures; Gravesande; Cote's Hydrostatics; Desaguliers; Muschenbroec; Keil's Introduction; Martin's Philosophy, Maclaurin's View of Sir Isaac Newton's Philosophy, Rohault per Clarke.

It appears that the instruction was given by lectures, the books of which the above is a partial list, were (says Dr. Smith) "to be consulted occasionally in the lectures, for the illustrations of any particular part; and to be read afterwards, for completing the whole." How closely this advanced curriculum of Dr. Smith was adhered to, and how nearly his ideal scheme came to be realized in the actual work of the college, we have no means of determining. This much is certain, that before the Revolution the institution attracted a large number of students. According to Dr. Smith, the attendance in the college alone went as high as one hundred, while the total attendance, including the pupils of the academy and charity schools, surpassed three hundred. Of the course of study which he planned for the institution, it has been said by competent judges that "no such comprehensive scheme of education then existed in the American colonies."

But there followed a reaction. Political troubles at the beginning of the Revolutionary War broke up the institution. The authorities of the college were accused of disloyalty, and in 1779 the charter was annulled by the Provincial Assembly, and the college estate vested in a new board. Dr. Smith was ejected, and in 1791 there was organized the "University of Pennsylvania." Many years elapsed before the institution regained the popularity it enjoyed before the war.

Self-taught Mathematicians.

The mathematicians mentioned in the previous pages were all men engaged in the profession of teaching. But, strange as it may seem, the most noted mathematician and astronomer of early times was not a professor in a college, nor had he been trained within college walls. We have reference to David Rittenhouse. He was born near Germantown, Pa., in 1732. Until about his eighteenth year, he was employed on his father's farm. The advantages for obtaining an education in rural districts were then exceedingly limited, but the elasticity of his genius was superior to the pressure of adverse fortune. At the age of twelve he came in possession of a chest of carpenter's tools, belonging to an uncle of his, who had died some years previously. This chest contained, besides the implements of trade, several elementary books treating of arithmetic and geometry. This humble coffer was to him an invaluable treasure, for the tools afforded him some means of exercising

the bent of his genius toward mechanics, while the books early led his mind to those pursuits for which it was pre-eminently fitted. While a boy he is said to have covered the fences and plows on his father's farm with geometrical figures. At the age of seventeen he constructed a wooden clock.

The delicacy of his constitution and the irresistible bent of his genius induced his parents to yield to his oft-repeated wish of giving up farming, and to procure for him the tools of a clock and mathematical instrument maker. Rittenhouse now worked diligently with his tools during the day, and at night spent a portion of his time which should have been passed in taking repose in the prosecution of his studies. His success seems to have been extraordinary, for his biographers assert that before the age of twenty he was able to read the Principia, and that he had discovered the method of fluxions without being aware that this had already been done by Newton and Leibnitz. In Sparks's American Biography we read that since Newton in his Principia "follows the synthetic method of demonstration and gives no clue to the analytic process by which the truth of this proposition was first discovered by him, * * * Rittenhouse began to search for the instrument which might be applied to the purpose of similar discoveries, and in his researches attained the principles of the method of fluxions."

Dr. Rush, in his eulogy on Rittenhouse, says in the same way: "It was during the residence of our ingenious philosopher with his father in the country that he made himself master of Sir Isaac Newton's Principia, which he read in the English translation of Mr. Motte. It was here, likewise, he became acquainted with the science of fluxions; of which sublime invention he believed himself, for a while, to be the author, nor did he know for some years afterwards that a contest had been carried on between Sir Isaac Newton and Leibnitz for the honor of the great and useful discovery. What a mind was here! Without literary friends or society, and with but two or three books, he became, before he had reached his four and twentieth year, the rival of two of the greatest mathematicians in Europe."

Our information concerning the studies of our young philosopher is so scanty, that we find it impossible to determine the exact range of his thoughts or the consequences that flowed from them. Not the slightest information as to the exact nature of his alleged invention has been preserved. He himself seems to have attached no weight to it. We are of the opinion that his invention, whatever it may have been, was not of sufficient importance to deserve the name of an "invention of fluxions." If Rittenhouse actually made an invention of such transcending magnitude before the age of twenty, and at a time when he had hardly begun his scientific studies, how is it that he made not the slightest approach to any similar discovery during the forty-four years of his maturer life? Though always a passionate lover of scientific pursuits, he made no original contributions whatever to the science of

pure mathematics. Science is indebteded to him chiefly for his orreries and the observations of the transit of Venus. We are, therefore, of the opinion that the alleged invention of fluxions was little more than a "rumor set afloat by idle gossip." It serves to show us, however, in what unbounded admiration he was held by his countrymen.

At the age of nineteen Rittenhouse made the acquaintance of Thomas Barton, a talented young clergyman who had been a student at the University of Dublin. An intimate friendship grew up between them, which proved advantageous to the mental improvement of both. Barton was able to furnish Rittenhouse with some books suitable for his instruction. The burning zeal with which our young scientist pursued his studies appears from the following extract of a letter he wrote to Barton on September 20, 1756, at the age of twenty-four: "I have no health for a soldier [the country was then engaged in war], and as I have no expectation of serving my country in that way, I am spending my time in the old trifling manner, and am so taken with optics, that I do not know whether, if the enemy should invade this part of the country, as Archimedes was slain while making geometrical figures on the sand, so I should die making a telescope."

As a mechanic, Rittenhouse became celebrated for the extreme exactness and finish of his workmanship. Especially celebrated were his chronometer clocks. It was while thus engaged in the manufacture of clocks that he planned and executed an instrument which brought into play both his mechanical and mathematical skill. This instrument was the *orrery*. Concerning this wonderful mechanism, he wrote to Barton January 28, 1767, as follows: "I do not design a machine which will give the ignorant in astronomy a just view of the solar system, but would rather astonish the skilful and curious observer by a most accurate correspondence between the situations and motions of our little representatives of our heavenly bodies and the situations and motions of those bodies themselves. I would have my orrery really useful by making it capable of informing us truly of the astronomical phenomena for any particular point of time, which I do not find that any orrery yet made can do." It was, indeed, intended to be a sort of a perpetual astronomical almanac, in which the results, instead of being exhibited in tables, were to be actually exhibited to the eye. His orrery greatly exceeded all others in precision. It attracted very general attention among well-informed persons, and the Legislature of Pennsylvania, in appreciation of the talents of Rittenhouse, voted that the sum of three hundred pounds be given to him.

There arose a lively competition between different colleges in this country for the possession of this orrery. While the College of Philadelphia was negotiating for its purchase, a committee from the College of New Jersey went to examine it, and concluded to buy it at once; and thus, much to the chagrin of Dr. William Smith, Princeton bore off the palm from Philadelphia in obtaining possession of the first orrery con-

structed by Rittenhouse. He afterwards made another one for the Philadelphia College. The author of The Vision of Columbus, a poem first published in 1787, alludes to the Rittenhouse orrery in Philadelphia and the mass of people crowding to the college hall to see it, in the following lines (Book VII):

> See the sage Rittenhouse, with ardent eye,
> Lift the long tube and pierce the starry sky;
> Clear in his view the circling systems roll,
> And broader splendours gild the central pole.
> He marks what laws th' eccentric wand'rers bind,
> Copies Creation in his forming mind,
> And bids, beneath his hand, in semblance rise,
> With mimic orbs, the labours of the skies.
> There wond'ring crowds with raptur'd eye behold
> The spangled heav'ns their mystic maze unfold;
> While each glad sage his splendid hall shall grace,
> With all the spheres that cleave th' ethereal space.

In August, 1768, Rittenhouse was appointed by the American Philosophical Society in Philadelphia as one of a committee to observe the transit of Venus on June 3d of the following year. A temporary observatory was built by him for the purpose near his residence at Norriton. Dr. William Smith aided him in procuring suitable instruments, and the preliminary arrangements were made with most scrupulous care. The approaching phenomenon was one of great scientific importance. Only two transits of Venus had been observed before his time, and of these, the first, in 1639, had been seen by only two persons. These transits happen so seldom that there cannot be more than two in one century, and in some centuries none at all. But the transits of Venus are the best means we have for determining the parallax of the sun. At the approach of the transit, Rittenhouse and his assistants in this observation, Dr. William Smith and Mr. Lukens, then surveyor-general of Pennsylvania, awaited the contacts in silence and anxiety. The observations were a success, and established for Rittenhouse the reputation of an exact and careful astronomer. The transit was observed in Boston by Professor Winthrop, and in Providence by Benjamin West, at almost all the observatories in Europe, and in various other parts of the globe. During the transit Rittenhouse saw one phenomenon which escaped the notice of all other astronomers. When the planet had advanced about half of its diameter upon the sun, that part of the edge of the planet which was off the sun's disc appeared illuminated, so that the outline of the entire planet could be seen. But a complete circle of light around Venus would indicate that more than half of Venus is illuminated. This can happen, as far as we know, only when the rays of light are refracted by an atmosphere. Hence, it would follow from the observations of Rittenhouse that Venus is surrounded, like the earth, by an atmosphere. But this appearance of a ring of light was not confirmed by other astronomers, and the state-

ment of Rittenhouse excited no attention for nearly a century, until his observation was, at last, confirmed by other astronomers.

An important invention made by Rittenhouse is that of the "collimator," "a device for obtaining a meridian mark without going far away; it has lately come back from Germany, where it was re-invented."*

The reputation which Rittenhouse had now acquired as an astronomer attracted the attention of the Government, and he was employed in several important geodetic operations. In 1779 he was named one of the commissioners for adjusting a territorial dispute between the States of Pennsylvania and Virginia; in 1786 he was employed in fixing the line which separates Pennsylvania from the State of New York, and in the following year he assisted in determining the boundary between New York and Massachusetts. In 1791 he was chosen successor of Dr. Franklin in the presidency of the American Philosophical Society. All his scientific communications were made to that society and published in its Transactions.

Rittenhouse came to be looked up to by his countrymen as an astronomer equalled by few and surpassed by none of his contemporaries. Listen, if you please, to Thomas Jefferson's estimate of him. In answer to the assertion of Abbé Raynal that "America had not yet produced one good poet, one able mathematician, one man of genius in a single art or a single science," Jefferson says: "When we shall have existed as a people as long as the Greeks did before they produced a Homer, the Romans a Virgil, the French a Racine and Voltaire, the English a Shakespeare and Milton, should this reproach be still true, we will inquire from what unfriendly causes it has proceeded, that the other countries of Europe and quarters of the earth shall not have inscribed any name in the roll of poets. * * * In war we have produced a Washington, whose memory will be adored while liberty shall have votaries, whose name shall triumph over time, and will in future ages assume its just station among the most celebrated worthies of the world. * * * In physics we have produced a Franklin, than whom no one of the present age has made more important discoveries, nor has enriched philosophy with more, or more ingenious, solutions of the phenomena of nature. *We have supposed Mr. Rittenhouse second to no astronomer living; that in genius he must be the first, because he is self-taught.* As an artist he has exhibited as great a proof of mechanical genius as the world has ever produced. He has not indeed made a world; but he has by imitation approached nearer its Maker than any man who has lived from the creation to this day."†

Such was Jefferson's estimate of Rittenhouse. James Renwick says that "he [Rittenhouse] had shown himself the equal in point of learning and skill as an observer to any practical astronomer then living." Dr. Bush, in his eulogy, exclaims: "What a mind was here! Without

*The Development of Astronomy in the United States, by Prof. T. H. Safford, p. 3.
† Jefferson's *Notes on Virginia*.

literary friends or society, and with but two or three books, he became, before he had reached his four-and-twentieth year, the rival of two of the greatest mathematicians in Europe!"

If we estimate Rittenhouse by what he *might have done* had he had a more rugged physical constitution and better facilities for self-development; had he had an observatory at his disposal such as those of his great contemporaries, Maskelyne and William Herschel in England, Lalande and Count Cassini in France, Tobias Mayer in Germany, then the above estimates may be correct. But if our astronomer be judged by the original contributions which, under existing adverse circumstances, he actually *did* make to astronomy and mathematics, then it must be admitted that he can not be placed in the foremost rank of astronomers then living. Friends will judge him by what *he might have done;* the world at large will judge him by what he *actually accomplished.* Our greatest indebtedness to Rittenhouse lies not in the original contributions he made to science, but rather in the interest which he aroused in astronomical pursuits, and in the diffusion of scientific knowledge in the New World which resulted from his efforts.

One who enjoyed, in his day, the reputation of being a "great mathematician," was Thomas Godfrey, of Philadelphia. He was a glazier by trade. Having met accidentally with a mathematical book, he became so delighted with the study that by his own unaided perseverance he mastered every book he could get on the subject. He pursued the study of Latin in order that he might read Newton's Principia. Optics and astronomy became his favorite studies, and the exercise of his thoughts led him in 1730 to conceive an improvement of the quadrant. In 1732 a description of his invention was sent to Dr. Hadley in England. Meantime, in 1731, Hadley had made a communication to the Royal Society of London, describing an improvement of the quadrant similar to that of Godfrey. The claims of both parties were afterwards investigated by the Royal Society, and both were entitled to the honor of invention. The instrument is still called "Hadley's quadrant," though of the two Godfrey was the first inventor. Afterwards it appeared that both had been anticipated in their invention by Newton.

Some of the personal characteristics of Godfrey are known to us through the writings of Benjamin Franklin. "I continued to board with Godfrey, who lived in part of my house with his wife and children, and had one side of the shop for his glazier's business, though he worked but little, being always absorbed in mathematics." In the autumn of 1727 Franklin formed most of his ingenious acquaintances into a club for mutual improvement, called *Junto.* It met Friday evenings. "One of the first members of our Junto," says Franklin, "was Thomas Godfrey, a self-taught mathematician, great in his way, and afterwards inventor of what is now called Hadley's Quadrant. But he knew little out of his way, and was not a pleasing companion, as, like most great mathematicians I have met with, he expected universal precision in

everything said, and was forever denying and distinguishing upon trifles, to the disturbance of all conversation."

This assertion of Franklin that all mathematicians he had met were insufferable from their trifling and captious spirit, has been extensively quoted by opponents of the mathematical sciences. It was quoted by Goethe, and afterwards by Sir William Hamilton, the metaphysician, when he was engaged in a controversy with Whewell, the celebrated author of the History of the Inductive Sciences, on the educational value of mathematical studies. Hamilton attempted to prove the startling proposition that the study of mathematics not only possessed no educational value, but was actually injurious to the mind. He must have experienced exquisite pleasure in finding that Franklin, the greatest physical philosopher of America, had made a statement to the effect that all mathematicians he had met were "forever denying and distinguishing upon trifles."

We shall not speak of this controversy, except to protest against any general conclusion being drawn from Franklin's experience of the captiousness of mathematicians. Take, for examples, David Rittenhouse and Nathaniel Bowditch, who were early American mathematicians, and, like Godfrey, self-taught men. Though Franklin's statement may be true in case of Thomas Godfrey, it is most positively unjust and false when applied to the other two scholars. The biographers of David Rittenhouse are unanimous and explicit in their assertion that in private and social life he exhibited all those mild and amiable virtues by which it is adorned. As to Nathaniel Bowditch, of whom we shall speak at length later on, we have the reliable testimony of numerous writers that he was a man remarkable for his social virtues, modest and attractive manners, and Franklinian common sense.

Mention should be made here of Benjamin Banneker, the self-taught "negro astronomer and philosopher," born in Maryland, who became noted in his neighborhood as an expert in the solution of difficult problems, and who, with the use of Mayer's Tables, Ferguson's Astronomy, and Leadbeater's Lunar Tables, made creditable progress in astronomy, and calculated several almanacs. His first almanac was for the year 1792. The publishers speak of it as "having met the approbation of several of the most distinguished astronomers in America, particularly the celebrated Rittenhouse." Banneker sent a copy to Mr. Jefferson, then Secretary of State, who said in his reply, "I have taken the liberty of sending your almanac to Monsieur de Condorcet, secretary of the Academy of Sciences at Paris, and member of the Philanthropic Society, because I considered it a document to which your whole color had a right for their justification against the doubts which have been entertained of them."* Banneker was invited by Andrew Ellicott to accompany "the Commissioners to run the lines of the District of Columbia" upon their mission.

* History of the Negro Race in America, by George W. Williams, p. 386.

II.

INFLUX OF ENGLISH MATHEMATICS, 1776–1820.

The Revolutionary War bore down so heavily upon the educational work in both elementary and higher institutions, that many of them, for a time, actually closed their doors. The majority of students and professors of Harvard and Yale were in the Army, or were in some other way rendering aid to the national cause. The buildings of Nassau (Princeton) College were for a time used as barracks. The business of Columbia College in New York was almost entirely broken up. The professors and students of Rutgers College at New Brunswick, N. J., were sometimes compelled by the presence of the enemy to pursue their academical studies at a distance from New Brunswick. The operations of Brown University in Providence, R. I., were discontinued during part of the war, the college building being occupied by the militia and the troops of Rochambeau. At William and Mary College the exercises were suspended in 1781 for about a year, and the building was occupied at different times by both British and American troops. The walls of the college were "alternately shaken by the thunder of the cannon at Yorktown and by the triumphant shouts of the noble bands who had fought and conquered in the name of American Independence." Academies and primary schools were either deserted or taught by women and white-haired men too old to fight. That the philosophic pursuits of scientific societies should have sunk very low is not surprising. Fifteen years elapsed between the publication of the first and second volumes of the Transactions of the American Philosophical Society in Philadelphia.

In spite of the financial depression and poverty which existed immediately after the war, much attention was paid to education. While in 1776 there existed in the colonies only seven colleges, the number was increased to nineteen before the close of the eighteenth century. Academies and grammar schools were established, and a large number of text-books were put through the press. Even during the war the printing-press sent out an occasional school-book. Thus, in 1778, while the war was raging most fiercely, an edition of Dilworth's spelling-book was printed, which contained in its preface the following patriotic passage: "At the beginning of the contest between the Tyrant and the States, it was boasted by our unnatural enemy, that, if nothing more, they could at least shut up our ports by their navy and prevent the importa-

tion of books and paper, so that in a few years we should sink down into barbarity and ignorance, and be fit companions for the Indians, our neighbors to the westward." These words, printed at the darkest period of the Revolutionary War, disclose a spirit far from submissive. The colonists were not quite ready to sink down into barbarity and ignorance. During the twenty-five years after the Declaration of Independence, more real progress was made in education than in the entire century preceding. Between 1776 and 1815 a large number of books on elementary and a few on higher mathematics were published in America. Many of them were reprints of English works, while others were compilations by American writers, modelled after English patterns. French and German authors were almost unknown. We may therefore call this the period of the "Influx of English Mathematics" into the United States. What little mathematics was studied in the colonies before the Revolution was, to be sure, gotten chiefly from English sources, but the scientific currents thither were then so very feeble and slow that we can hardly speak of an "influx."

Elementary Schools.

It is a significant fact that of the arithmetics used before the Revolution, but one work in the English language was written by an American author. It is equally significant that with the close of the great struggle for liberty, there began a period of activity in the production of new school-books. The second book devoted exclusively to arithmetic, compiled by an American author, and printed in the English language, was the New and Complete System of Arithmetic by Nicholas Pike, (Newburyport, 1788.)*

Nicholas Pike (1743-1819) was a native of New Hampshire, graduated at Harvard College in 1766, and was for many years a teacher and afterward a magistrate at Newburyport in Massachusetts. His arithmetic received the approbation of the presidents and professors of the leading New England colleges. A recommendation from Harvard professors contains the following timely remark: "We are happy to see so useful an American production, which, if it should meet with the encouragement it deserves, among the inhabitants of the United States, will save much money in the country, which would otherwise be sent to

* It appears that Greenwood's Arithmetic, published nearly sixty years previously, was at this time not known to exist. Pike's Arithmetic was called the *first* American work of its kind. Dr. Artemas Martin has sent the writer the American Antiquarian, (Vol. IV, No. 12, New York, May, 1888) giving an account of Pike's book. It gives a letter written by George Washington at Mount Vernon, June 20, 1788, to Nicholas Pike, in which the former politely acknowledges the receipt of a copy of Pike's Arithmetic. We quote from the letter the following passage:

"Its merits being established by the approbation of competent judges, I flatter myself that the idea of its being an American production and the *first of the kind* which has appeared, will induce every patriotic and liberal character to give it all the countenance and patronage in his power."

Europe, for publications of this kind." Pike's arithmetic passed through many editions, was long the standard mathematical manual in New England schools, and formed the basis for other arithmetics. It was a very extensive and complete book for that time. A large proportion of the rules were given without demonstration, while some were proved algebraically. In addition to the subjects ordinarily found in arithmetics, it contained logarithms, trigonometry, algebra, and conic sections, but these latter subjects were so briefly treated as to possess little value. After the appearance of Webber's, Day's, and Farrar's Mathematics for colleges, which elaborated these subjects at greater length, they were finally omitted in the fourth edition of Pike's Arithmetic, in 1822.

In 1788, when the first edition appeared, English money was still the prevalent medium of exchange in the United States. To be sure, Federal money was adopted by Congress as early as 1786, but previous to 1794 there was no United States coin of the denomination of a dollar. It was merely the money of account, based upon the Spanish dollar, which had long been in use in this country. Congress passed a law organizing a mint in 1792, but permitting the circulation of foreign coins for three years, by which time it was believed the new coinage would be ready in sufficient amount. When dollars and cents began to replace pounds and shillings, it became desirable that the Federal currency be explained in arithmetics and taught in schools. In consequence of this, the sterling notation was changed to Federal in the third edition of Pike's arithmetic, which was brought out in Boston in 1808 by Nathaniel Lord. Similar changes were made in other arithmetics.*

Down to the year 1800, the only arithmetic written by an American, which enjoyed wide-spread and prolonged popular favor, was the one of Nicholas Pike. In 1800 appeared a second successful arithmetic, The School-master's Assistant, by Nathan Daboll, a teacher in New London

* Contemporaneously with Pike's Arithmetic there appeared in Philadelphia the Elementary Principles of Arithmetic, by Thomas Sarjeant. This book, as well as the Federal Arithmetic, or the Science of Numbers (Philadelphia, 1793), by the same author, had only an ephemeral reputation. John Gough's Treatise on Arithmetic in Theory and Practice, edited by Benjamin Workman (Boston, 1789), as well as Gough's American Accountant, or School-master's New Assistant, abridged by Benjamin Workman and revised by Patterson (Philadelphia, 1796), had a rather limited circulation. Nor did John Vinall's Arithmetic (Boston, 1792), enjoy better success. After having been a teacher in Newburyport for seventeen years, Vinall at last became writing-master in a school in Boston, his native city. He is said to have been coarse in speech and, like his book, unpopular. Gordon Johnson wrote an arithmetic (Springfield, 1792), which never had more than a passing local reputation. Somewhat more successful was the Introduction to Arithmetic (Norwich, Conn., 1793), by Erastus Root, a graduate of Dartmouth, for several years a teacher, and afterward an active politician and member of Congress.

Our list of arithmetics printed previously to the year 1800 includes the names of several other "quaint and curious volumes," which, after an ephemeral reputation,

(born 1750 and died 1818). This work passed through numerous editions. Though Daboll had to compete with Pike's Abridged Arithmetic and with the celebrated Scholar's Arithmetic of Daniel Adams, it nevertheless acquired an extensive popularity. The expression, "according to Daboll," came to be a synonym for "mathematical correctness." It pushed aside the less favorite works. The main element of popularity of Daboll's School-master's Assistant lay in the fact that it introduced Federal money immediately after the addition of whole numbers, and showed how to find the value of goods therein immediately after simple multiplication. This arrangement, says the author, may be of great advantage to many who perhaps will not have an opportunity to learn fractions. Decimal fractions were wisely made to precede vulgar fractions. In the "Recollections" by Peter Parley, of the town of Ridgefield, Conn., are found the following interesting remarks: "We were taught arithmetic in Daboll, then a new book, and which, being adapted to our measures of length, weight, and currency, was a prodigious leap over the head of poor old Dilworth, whose rules and examples were modelled upon English customs. In consequence of the general use of Dilworth in our schools for perhaps a century, pounds, shillings, and pence were classical, and dollars and cents vulgar for several succeeding generations. 'I would not give a penny for it' was genteel; 'I would not give a cent for it' was plebeian."

Since the adherence to pounds and shillings came to be offensive to the people of the young republic, Mr. Hawley, in 1803, undertook to revise the work and alter all the problems to Federal currency. He called the new work "Dilworth's Federal Calculator," but after this change the book was so completely different from the original that the use of Dilworth's name in the title seemed hardly justifiable. Be that as it may, the Federal Calculator was not a success.

At the beginning of the nineteenth century there were three "great arithmeticians" in America, namely, Nicholas Pike, Nathan Daboll, and

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passed into forgetfulness, never to be resurrected to memory except by the curiosity of some inquiring lovers of "forgotten lore." To the above names we add: The American Tutor's Assistant, by John Todd, Philadelphia, third edition, 1797; Arithmetic by Zachariah Jess, Philadelphia, 1797; American Arithmetic, by David Cook, New Haven, Conn., 1799; The Usher, comprising arithmetic in whole numbers, Federal money, mensuration, surveying, etc., by Ezekiel Little, Exeter, 1799; Usher's Arithmetic, abridged, by Ezekiel Little, 1804; The American Accountant, by Chauncey Lee, Lansingburg, 1797; The American Accountant, by William Milne, New York, 1797, in which, instead of the answers to the problems, which were usually given, the author gave the remainders, after casting out the nines from the answers. A curious little volume is the following: "The Young Gentleman's and Lady's Assistant, containing Geography, Natural Philosophy, Rhetoric, Miscellaneous, to which is added a short and complete system of Practical Arithmetic, wherein the money of the United States of America is rendered easy to the perception of youth. The whole divided into small sections for the convenience of schools, by Donald Fraser, author of the Columbian Monitor, New York, 1706."

Daniel Adams. Having noticed the first two, we shall briefly speak of the third. Daniel Adams published in 1801 the Scholar's Arithmetic, a work which in point of merit towers far above the mass of contemporary school-books. Adams was a native of Massachusetts, graduated at Dartmouth College in 1797, and then became teacher, physician, and editor. He taught school in Boston from 1806 to 1813, then removed to New Hampshire, where he afterward served as State Senator. Though engaged in various lines of thought, arithmetical studies were his favorite. He furnished the school-boys' satchels not only with the Scholars' Arithmetic, but also with the Primary Arithmetic, and, in 1827, with the New Arithmetic, which passed through numerous editions. The New Arithmetic differed from the Scholars' Arithmetic in being analytic instead of synthetic in treatment. The analytic or inductive method of teaching, introduced into Switzerland by Pestalozzi, was gaining ground rapidly in this country at the beginning of the second quarter of the present century.

Between 1800 and 1820, a large number of arithmetics sprang into existence. Most of them enjoyed only a mushroom popularity. Among the more successful of the new aspirants to arithmetical fame were the following: Jacob Willetts, of Poughkeepsie, N. Y., the author of the Scholar's Arithmetic, 1812; William Kinne, of Maine, who graduated at Yale in 1804, and subsequently became teacher in his native State and the author of A Short System of Practical Arithmetic, Hallowell, second edition, 1807; Michael Walsh, the author of A New System of Mercantile Arithmetic, adapted to the commerce of the United States, Newburyport, 1801; Stephen Pike, whose arithmetic was published in Philadelphia in 1813; Samuel Webber's Arithmetic, 1810, which was used chiefly by students preparing for Harvard.*

Our list of arithmetics published during the first twenty years of this century is doubtless very imperfect. Of the larger number of publications, the great majority had only an ephemeral reputation. Excepting those of Pike, Adams, and Daboll, hardly any have survived the recollection even of the aged.

* Less widely used were the following books: Jonathan Grout's Guide to Practical Arithmetic, 1802; Caleb Alexander's New System of Arithmetic, Albany, 1802; W. M. Finlay's Arithmetical Magazine, or Mercantile Accountant, New York, 1803; James Noyes's Federal Arithmetic, 1804; the American School-master's Assistant, by Jesse Guthrie, of Kentucky, Lexington, 1804; Samuel Temple's System of Arithmetic in Federal Currency, Boston, 1804; the Youth's Arithmetical Guide, by Tandon Addington and Watson, Philadelphia, 1805; Mathematical Manual for the Use of St. Mary's College of Baltimore, containing arithmetic and algebra, by L. I. M. Chevigne, Baltimore, 1806; Kimber's Arithmetic Made Easy for Children, second edition, 1807; Ballard's Gauging Unmasked, 1806; Robert Patterson's Treatise on Arithmetic, Philadelphia, 1819; Daniel Staniford's Practical Arithmetic, Boston, 1818; George Fenwick's Arithmetical Essay, Alexandria, 1810; Compendium of Practical Arithmetic, by Osgood Carleton, Boston, 1810; the American Arithmetic, by Oliver Welch of New Hampshire; The Teacher's and Pupil's Assistant, by Dale Tweed of northern New York, 1820; the Arithmetic of Leonard Loomis; The Columbian Tutor's Assistant, by D. McCurdy, Washington, 1819; The First Lines of Arithmetic, 1818, by De Wolf and

Between 1815 and 1820 a reform in mathematical teaching was inaugurated in this country. Foremost among the leaders in this new movement was John Farrar of Harvard, who translated into English for the use of colleges a number of French works. The French books of that time were far in advance of the English. This reform in the teaching of the more advanced mathematics was accompanied by a similar reform in arithmetical teaching. The new ideas of Pestalozzi were vigorously forcing their way from Switzerland to all parts of the civilized world. Among the earliest fruits they bore in this country were the First Lessons in Arithmetic, by Warren Colburn, 1821. This primer contained points of great excellence, and it had a sale such as no other arithmetic ever had before.

We do not now speak of these reforms, except simply for the purpose of marking the end of an old period and the beginning of a new one.

Having enumerated the text-books published in the United States after the Revolution and preceding the year 1820, we shall now briefly examine their contents. The leading characteristics which we observed in Dilworth's School-master's Assistant, are found to exist in the books of this period. The arithmetics of this time were little more than Pandora's boxes of ill-formed rules to be committed to memory. Reasoning was exiled from the realm of arithmetic, and memory was made to rule supreme. A science chiefly intended to cultivate the understanding was offered to the exercise merely of memory.

This banishment of demonstration and worship of memory did not, I am glad to say, originate in this country. As already remarked, it came from England. About the middle of the seventeenth century there arose in England the commercial school of arithmeticians. To this school, says De Morgan, "we owe the destruction of demonstrative arithmetic in this country, or rather the prevention of its growth. It never was much the habit of arithmeticians to prove their rules; and the very word *proof*, in that science, never came to mean more than a

Brown, teachers in Hartford, Conn.; the Arithmetic of Zachariah Jess of Delaware(?); The Scholar's Guide to Arithmetic, by Plinehas Merrill of New Hampshire; Collection of Arithmetical Tables, Hartford, 1812; Arithmetic Simplified, 1818, by John J. White; and The Youth's Guide, by Mordecai Stewart, Baltimore, 1818; Rev. John White's Mental Arithmetic, Philadelphia, 1818; "The American Youth: being a New and Complete Course of Introductory Mathematics: designed for the use of Private Students, by Consider and John Sterry." Vol. 1, Providence.

Besides these American works there were a number of foreign books republished in this country. Among these are, The Tutor's Guide, by Charles Vyse, London, 1770, which reached the thirteenth American edition in Philadelphia in 1806; A Complete Treatise on Arithmetic, by Charles Hutton, Edinburgh, 1802, first American edition, New York, 1810; A System of Practical Arithmetic, by Rev. J. Joice, London, 1816, and adapted to the commerce of the United States by J. Walker, Baltimore, 1819; The Scholar's Guide to Arithmetic, by John Bonnycastle, London, 1786, Philadelphia, 1818. These English books can hardly be said to have excelled our American arithmetics; nor did they attain to any remarkable success in the New World.

test of the correctness of a particular operation, by reversing the process, casting out the nines, or the like. As soon as attention was fairly diverted to arithmetic for commercial purposes alone, such rational application as had been handed down from the writers of the sixteenth century began to disappear, and was finally extinct in the work of Cocker or Hawkins, as I think I have shown reason for supposing it should be called. From this time began the finished school of teachers, whose pupils ask, when a question is given, what rule it is in, and run away, when they grow up, from any numerical statement, with the declaration that anything may be proved by figures—as it may, to them."*

Such is the history of the commercial school of arithmeticians in England. In America this school became firmly established wherever arithmetic was taught. Thus the sins of the early English pedagogues were visited upon the children in England and America unto the third and fourth generations. As late as 1818, one of our American compilers of text-books, Daniel Staniford, actually stated in the preface, as a recommendation to his book, that "all mathematical demonstrations are purposely omitted, to clear illustrations of the rules by easy examples and such as tend to prepare the scholar for business." Not only was this method adopted in practice, but even advocated in theory. In both English and American arithmetics the rules were ill-arranged and disconnected. The pupil had to learn a dozen rules which might have been reduced conveniently to two or three principles. The continuity in the reasoning upon quantities expressed by integers and those expressed in common or decimal fractions was often so completely disguised that it became necessary to repeat the rules. Thus Dilworth and Bonnycastle give in their arithmetics three distinct rules, as follows:

Rule of three for integers.
Rule of three for vulgar fractions.
Rule of three for decimal fractions.

Nicholas Pike, Daniel Staniford, and John Vinall each give "Rules of interest," and later on again "Rules of interest by decimals." The result of this cumbrous rule-system is that the scholar acquires the art of solving problems, provided he knows what rule it falls under, which is not always sure to be the case, for the first practical problem which will arise may be one requiring not one rule, but a combination of rules, which can therefore not be solved directly by the rules in his book. And here he is fairly aground, for he has no mastery of principles, but is the abject slave of rules. Such a system of arithmetic has been very appropriately called *ciphering*, since intellect goes for nothing throughout.

Among other features which characterize old American arithmetics are the following:

(1) The total absence of exercises in mental arithmetic.

* Arithmetical Books, p. xxi.

(2) The meagre treatment of fractions. The number of exercises was so very limited that it was impossible for the student to acquire a mastery of fractions without additional drill.

(3) The process of "cancellation," which shortens calculations so much, was entirely unknown. Strange as it may seem, it is less than fifty years since cancellation was introduced into our arithmetics. One of the first books containing this process was published in 1840 by C. Tracy, in New Haven, entitled "A New System of Arithmetic, in which is explained and applied to practical purposes * * * the principle of cancelling. * * *"

(4) The system of numeration in early American arithmetics was not the French now generally used, but the English, in which the digits of a number are distributed in periods of six, and consequently proceed by millions. This method was first adopted by the Italians. Lucas de Burgo gives it in a work written in 1494. The method of reckoning by three places, as used in this country and on the Continent, seems to have originated with the Spanish.

(5) The subject which we now call proportion was then called the "Rule of Three." It was taught as a mere rule. The principle underlying it was ignored completely. That a proportion is the expression of the equality of two ratios was then not even hinted at. This fact goes to explain a point which otherwise would seem mysterious. If proportion is the equality of ratios, then why was not the usual symbol used to express that equality? Why were four dots used instead of the two horizontal lines? The answer seems to be that arithmeticians were not in the habit of thinking of a proportion as the *equality* of two ratios. A ratio was expressed by two dots, and the four dots were placed between the ratios simply to disjoint the terms, and to show that the second and third terms of the proportion were not in the same relation to each other as the first and second or third and fourth.

(6) The old arithmetics contain two methods of solving problems which are but rarely given in modern arithmetics. The methods I refer to were first used by the Hindoos on the far off banks of the Ganges, then borrowed from the Hindoos by the Arabs, who transmitted them to the Europeans. They are called the methods of "single position" and "double position." They teach us how to resolve questions by making one or two suppositions of false numbers, and then making corrections for the resulting errors.

We have seen that previous to the year 1820 a large number of arithmetics were published in this country; counting both American and foreign, there were to our knowledge over sixty different authors. Notwithstanding this fact, the majority of schools had an inadequate supply of arithmetics. In country schools especially, books were scarce and of a rather miscellaneous character, such as had been in families perhaps half a century. Johnnie Smith would, perhaps, bring to school a dilapidated copy of Dilworth's Arithmetic, which had been used once

by his father. His classmate, Billy Brown, would carry in his satchel a copy of Nicholas Pike's Abridged Arithmetic; the curly-headed Jimmy Jones would express his preference for Daboll's School-masters' Assistant, while the majority of the boys had no books at all. Whenever the supply of arithmetics was insufficient, manuscript books were resorted to. Arithmetics were sometimes covered with sheepskin, in the economical expectation that they might be made to lead not only one boy, but also his younger brothers in succession, to the golden science of numbers.

We have now spoken of the most popular arithmetics once used in this country. We have also briefly examined their contents. Our next task will be to ascertain, as far as possible, the manner in which arithmetic was taught in the school-room.

One of the first inquiries in this connection is regarding the quality of the teachers. The best teachers in elementary schools that our forefathers knew were young students who taught school for money to finish their courses in theology, medicine, or law. But this class of school-teachers was not large in early days. The representative school-masters of by-gone times were the itinerant school-masters. They were mostly foreigners. Their qualifications seemed to be the inability to earn anything in any other way. They were generally without families and had no fixed residence; they kept school first in one place and then in another, and wandered about homeless. Many were given to drinking and gambling. As a class, their knowledge was limited to the merest elements. We are told that as late as 1822, in a town in the State of Connecticut, six out of fifteen applicants for positions as teachers were rejected because they did not understand notation and numeration of numbers. And yet these candidates came well recommended as having taught school acceptably in other towns for one, two, or three winters. If this story be true, then it will not seem strange to hear that it was a common practice for teachers in those early days to have their scholars "skip" fractions. This omission was justified on the ground that "fractions were rarely used in business," but there were generally other good and untold reasons for "skipping" the subject. There were few schools that carried the students beyond the rule of three or proportion.

We have seen the great defects in the old arithmetics. The statement of rules took the place of explanations and reasoning. If the school-masters had been competent and well trained, then the defects of bad books might have been remedied by skillful teaching, but the teaching was generally of the poorest kind. The truth of this assertion will be strikingly illustrated by a few examples. Joseph T. Buckingham tells us how, in 1790 or 1791, when he was about twelve years old, he began to learn arithmetic. I quote his exact words: "I told the master I wanted to learn to cipher. He set me a *sum* in simple ad-

dition, five columns of figures and six figures in each column. All the instruction he gave me was—add the figures in the first column, carry one for every ten, and set the overplus down under the column. I supposed he meant by the first column the left-hand column, but what he meant by carrying one for every ten was as much a mystery as Samson's riddle was to the Philistines. I worried my brains an hour or two, and showed the master the figures I had made. You may judge what the amount was when the columns were added from left to right. The master frowned and repeated his former instruction—add up the column *on the right*, carry one for every ten, and set down the remainder. Two or three afternoons (I did not go to school in the morning) were spent in this way when I begged to be excused from learning to cipher, and the old gentleman with whom I lived thought it was time wasted; and if I attended the school any further at that time reading, spelling, and a little writing were all that was taught." Next winter a more communicative teacher had the school, "and under him some progress was made in arithmetic, and I made tolerable acquisition in the first four rules, according to Dilworth's School-master's Assistant, of which the teacher and one of the oldest boys had a copy."

An experience similar to that of the writer just quoted was that of Warren Burton. In his book entitled "The District School as it was, by one who went to it," he says that simple addition was easy; "but there was one thing I could not understand—that carrying of tens. It was absolutely necessary, I perceived, in order to get the right answer, yet it was a mystery which that arithmetical oracle, our school-master, did not see fit to explain. It is possible that it was a mystery to him. Then came subtraction; the borrowing of ten was another unaccountable operation. The reason seemed to be then at the very bottom of the well of science; and there it remained for that winter, for no friendly bucket brought it up to my reach."

Mr. William B. Fowle gives an interesting account of John Tileston, who was chief writing-master in a reading school in Boston about the year 1790. It illustrates both the modes of teaching and the competency of teachers. One regulation of that school required the writing-master to teach "writing, arithmetic, and the branches usually taught in town schools, including vulgar and decimal fractions." Mr. Fowle speaks of Tileston as follows:*

"He loved routine. * * * Printed arithmetics were not used in the Boston schools until after the writer left them, and the custom was for the master to write a problem or two in the manuscript of the pupil every other day. No boy was allowed to cipher till he was eleven years old, and writing and ciphering were never performed on the same day. Master Tileston had thus been taught by Master Proctor [his predecessor], and all the sums he set for his pupils were copied exactly from his

* Barnard's Journal, Vol. V, p. 336.

old manuscript. Any boy could copy the work from the manuscript of any other further advanced than himself, and the writer never heard of any explanation of any principle of arithmetic while he was at school. Indeed, the pupils believed that the master could not do the sums he set for them. * * * It is said that a boy who had done the sum set for him by Master Tileston carried it up, as usual, for examination. The old gentleman, as usual, took out his manuscript, compared the slate with it, and pronounced it wrong. The boy went to his seat and reviewed his work, but finding no error in it, returned to the desk, and asked Mr. Tileston to be good enough to examine the work, for he could find no error in it. This was too much to require of him. He growled, as his habit was when displeased, but he compared the sums again, and at last, with a triumphant smile, exclaimed, 'See here, you *nurly* (gnarly) wretch, you have got it, "If four tons of hay cost so much, what will seven tons cost?" when it should be, "If four tons of *English* hay cost so and so." Now go and do it all over again.'" In this story, it may be remarked, some allowance must doubtless be made for the genius of the narrator.

The illustrations which have been given of the incompetency of teachers may appear to be exaggerations, and we certainly wish that for the good name of our early educators they were exaggerations. But the more we inquire into this subject and the more evidence we accumulate, the stronger the conviction becomes that most of them are not exaggerations, but fair samples of the teaching done by the average school-master in elementary schools eighty or one hundred years ago.

In view of these facts, the most obstinate pessimist will be forced to admit that, within the last one hundred years, *progress* has been made. We have better books and abler teachers. Our methods of teaching arithmetic, though still imperfect, are a prodigious leap in advance of those of olden times. We boast of our material progress, and we certainly have great reason for doing so, but the progress in intellectual fields, and in education in particular, though less ostentatious, is none the less instructive.

Not without interest are the following two stanzas of a poem, entitled "A Country School," which was anonymously contributed to the New Hampshire Spy, and preserved in E. H. Smith's collection of American poems (1793):

Will pray Sir Master mend my pen?
Say, Master, that's enough. Here, Ben,
Is this your copy? Can't you tell?
Set all your letters parallel.
I've done my sum—'tis just a groat—
Let's see it. *Master, m' I g'out?*
Yes, bring some wood in. What's that noise?
It isn't I, sir, it's them boys.

Come Billy, read. What's that? That's A—
Sir, Jim has snatch'd my rule away—
Return it, James. Here rule with this—
Billy, read on—that's crooked S.
Read in the spelling-book. Begin—
The boys are out. Then call them in—
My nose bleeds, mayn't I get some ice,
And hold it in my breeches? Yes.
John, keep your seat. *My sum is more—*
Then do't again—divide by four,
By twelve, and twenty—mind the rule.
Now speak, Manasseh, and spell tool.
I can't. Well try. T, W, L.
Not wash'd your hands yet, booby, ha?
You had your orders yesterday.
Give me the ferule, hold your hand,
Oh! Oh! there—mind my next command.

COLLEGES.

Before proceeding to the history of mathematics in higher institutions, we shall speak of American reprints of English mathematical works for colleges. First of all comes that good old Greek geometry of Euclid, of which the English made excellent translations. An edition of Euclid appeared in Worcester in 1784. This seems to be the earliest American edition. After the beginning of this century numerous editions of it were published. In 1803, Thomas and George Palmer, in Philadelphia, published Robert Simson's Euclid, together with the book of Euclid's Data, and the Elements of Plane and Spherical Trigonometry. The book was sold "at the book-stores in Philadelphia, Baltimore, Washington, Petersburg, and Norfolk."

Prof. S. Webber says, in his "Mathematics," that a good American edition of Playfair's Elements of Euclid, containing the first six books, with two books on the geometry of solids, was given by Mr. F. Nicholls, of Philadelphia, 1806. John D. Craig, teacher of mathematics in Baltimore, brought out an edition of Euclid in 1818. Robert Simson's Euclid was republished in Philadelphia in 1821; Playfair's in New York in 1819 and 1824, and in Philadelphia in 1826. In 1822 appeared the following work: "Euclid's Elements of Geometry, the first six books, to which are added the Elements of Plane and Spherical Trigonometry, a System of Conic Sections, Elements of Natural Philosophy as far as it relates to Astronomy, according to the Newtonian System, and Elements of Astronomy, with notes by Rev. John Allen, A. M., professor of mathematics at the University of Maryland." John D. Craig, in a notice of this book, says that Newton's work at this day is "almost a locked treasure among us," owing to the "scarcity of tracts giving the necessary preparatory knowledge." The object of this volume was to supply that want.

An English mathematician, whose works found their way across the ocean, was John Bonnycastle, professor of mathematics at the Royal

Military Academy, Woolwich. His Introduction to Algebra (London, 1782) was revised and edited in this country by James Ryan in 1822. Bonnycastle was a teacher of rules rather than principles.

An English author well-known in this country was Thomas Simpson. His Treatise on Algebra was published in Philadelphia in 1809. The second American from the eighth English edition, revised by David McClure, teacher of mathematics, came out in Philadelphia in 1821. As was frequently the case in those days, all demonstrations are here given by themselves in the manner of notes placed below a horizontal line on the page. They could be taken or omitted by teacher and pupil at pleasure, and were generally omitted. The author's explanations and demonstrations wanted simplicity, and we need not wonder if they were "looked upon, by some, as rather tending to throw new difficulties in the way of the learner than to the facilitating of his progress."

Another English algebra reprinted here was that of B. Bridge, fellow of St. Peter's College, Cambridge (second American edition from eleventh London, Philadelphia, 1839). We are informed that this work was introduced into the University of Pennsylvania, the Western University, Pittsburg, in Gummere's School at Burlington, the Friends' College at Haverford, and "a great number of the best schools in the United States." The Three Conic Sections, by the same author, was also patronized by some of our colleges. This subject was here treated purely synthetically, as was the case also in Robert Simson's Conic Sections, reprinted here in 1809 (?), and in all other English treatises of that time. Analytic methods, which proved so powerful in the hands of mathematicians on the Continent, were still underrated in England. The exclusive adherence to the synthetic method was due to an excessive worship of the views of Newton, who favored synthesis and employed it throughout his Principia.

We next mention Rev. Samuel Vince's Fluxions, printed in Philadelphia in 1812, or about twenty years after its first appearance in England. This seems to be the only work devoted exclusively to fluxions which was ever published in this country. Before the introduction of the Leibnitzian notation it was the text-book most generally used in our colleges, whenever fluxions were taught. An edition of Vince's Astronomy came out in Philadelphia in 1817. Of his other works, his "Conic Sections, as preparatory to the reading of Newton's Principia," was best known in America. Vince held the position of Plumian professor of astronomy and experimental philosophy in the University of Cambridge, England. His works generally lacked elegance, and failed to teach the more modern and improved forms of the mathematical science.

More prominent than any of the English authors here mentioned was Charles Hutton. His Course of Mathematics was edited in America by Robert Adrain, and will be spoken of again later.

HARVARD COLLEGE.

It has already been stated that the chair of mathematics and natural philosophy at Harvard was occupied from 1779 to 1788 by Rev. Samuel Williams, a pupil of John Winthrop. Williams wrote manuscript books on astronomy, mathematics, and philosophy. His mathematical manuscripts were probably studied in place of Ward's Mathematics, which had been used by his predecessor, John Winthrop. We possess hardly any information on the mathematical instruction during his time. The following is taken from the diary of a student who, in 1786, applied for admission to the third term of the Junior year: "Mr. Williams asked me if I had studied Euclid and arithmetic."* This question having been answered, apparently, in the affirmative, he was admitted. From this it would seem that at that time Euclid and arithmetic were the only mathematical studies pursued previous to the close of the Junior year. The fourth year, says Amory, seems to have been principally occupied in the study of mathematics. From a quotation given by Amory, we infer that algebra was a college study at this time. It had probably been so during the last fifty years, but we possess no data from which this could be positively affirmed.

A ray of light upon the inner workings of the college is thrown by quotations from the diary of a student who was at Harvard in 1786. They show that the tutors of the college failed to command the esteem and respect of the students. Complaints were made that the *Greek* tutor was too young. "Before he took his second degree, which was last commencement, he was chosen a tutor of *mathematics*, in which he betrayed his ignorance often." Of another tutor it is remarked: "We recite this week to our own tutor in Gravesand's Experimental Philosophy. This gentleman is not much more popular than the rest of the tutors." Whoever has observed the freedom with which college boys speak of their instructors, knows that statements like these must be taken with some allowance. But the practice alluded to above, of selecting graduates who had excelled mainly in *classical studies* as tutors in *mathematics*, seems absurd. And yet it is well known that this custom was continued even in some of our best colleges down to a comparatively recent date. The objections to the custom which existed at Harvard previous to 1767 are still more obvious. In the early days of Harvard each tutor taught all branches to the class assigned to him, throughout the whole collegiate course. But in 1767 the rule was introduced that one tutor should not teach all the subjects, but only one subject, such as Greek; another tutor should have Latin; another mathematics, physics, natural philosophy, etc.

In 1789 Samuel Williams was succeeded in the professorship of mathematics and natural philosophy by Samuel Webber. Webber engaged, while a boy, in agricultural pursuits, and at the advanced age of

* "Old Cambridge and New," by Amory, in North American Review, Vol. 114, 1872.

twenty, in 1780, he entered Harvard College. After graduating he remained two years at the college, studying theology. He then held a tutorship till his appointment to the Hollis professorship, in which office he spent seventeen of the most important years of his life. In 1806 he was elected president of Harvard. He occupied this position till his death, in 1810. Henry Ware, in his eulogy of Webber, says: "As a scholar his attainments were substantial, embracing various branches of learning, but, mathematical science being most congenial to his taste and habits, he quitted his professorship for the presidency with reluctance. In communicating instruction, he united patience and facility with a thorough acquaintance with his subject." Edward Everett, who was a student at Harvard in Webber's time, makes a somewhat different estimate of him, saying that Webber was "reputed a sound mathematician of the old school, but rather too much given to routine."* In another place, Everett speaks of him in the same way as "a person of tradition and routine." Judge Story says of him, "Professor Webber was modest, mild, and quiet, but unconquerably reserved and staid."†

In 1787, just before Samuel Webber was elected professor, the course of studies at Harvard was revised, with a view of raising the standard of learning. According to the new scheme, the classics "formed the principal study during the first three college years. The Freshmen were instructed, also, in rhetoric, the art of speaking, and *arithmetic*; the Sophomores in *algebra, and other branches of mathematics*; the Juniors in Livy, Doddridge's Lectures, and once a week, the Greek Testament; the Seniors in logic, metaphysics, and ethics."‡

The elementary mathematics were now studied in the first half of the college course instead of the second half. The Freshmen and Sophomores now began taking mathematics, though, we fear, only in ineffectual homœopathic doses. If arithmetic was begun in the Freshman year, then we may be sure that no very extensive course could have been given before the close of the Sophomore year. According to Judge Story, Saunderson's Algebra was used in 1795 or 1796. The original work of this blind mathematician was extensive, and in two volumes. The book used at Harvard consisted most likely of "Selected parts of Professor Saunderson's Elements of Algebra," published in one volume.§

W. Williams, a classmate of Channing (class of 1798), says: "The Sophomore year gave us Euclid to measure our strength. Many halted at the '*pons asinorum*.' But Channing could go over clear at the first trial, as could some twelve or fifteen of us. This fact is stated to show that he had a mind able to comprehend the abstrusities of mathematics,

* Old Cambridge and New, Vol. IV, p. 199.

† Memoir of W. E. Channing, by W. F. Channing, fourth ed., 1850, Vol. I, p. 47.

‡ Quincy's History of Harvard University, Vol. II, p. 279.

§ Third edition, London, 1771.

though, to my apprehension, he excelled more decidedly in the Latin and Greek classics, and had a stronger inclination to polite literature."

We are, moreover, told of Channing: "He delighted, too, in geometry, and felt so rare a pleasure in the perception of its demonstrations that he took the fifth book of Euclid with him as an entertainment during one vacation." Such experiences are frequent with a student of advanced mathematics, but, unfortunately, too rare with pupils studying the elements.

If the course given by Quincy be supposed complete, then no mathematics was studied after the Sophomore year. This was probably not true; it certainly was not true ten years later. In 1797, at least some of the students pursued the more advanced mathematics during the latter part of the college course. A glimpse of light on this subject is thrown by the following quotation from the eulogy of John Pickering: "Great as was his enthusiasm for classical learning, he had in college as real a love for the study of mathematics, and highly distinguished himself in this department. Near the close of his Senior year he received the honor of a mathematical part, which appeared to give him more pleasure than all his other college honors. It afforded him an opportunity to manifest his profound scholarship in a manner most agreeable to his feelings. When he had delivered the corporation and overseers this part, containing solutions of problems by fluxions, he had the rare satisfaction to be told that one of them was more elegant than the solution of the great Simpson, who wrote a treatise on fluxions, in which the same problem was solved by him." It follows from this that provisions were made for the study of fluxions, at least for students who may have desired to study them.

Of the mathematical theses, written by Juniors and Seniors, which have been deposited in the Harvard Library, one hundred and thirty-three were written during the period from 1781 to 1807. Of these, the great majority are on the calculation and projection of eclipses. Surveying and the algebraic solution of problems receive also a large amount of attention. Of the one hundred and thirty-three theses only seven show by their titles that they contain "fluxionary problems." Their dates are, 1796, 1803 (two), 1804, and 1806 (three). After 1807, theses containing solutions of problems on fluxions are quite numerous. It may be of interest to state that John Farrar, the future professor of mathematics at Harvard, wrote in 1803 a thesis on the "Calculation and Projection of a Solar Eclipse." James Savage, the great authority on American genealogy, furnished a colored view of churches and college buildings; Everett, the diplomatist, a colored "*Templi Episcopalis Delineatio Perspectiva.*" One thesis is on the "Calculation and Projection of a Solar Eclipse which took place in the year of the Crucifixion."*

* Biographical Contributions of the Library of Harvard University, No. 32.

In 1802 the standard for admission to Harvard College was raised. In mathematics a knowledge of *Arithmetic* to the "Rule of Three" was required. Thus, in 1803, for the first time had it become necessary, according to regulations, for a boy to know something about arithmetic before he could enter Harvard. We surmise, however, that the requirements in arithmetic were very light, for we know from the diary of a student in the Freshman class in 1807 that arithmetic continued to be taught during the first year at college.* After 1816 the whole of arithmetic was required for admission.

From the beginning of the nineteenth century on, we can get more definite information regarding the extent to which the mathematical studies were pursued. We need only examine the college text-books which began then to be printed in this country. The earliest mathematical text-books for colleges, written by an American author, are those of Samuel Webber. In 1801 were published in two volumes his "Mathematics, compiled from the best authors and intended to be a text-book of the course of private lectures on these sciences in the University of Cambridge." A second edition appeared 1808. These works were for a time almost exclusively used in New England colleges, but they finally gave place to translations from French works, executed by John Farrar, the successor of Webber in the professorship of mathematics.

Within two volumes, each of 460 pages and in large print, are embraced the following subjects: arithmetic, logarithms, algebra, geometry, plane trigonometry, mensuration of surfaces, mensuration of solids, gauging, heights and distances, surveying, navigation, conic sections, dialing, spherical geometry, and spherical trigonometry. Some idea of the extent to which each branch of mathematics was carried may be obtained, if we state that in Webber's works 124 pages were given to algebra, while Newcomb's Algebra, for instance, numbers 545 pages. The subject of conic sections was disposed of by Webber within 68 short pages, while Wentworth's Analytic Geometry covers 273 crowded pages. Comparatively much space was given by Webber to the applications of mathematics, such as gauging, heights and distances, surveying, navigation, and dialing. These practical subjects received much more attention then than they are now receiving in the academic department in the majority of our colleges. Webber devotes only 47 pages to the important and extensive subject of geometry, and gives solutions of geometrical problems, but no theorems. This apparent neglect of the oldest and most beautiful of mathematical sciences is explained by Webber in the second edition of his work. In a foot-note (p. 339) he says that "A tutor teaches, in Harvard College, Playfair's Elements of Geometry, containing the first six books of Euclid, with two books on the geometry of solids. Of this work Mr. F. Nichols, of Philadelphia, has given a good American edition" (1806). Webber's chapter on geometry was, therefore, intended simply as a book with problems to accompany or

* "Old Cambridge and New," by Amory, North American Review, Vol. 114, p. 118.

follow Euclid. If the course in elementary geometry was taught as here indicated, then it can hardly be said that this subject was neglected. Wherever Euclid is diligently studied, there geometry is not slighted.

Of John Farrar, the successor of Webber in the chair of mathematics and natural philosophy, we shall speak when we consider the influx of French mathematics into the United States.

Before leaving Harvard College we shall quote two short passages taken from the Harvard Lyceum. This journal was a publication by the students, and was the earliest of that kind at this college. The quotations about to be given disclose an effort to arouse interest among students in mathematical studies. In the first number, which appeared July 14, 1810, we read as follows: "The dry field of mathematics has brought forth most ingenious and elegant essays, most curious and entertaining problems. It is our wish to construct or select such questions in their various branches as may exercise the skill of our correspondents in their solution." This promise was not strictly kept. Mathematical enthusiasm could not be aroused quite so easily. There is to be found, to be sure, an ingenious essay on mathematical learning, presumably written by a Sophomore, in which we read: "Perhaps no science has been so universally decried by the overweeningly dull as the mathematics. Superficial dabblers in science, contented to float in doubts and chimeras, and unable to see the advantage of demonstrable truth, turn back before they have passed the narrow path which leads to the firm ground of mathematical certainty, and not willing to have others more successful than themselves, like the Jewish spies, they endeavor to deter them from the way by horrid stories of giant spectres in the promised land of demonstration, and scarcely a Caleb is found to render a true account of its beauties." But the Jewish spies were too eloquent, and there was no Caleb to furnish curious and entertaining problems.*

YALE COLLEGE.

The chair of mathematics and natural philosophy at Yale College was established in 1770. Its first incumbent was Nehemiah Strong, who occupied it till 1781. In 1794 Josias Meigs was appointed to the position. Meigs was graduated at Yale in 1778, and served as tutor in mathematics, natural philosophy, and astronomy, from 1781 to 1784. In 1783 he was admitted to the bar, and some years later practiced in Bermuda. He appeared as a defender of American vessels that were captured by British privateers, and was, in consequence, tried for treason. He was professor at Yale until 1801, teaching mathematics, natural philosophy, and chemistry.† He then became president of the

* The Harvard Book, by F. O. Vaille and H. A. Clark, Vol. II, p. 174.

† Prof. Benjamin Silliman (class of 1796) says, that on November 4, 1795, "Mr. Meigs heard the class recite at noon, as Dr. Dwight is out of town. Although Mr. Meigs is a very sensible man and very well calculated for the office which he now fills, still it is very easy to make a contrast between him and the president; but I am

University of Georgia. In 1812 he was appointed surveyor-general, and two years later, Commissioner of the General Land Office of the United States.

An important event at this period was the growth, among students, of a love for rhetoric and literature. English literature had hitherto been quite neglected. A taste for this study was excited by two young men, John Trumbull and Timothy Dwight, who were elected tutors in 1771. John Trumbull published, during the first year of his tutorship, a poem entitled the "Progress of Dulness," a satire, intended to expose the absurdities then prevailing in the system of college instruction. Ancient languages, mathematics, logic, and divinity received, in his opinion, an altogether disproportionate amount of time. In his poems, he introduces "Dick Hairbrain," a town fop, ridiculous in dress and empty of knowledge, and speaks of him as follows:

"What though in algebra, his station
Was negative in each equation;
Though in astronomy survey'd,
His constant course was retrograde;
O'er Newton's system though he sleeps,
And finds his wits in dark eclipse!
His talents proved of highest price
At all the arts of card and dice;
His genius turn'd with greatest skill,
To whist, loo, cribbage, and quadrille,
And taught, to every rival's shame,
Each nice distinction of the game."

Timothy Dwight's love for literature did not entirely displace his interest for mathematics. On the contrary, we read in a life of him by his son that, "In addition to the customary mathematical studies, he carried them [his students] through spherics and fluxions, and went as far as any of them would accompany him into the Principia of Newton." "This, however, must have been a very rare thing," says President Woolsey. Dwight was tutor at the college for six years. To exhibit his continued interest in mathematics during that time we quote from the biography of him the following passage: "At a subsequent period, during his residence in college as a tutor, he engaged deeply in the study of the higher branches of the mathematics. Among the treatises on this science to which his attention was directed, was Newton's Principia, which he studied with the utmost care and attention, and demonstrated, in course, all but two of the propositions in that profound and elaborate work. This difficult but delightful science, in which the mind is always guided by *certainty* in its discovery of truth, so fully engrossed his attention and his thoughts that, for a time, he

doubtful whether the comparison is not a false one, because the president is one of those characters which we very seldom meet with in the world, and who form its greatest ornaments." (Barnard's Educational Journal, vol. 25, 1876, International Series, vol. 1, p. 230.)

lost even his relish for poetry; and it was not without difficulty that his fondness for it was recovered."

It will be remembered that the Mathematics of Ward had been introduced before the Revolution. In 1788 Nicholas Pike's arithmetic was adopted. Soon after 1801 Samuel Webber's Mathematics displaced the works previously used, excepting Euclid, which was presumably used during this whole period as the text-book in geometry. At about the beginning of this century the mathematical course was as follows: Freshmen, Webber's Mathematics; Sophomores, Webber's Mathematics and Euclid's Elements; Juniors, Enfield's Natural Philosophy and Astronomy, and Vince's Fluxions; Seniors, natural philosophy and astronomy.

A strong impetus to the study of mathematics in American colleges was given by Jeremiah Day. He graduated from Yale in 1795, became tutor there in 1798, and was elected professor of mathematics and natural philosophy in 1801. Feeble health prevented him from entering upon the duties of his professorship till 1803; but after that he continued in them till 1817, when he succeeded President Dwight in the presidency of the college. The chair was then given to Alexander Metcalf Fisher.

At the beginning of this century the great want of this country in the department of pure mathematics was adequate text-books. Professor Webber, of Harvard, was the first who attempted to supply this want. In those colleges in which a single system of mathematics had been adopted, preference was generally given to the "Mathematics" of Webber. But his compilation was rather imperfectly adapted to the purposes for which it was made. It was not sufficiently copious. Many topics, though strictly elementary and practically important, were passed over in silence. The method of treatment was too involved and the style not sufficiently clear to make the subject attractive to the young student. Accordingly Professor Day set himself to work to write a series of books which should supply more adequately the needs of American colleges. In 1814 appeared his Algebra, and his Mensuration of Superficies and Solids, in 1815 his Plane Trigonometry, and in 1817 his Navigation and Surveying. It was the original intention of the author to prepare also elementary treatises on conic sections, spherics, and fluxions, but on his elevation to the presidency he abandoned this design. His Algebra passed through numerous editions, the latest of which was issued in 1852, by the joint labors of himself and Professor Stanley.

Day's books are very elementary, and introduce the student by easy and gradual steps to the first principles of the respective branches. To us of to day, they appear too elementary for college use, but it must not be forgotten that at the time they were prepared, they were just what was needed to meet the demands of the times. Students applying for matriculation in those days had received very defective prepara-

tory training, especially in mathematics. With such unwrought material before him, it was natural for the teacher to show his preference for a text-book in which every process of development and reasoning was worked out patiently and minutely through all its successive steps. Day took for a model the diffuse manner of Euler and Lacroix, rather than the concise and abridged mode of the English writers. The great danger in this course is that no obstacles are left to be removed by the student through his own exertion. In the opinion of some teachers, Day has laid himself open to criticism by carrying the principle of making mathematics *easy* somewhat too far. It is no little praise for a book written at that time to say that, unlike most books of that period, Day's mathematics did not encourage the cramming of rules or the performing of operations blindly. On the contrary, the diligent student acquired from them a rational understanding of the subject.

Day's mathematics were at once everywhere received with eagerness. They were introduced in nearly all our colleges. Even at the end of a period of fifty years they still held their place in many of our schools. In view of these facts, "it may safely be said that the value of what their author did by means of them for the college and for the country at large, while holding the office of professor from 1803 to 1817, the time when he succeeded Dr. Dwight, was not surpassed by anything in science and literature which he did subsequently during his long term of office as president of the college."*

As a teacher and writer, President Day was distinguished for the simplicity and clearness of his methods of illustration. His kind-heartedness and urbanity of demeanor secured the love and respect both of friends and pupils.

He was succeeded in the chair of mathematics and natural philosophy by Alexander Metcalf Fisher, who held it until his death by drowning in 1822, at the shipwreck of the *Albion*, off the Irish coast. Fisher possessed extraordinary natural aptitudes for learning. He had prepared a full course of lectures in natural philosophy, both theoretical and experimental, which were marked for their copiousness and their exact adaptation to the purpose of instruction. His clear conception of what a text-book should be is well shown in his review of Enfield's Philosophy.†

Regarding the course in natural philosophy at Yale, it may be remarked, that in 1788 Martin's Philosophy, which had gone out of print, was succeeded by Enfield's Natural Philosophy, first published in 1783. William Enfield was a prominent English dissenter. He preached in

* Yale College: A Sketch of its History, by William Kingsley, Vol. I, p. 115.

† American Journal of Science, Vol. III, 1821, p. 125. In Vol. V, p. 83, of the same journal, is an article by him, "On Maxima and Minima of Functions of Two Variable Quantities." He contributed solutions to questions in the American Monthly Magazine, and in Leybourn's Mathematical Repository (London). The fourth volume of the Memoirs of the American Academy of Arts and Sciences contains observations by him on the comet of 1819 and calculations of its orbit.

Unitarian churches and published several volumes of sermons. Being engaged chiefly in theological studies, comparatively little attention was paid by him to the exact sciences. Nevertheless, he succeeded in compiling a work on natural philosophy which possessed elements of popularity and was used in our American colleges for four decennia. In 1820 appeared the third American edition of this work, which was then used by nearly all the seminaries of learning in New England, notwithstanding the fact that, excepting in electricity and magnetism and a few particulars in astronomy, it presented hardly any idea of the progress made in the different branches of philosophy since the period of Newton.

UNIVERSITY OF PENNSYLVANIA.

The University of Pennsylvania, which had such a remarkable growth under the administration of Dr. William Smith, before the Revolutionary War, had a comparatively small attendance of students after the war, and the college department is said to have been quite inferior to that of the leading American colleges of that time.

An educator who was long and prominently connected with this institution and whose activity was directed towards maintaining and raising its standard, was Robert Patterson, the elder. He was born in 1743 in Ireland, and at an early age showed a fondness for mathematics. In 1768 he emigrated to Philadelphia. He first taught school in Buckingham, and one of his first scholars was Andrew Ellicott, who afterward became celebrated for his mathematical knowledge displayed in the service of the United States.

About this time Maskelyne, the astronomer royal of England, compiled and published regularly the Nautical Almanac. This turned the attention of the principal navigators in American ports to the calculations of longitude from lunar observations, in which they were eager to obtain instruction. Patterson removed to Philadelphia, began giving instruction on this subject, and soon had for his scholars the most distinguished commanders who sailed from this port. Afteward he became principal of the Wilmington Academy, Delaware, and in 1779 was appointed professor of mathematics and natural philosophy at the University of Pennsylvania, which post he filled for thirty-five years. He was also elected vice-provost of that institution.*

Robert Patterson communicated several scientific papers to the Philosophical Transactions (Vols. II, III, and IV), and was a frequent contributor of problems and solutions to mathematical journals. He edited James Ferguson's Lectures on Mechanics (1806), and also Ferguson's "Astronomy explained upon Sir Isaac Newton's principles and made easy to those who have not studied mathematics" (1809). Fer-

* Transactions of American Philosophical Society, Vol. II, New Series, Obituary Notice of Robert Patterson, LL. D., late President of the American Philosophical Society.

guson was a celebrated lecturer on astronomy and mechanics in England, who contributed more than perhaps any other man there to the extension of physical science among all classes of society, but especially among that largest class whose circumstances preclude them from a regular course of scientific instruction. His influence was strongly felt even in this country, as is seen from the American editions by Robert Patterson of his astronomy and mechanics. Patterson wrote a small astronomy, entitled the Newtonian System, which was published in 1808. Ten years later he published an arithmetic, elaborated from his own written compends, previously used in the University. Though lucid and ingenious, this arithmetic was rather difficult for beginners, and never reached an extended circulation.

It is believed by many that mathematicians generally possess a strong memory for numbers. This was certainly not true of Patterson, for we are told that he could not remember even the number of his own house. He met this dilemma by devising a *mnemonic*, which was indeed worthy of a mathematician. The number of his residence was 285, which answered to the following conditions: "The second digit is the cube of the first, and the third the mean of the first two." It is to be wondered that, during some fit of intense abstraction, the learned professor did not pronounce 111 to be the number of his house, instead of 285; for 111 is a number satisfying the above conditions quite as well as 285.

When Robert Patterson resigned his position at the University of Pennsylvania in 1814, he was succeeded by his son, Robert M. Patterson. The latter was graduated at the University in 1804. After receiving the degree of M. D., in 1808, he pursued professional studies in Paris and London. In 1814 he was appointed professor of mathematics and natural philosophy, which office he filled until 1828, when he accepted the chair of natural philosophy at the University of Virginia. Robert M. Patterson published no mathematical books.

From 1828 to 1834 the chair of mathematics and natural philosophy was filled by Prof. Robert Adrain. The days of greatest activity of this most prominent teacher of mathematics were spent at other institutions, but we take this opportunity of introducing a sketch of his life.*

Robert Adrain was born in Ireland. At the age of fifteen he lost both his parents, and thenceforward he supported himself by teaching. At the end of an old arithmetic he found the signs used in algebra. His curiosity becoming greatly excited to discover their meaning, he gave himself no rest until at last he found out what they meant. In a short time he was able to resolve any sum in the arithmetic by algebra. Thenceforth he devoted himself with enthusiastic ardor to mathematics. He took part in the Irish rebellion of 1798, received a severe wound, and

* This sketch is extracted from an article in the Democratic Review, 1844, Vol. XIV.

escaped to America. Immediately after his arrival he began teaching in New Jersey. After two or three years he became principal of the York County Academy in Pennsylvania. He then began contributing problems and solutions to the Mathematical Correspondent, a journal published in New York. This was the means of bringing his mathematical talents before the public. He obtained several prize medals, awarded for the best solutions.

In 1805 he moved to Reading, Pa., to take charge of the academy of that place. He started there a mathematical journal called the Analyst. The first number was published in Reading, but its typographical execution disappointed him so much that he employed a publisher at Philadelphia and incurred the extra expense of a republication. We shall speak of this journal again later.

In 1810 he was called to the professorship of mathematics and natural philosophy at Queen's (now Rutgers) College; and, in 1813, to the professorship of mathematics at Columbia College. In New York he became the center of attraction to those pursuing mathematical studies. A mathematical club was established, in which he shone as the great luminary among lesser lights. As a teacher, he had a most happy faculty of imparting instruction.

In 1826 the delicate state of his wife's health induced him to leave Columbia College in New York and to remove to the pure air and healthful breezes of the country near New Brunswick. About two years later he was induced to accept the professorship of mathematics at the University of Pennsylvania, a position which had been held at the beginning of the century by the well-known Robert Patterson. Adrain became also vice-provost of this institution.

He resigned this position in 1834 and returned to his country seat near New Brunswick, intending to pass his time with his family and in study. But he did not remain there long, for the habit of teaching had become too strong easily to be resisted. He moved to New York and taught in the grammar school connected with Columbia College until within three years of his death. At this time his mental faculties began very perceptibly to fail. He greatly lamented their decay, and, one day when a friend called in to see him, he had a volume of La Place on his lap endeavoring to read it. "Ah," said he, in a melancholy tone of voice, "this is a dead language to me now; once I could read La Place, but that time has gone by." He died in 1843.

Among American mathematicians of his day, Robert Adrain was excelled only by Nathaniel Bowditch. Of his many contributions to mathematical journals, one of the earliest was an essay published in 1804 in the Mathematical Correspondent on Diophantine analysis. This was the earliest attempt to introduce this analysis in America.

In 1808 Adrain began editing and publishing the Analyst, or Mathematical Museum. At that time he had not yet entered upon his career as college professor. The above periodical contained chiefly solutions

to mathematical questions proposed by the various contributors. It was a small, modest publication, which had only a very limited circulation in this country, and was unknown to foreign mathematicians. It lived, moreover, only a very short time, for only five numbers ever appeared. And yet, this apparently insignificant little journal, edited by a teacher at an ordinary academy, contained one article which was an original contribution of great value to mathematical science. It was, in fact, the first original work of any importance in pure mathematics that had been done in the United States. I refer to Robert Adrain's deduction of the Law of Probability of Error in Observation. The honor of the first statement in printed form of this law, commonly known as the Principle of Least Squares, is due to the celebrated French mathematician Legendre, who proposed it in 1805 as an advantageous method of adjusting observations. But upon Robert Adrain falls the honor of being the first to publish a demonstration of this law. He does not use the term "least squares," and seems to have been entirely unacquainted with the writings of Legendre. It follows, therefore, that not only the two deductions of this principle given by Adrain were original with him, but also the very principle itself.

We now give the history of this discovery by Adrain. Robert Patterson, of the University of Pennsylvania, proposed in the Analyst the following prize question: "In order to find the content of a piece of ground, * * * I measured with a common circumferentor and chain the bearings and lengths of its several sides, * * * but upon casting up the difference of the latitude and departure, I discovered * * * that some error had been contracted in taking the dimensions. Now, it is required to compute the area of this inclosure *on the most probable supposition* of this error." This was proposed in No. II of the Analyst, and after being a second time renewed as a prize question in No. III, it was at length, in No. IV, solved by a course of special reasoning by Nathaniel Bowditch, to whom Adrain awarded the prize of ten dollars. Immediately following Bowditch's special solution, the editor, Adrain, added his own solution of the following more difficult general problem: "Research concerning the probabilities of the errors which happen in making observations."* This paper is of great historical interest, as containing the first deduction of the law of facility of error.

$$\varphi(x) = ce^{-hx^2}$$

$\varphi(x)$ being the probability of any error x, and c and h quantities de-

* Analyst, No. IV, pp. 93–97. Copies of this journal are very rare. No. IV is to be found in the Congressional Library in Washington; No. III and No. IV are in the Library of the American Philosophical Society, Philadelphia. Adrain's first proof of the Principle of Least Squares was re-published by Cleveland Abbe in the American Journal of Science and Arts, third series, 1871, pp. 411–415. Adrain's second proof was re-published by Mansfield Merriman in the Transactions of the Connecticut Academy, Vol. IV, 1887, p. 164; also in the Analyst (edited and published by J. E. Hendricks, Des Moines, Iowa), Vol. IV, No. II, p. 33.

pending on the precision of the measurement. Adrain gives two proofs of this law. The first proof depends upon the "self-evident principle," as he calls it, that the true errors of measured quantities are proportional to the quantities themselves. The arbitrary nature of this assumption is pointed out by J. W. L. Glaisher in the Memoirs of the Royal Astronomical Society, Part II, vol. 39, 1871–72. "This," says Glaisher, "seems very far from being evident, not to say very far from being true, generally. One would expect a less relative error in a greater distance." Glaisher raises other objections to Adrain's first proof, and then pronounces it entirely inconclusive. Adrain's second proof, which is essentially the same as that given later by John Herschel, and usually called Herschel's proof, is likewise defective, as has been pointed out by Prof. Mansfield Merriman.

In order to place these criticisms on Adrain's two demonstrations in the proper light, it should be remarked here that the subject of which they treat is one of great difficulty. There has been great difference of opinion among leading mathematicians as to what *assumptions* regarding the nature of errors can be safely and legitimately made, and taken as a basis upon which to construct demonstrations and what ones should be rejected as being false or as demanding demonstration.

Subsequently to Adrain's paper, proofs were published by Gauss, La Place, Bessel, Ivory, John Herschel, Tait, Donkin, and others. Altogether, there appeared over a dozen distinct proofs, but all of these "contain, to say the least, some point of difficulty" (Glaisher). If, therefore, it be said that Adrain's proofs are inconclusive, we must remember that all other proofs hitherto given possess to a greater or less degree the same defect.

The number of the Analyst which gives Adrain's two proofs contains also the following applications of this method: (1) To find the most probable value of any quantity of which a number of direct measurements is given; (2) to find the most probable position of a point in space; (3) to correct dead-reckoning at sea; to correct the bearings and distances of a field survey.

At the close of the article he says: "I have applied the principles of this essay to the determination of the most probable value of the earth's ellipticity, etc., but want of room will not permit me to give the investigation at this time." It was published nine years later in Volume I, new series, of the Transactions of the American Philosophical Society (papers No. IV and XXVII). In the first paper he finds the earth's ellipticity to be $\frac{1}{313}$ instead of $\frac{1}{376}$, as was erroneously given by La Place (La Mécanique Céleste, Vol. III). In the second paper Adrain applies his rule to the evaluation of the mean diameter of the earth, which he finds to be 7,918.7.

His rule for correcting dead-reckoning at sea was adopted by Dr. Bowditch in his last edition of his Practical Navigator. Adrain's rule for correcting a survey is referred to by John Gummere in his Survey-

ing, as having been given and demonstrated by Bowditch and Adrain in the Analyst.

It thus appears that these rules of Adrain were made use of by at least some of the contemporary American mathematicians, but the principles from which these rules were deduced and the demonstrations of these principles appear to have excited little attention, and to have been soon forgotten. Foreign mathematicians never heard of Adrain's investigations on the subject of least squares until Adrain's first proof and extracts from other papers were reprinted by Cleveland Abbe in the American Journal of Science and Arts in 1871, or sixty years after their first publication in the Analyst. By a very strange oversight Cleveland Abbe does not even mention Adrain's second proof. The existence of this proof was pointed out, however, a few years later by Prof. Mansfield Merriman.

It is not much to the credit of American mathematicians that they should have permitted theoretical investigations of such great value to remain so long in obscurity. Let justice be done to Adrain, and let him be credited "with the independent invention and application of the most valuable arithmetical process that has been invoked to aid the progress of the exact sciences."

By the numerous elegant solutions which Adrain contributed to mathematical journals in this country, by his labors as teacher at Rutgers College, Columbia College, and the University of Pennsylvania, by his editions of Hutton's Mathematics, he contributed powerfully to the progress of mathematical studies in the United States. His first edition of Hutton's Mathematics was followed in 1812 by a second edition, and in 1822 by a third edition, in which he added an elementary treatise of sixty pages on descriptive geometry, "in which the principles and fundamental problems are given in a simple and easy manner." Other editions came out later. Adrain's edition of Hutton contained improvements in method and important corrections, the value of which was recognized by Mr. Hutton himself.

It may be well to call to mind at this place that Charles Hutton was professor of mathematics at the Royal Military Academy at Woolwich from 1773 to 1805. His course of mathematics was published in London, 1798–1801. In its day this work was doubtless the best of its kind in the English language. But at that time the English were far behind the French in the cultivation and teaching of mathematics. Hutton's course was plain and simple, but defective both in extent and analysis. The English works of that day generally contained rules without principles, and were decidedly inferior to the explanatory treatises of Lacroix and Bourdon, then used in France. Hutton's mathematics were used once at our own Military Academy at West Point, but were soon exchanged for the more analytical and copious treatises by French authors.

We close our remarks on Robert Adrain with the following quotation

from the Democratic Review of 1844, Vol. XIV: "He published little, because he was too severe a critic upon his own writings. He would revise and re-revise. It is said that while at Columbia, he had a treatise on the differential and integral calculus all written out and ready for the press; but upon giving it a further revision he became dissatisfied with some parts of it, and committed the whole to the flames." He left a number of manuscripts with commentaries on the Mécanique Analytique of Lagrange and the Mécanique Céleste of La Place.*

COLLEGE OF NEW JERSEY (PRINCETON).

The College of New Jersey first opened at Elizabethtown, in 1746. Soon after, it was transferred to Newark, and in 1756 to Princeton. About seventy students moved from Newark to Princeton. The first president died after having been in office less than a year. His successor, Aaron Burr, the elder, held the post for ten years. He was an incessant worker and toiler. Though he was assisted by two tutors, he was himself teacher in Greek, logic, ontology, natural philosophy, *and in the calculation of eclipses.*† The courses in physics were illustrated by apparatus which had been obtained from Philadelphia. Popular lectures were delivered on the new electricity, and both president and students repeated Franklin's experiment on the influence of pressure on the boiling point, with glass globes of their own.

At first the college had no professors with fixed functions and permanent salaries. The instruction in classics and mathematics was committed to tutors who had lately graduated and were preparing for the ministry. They taught generally for but few years.

The first professor of mathematics and natural philosophy was William Churchill Houston. In early manhood he entered Princeton College, taught in the college grammar school, and was graduated in 1768. He was then appointed tutor, and, in 1771, elected professor. At the beginning of the Revolutionary War, he and Dr. Witherspoon were the only professors in the college. When Princeton was invaded in 1776, and the college was closed, he took active part in the war. As soon as quiet was restored at Princeton, he resumed his college duties. Soon after he was sent as a representative to Congress. He resigned his chair in 1783. In midst of his many duties, he had acquired a sufficient knowledge of law to be admitted to the bar. As a lawyer he soon acquired great reputation.

John Adams visited Princeton College in the opening days of the Revolution, when he was on his way to the Continental Congress. In his diary of August 26, 1774, he says: "Mr. Euston,‡ the professor of mathematics and natural philosophy, showed us the library; it is not large, but has some good books. He then lead us into the apparatus

*For want of the necessary material, our sketch of the University of Pennsylvania will not be continued.

† The College Book, by Charles F. Richardson and Henry A. Clark, 1878, p. 37.

‡ *Mr. Houston* was probably intended.

room; here we saw a most beautiful machine—an orrery or planetarium, constructed by Mr. Rittenhouse, of Philadelphia." It will be remembered that both the University of Pennsylvania and Princeton College had been negotiating for the first orrery made by Rittenhouse, and that Princeton carried it off, much to the chagrin of Dr. William Smith, the president of the University of Pennsylvania.

The chair of mathematics and natural philosophy was filled two years after the resignation of Houston by the appointment of Ashbel Green. He was a native of New Jersey, and was graduated at Princeton College in 1783. He entered the ministry, and was professor for the two years succeeding 1785. Later, he became president of the institution.

Green's successor was Dr. Walter Minto, a Scotch mathematician of eminence. He was graduated at the University of Edinburgh, and then became tutor to the sons of George Johnstone, a member of Parliament. With them he travelled over much of Europe. In Pisa he became acquainted with Dr. Slop, the astronomer, and through him with the then novel application of the higher analysis to planetary motion. After returning to Scotland he became teacher of mathematics at Edinburgh. He came to the United States in 1786, and one year after became connected with Princeton College. Before coming to America he published a Demonstration of the Path of the New Planet; Researches into Some Parts of the Theory of Planets; and (with Lord Buchan) an Account of the Life, Writings, and Inventions of Napier of Merchiston.

While at Princeton, he delivered on the evening preceding the annual commencement of the year 1788 "an inaugural oration on the Progress and Importance of the Mathematical Sciences." In this address he traces the history of mathematics down to the time of Newton, then directs his remarks to the students and trustees, emphasizing the importance of mathematical study. "The genius of Newton," says he, "had he been born among the Indians, instead of discovering the laws of the universe, would have been limited to the improvement of the instruments of hunting, or to the construction of commodious wigwams." At the time when this address was delivered he had been professor at Princeton about a year. Near the close of his oration he says: "It gives me a deal of pleasure, gentlemen, to have occasion to observe, in this public manner, that a considerable majority of those of you who have studied the mathematics under my direction have acquitted yourselves even better than my expectations, which, believe me, were very sanguine." This inaugural address is his only publication while he was connected with Princeton College, but the college library contains some careful and curious MSS. on mathematical analysis written by him. Doctor Minto died at Princeton in 1796.

The mathematical duties were now assigned to Dr. John Maclean, a native of Scotland. In his day he was one of the most distinguished professors at Princeton, "the soul of the faculty." His specialty was chemistry, which he had studied in Paris. He is said to have been one

of the first to reproduce in America the views of the new French school in chemistry. During seven years he was professor not only of chemistry, but also of natural history, mathematics, and natural philosophy; and after a short interval of four years, during which he was relieved from mathematical instruction by the appointment of Dr. Andrew Hunter to those duties, he again assumed charge of all the scientific instruction given to the students. He died in 1814.

From 1812 to 1817, Elijah Slack, a graduate of Princeton and a minister, was professor of natural philosophy and chemistry. He taught also mathematics. He was afterwards president of Cincinnati College. Henry Vethake taught mathematics from 1817 to 1821. In 1823, Mr. John Maclean, a young man of only twenty-three years, was made professor of mathematics.

It may here be remarked that in the library of Princeton College there is a folio volume of great interest and value, containing a copy of the first printed edition of Euclid's Elements in Greek (Basle, 1533); the commentary of Proclus on the First Book of Euclid (Basle, 1533); a twofold Latin translation (Basle, 1558)—one the Adelard–Campanus version, from the Arabic, the other the first translation into Latin from the Greek from Theon's Revision. This folio was once the property of Henry Billingsley, who three hundred years ago made the first translation of Euclid into English. By the examination of this folio, Dr. G. B. Halsted was able to show that the first English translation was made from the Greek, and not, as was formerly supposed, from any of the Arabic–Latin versions.*

DARTMOUTH COLLEGE.

Dartmouth College, at Hanover, was chartered in 1769. Wheelock was the first president, and his first associate in instruction as tutor was Bezaleel Woodward, who had graduated at Yale in 1764, during the presidency of Clap, of whom it was said that in mathematics and natural philosophy "he was not equalled by more than one man in America."

Three of Dartmouth's first class were prepared for college at the "Indian Charity School" in Lebanon, and passed their first three years at Yale.

The facilities for acquiring a classical and scientific education appear to have been substantially the same at Dartmouth, at the outset, as in other American colleges of that period.† Some notion regarding the mathematical course at this college may be drawn from a letter written in 1770 by Nathan Teasdale, a learned and indefatigable teacher in eastern Connecticut, to Dr. Wheelock, the president of the college, introducing one of Teasdale's pupils, who applied for admission to the Senior year. The young man is described as "a genius somewhat

*Note on the First English Euclid, American Journal of Mathematics, Vol. II, 1879.

† History of Dartmouth College, by B. P. Smith, p. 58.

better than common," who "had made excellent progress." "In arithmetic, vulgar and decimal, he is well versed. I have likewise taught him trigonometry, altimetry, longimetry, navigation, surveying, dialing, and gauging." "He likewise studied Whiston's Astronomy, all except the calculations." We are, probably, not far from the truth, if we conclude that the studies here enumerated were, in substance, the mathematics then studied at Dartmouth during the first three years.

The first twelve or thirteen years were years of very great trial for Dartmouth. The funds of the college were small and the students few. The Revolutionary War, though it did not interrupt the college exercises and disperse the students, must have diminished their number and affected their spirits. As in other localities, so in New Hampshire, the means of fitting for college were very imperfect and many of the college studies were inadequately pursued. "I remember," says Samuel Gilman Brown,* "hearing one of the older graduates say that the first lesson of his class in mathematics was twenty pages in Euclid, the instructor remarking that he should require only the captions of the propositions, but if any doubted the truth of them he might read the demonstrations, though for *his* part his mind was perfectly satisfied." In stories like this, however, we must allow something for the genius of the narrator. This story, if not true, is certainly of the *ben trovato* sort. The requirements for admission to American colleges in those days were low, and the system of choosing the tutors, to whose care the Freshmen and Sophomore classes were entirely committed, was enough to destroy any chances of rectifying the errors of bad and insufficient preparation. Not unfrequently a fresh graduate who had excelled in classics alone, with very little taste for mathematics, would be chosen to fill a tutorial vacancy requiring him to teach mathematics, and *vice versa*. The bad consequences of such a system need not be dwelt upon here.

We see from the above that Euclid's geometry had been introduced in the early days of the college.

In 1790 the studies in college were as follows:†

"The Freshmen study the learned languages, the rules of speaking, and the elements of mathematics. The Sophomores attend to the languages, geography, logic, and mathematics. The Junior Sophisters, beside the languages, enter on natural and moral philosophy and composition. The Senior class compose in English and Latin; study metaphysics and the elements of natural and political law.

"The books used by the students are Lowth's English Grammar, Perry's Dictionary, *Pike's Arithmetic*, Guthrie's Geography, *Ward's Mathematics*, Atkinson's Epitome, *Hammond's Algebra*, *Martin's and Enfield's Natural Philosophy*, *Ferguson's Astronomy*, Locke's Essay,

*Address before the Society of Alumni of Dartmouth College, 1855, p. 17.

†Barnard's Journal of Education, Vol. 26, 1876, International Series, Vol. I, p. 278, quoted by Judge Crosby from Belknap's History of New Hampshire, p. 296.

Montesquieu's Spirit of Laws, and Burlamaqui's Natural and Political Law." Hammond's Algebra was, we believe, an English work. In a catalogue of old, second-hand books we find, "Hammond, N., Elements of Algebra in a new and easy method, etc., 8vo. calf, 1742."

Of the early graduates of Dartmouth we would mention Daniel Adams, of the class of 1797, who furnished the schoolboy's satchel with the Scholar's Arithmetic, one of the best and most popular books of the time.

Another graduate somewhat distinguished in the mathematical line was John Hubbard, of the class of 1785. After studying theology, he became preceptor of the New Ipswich and Deerfield academies in Massachusetts. Afterwards he was judge of probate of Cheshire County, N. H. In 1804 he succeeded B. Woodward in the chair of mathematics and natural philosophy at Dartmouth, and filled it till his death in 1810. He published an Oration, Rudiments of Geography, The American Reader, and an essay on Music, but nothing on mathematics.

For twenty-three years, beginning in 1810, Ebenezer Adams was professor of mathematics and natural philosophy. In 1833 he was made professor emeritus.

For 1824 the mathematical studies as indicated in the catalogue of the college, was as follows: The Freshmen reviewed "arithmetick" and then studied algebra during the third term. No mathematical studies are given for the first two terms. The Sophomores were put down for six books of Euclid during the first term, plane trigonometry and its usual applications during the second term, and the completion of Euclid during the third term. The Juniors studied "conick" sections, and "spherick" geometry and trigonometry during the first term. The rest of the year was given to natural philosophy and astronomy. No mathematics in the last year.

In 1828 the college course was the same, but algebra to the end of simple equations was added to the terms for admission.

BOWDOIN COLLEGE.*

When Bowdoin College was first organized, in 1802, the requirements for admission were an acquaintance with the "fundamental rules of arithmetic." Later, the expression "well-versed in arithmetic" is used. The first definite increase in the requirements did not occur till 1834, when part of algebra was added.

During the first three years of its existence the college had no regular professor of mathematics. But in 1805 the faculty was reinforced by the arrival of Parker Cleaveland, who six years before had graduated at Harvard first in his class and had been tutor in the university. The department of mathematics and natural philosophy was assigned to the

* For the greater part of the material used in writing this sketch, the writer is indebted to Prof. George T. Little of Bowdoin College.

youthful instructor. He remained at Bowdoin till his death in 1868 and earned for himself the enviable reputation of "Father of American mineralogy." Cleaveland was professor of mathematics from 1805 till 1835. One of the first books used was Michael Walsh's arithmetic, published at Newburyport in 1801. Webber's mathematics were taught for many years, until they were displaced by Farrar's "Cambridge mathematics."

The course in mathematics at the beginning and for twenty years after, was, in the Freshman year, arithmetic; Sophomore year, algebra, geometry, plane trigonometry, mensuration of surfaces and solids; Junior year, heights and distances, surveying, navigation, conic sections; Senior year, spherical geometry and trigonometry with application to astronomy. Algebra was gradually forced back to the Freshman year, but a part of the first term of this year was given to arithmetic as late as 1850.

In regard to the instruction in mathematics during the professorship of Parker Cleaveland, Professor Little sends us a copy of a letter from their oldest living graduate, the Rev. Dr. T. T. Stone, of the class of 1820. Says he: "Until near the close of our college life we had but one professor with the president and two tutors. Professor Cleaveland added to his duties as teacher of the natural sciences, in particular chemistry, mineralogy, and such as were contained in Enfield's Natural Philosophy, those of instructor in mathematics; although, I think, in the latter, that is, in mathematics, one of the tutors took part. Of the tutors who had most to do with this department, I remember Joseph Huntington Jones, afterward a Presbyterian minister in Philadelphia; Samuel Greene, well known in later days as minister of—I think, the Essex Street Church, Boston; and Asa Cummings, minister soon after of the First Church in North Yarmouth (the North since dropped off), and, later still, editor for many years of the Christian Mirror, and, if I am not mistaken, other tutors sometimes assisted in the department, as Mr. Newman, who from tutor became professor of the ancient languages in the spring of 1820, and afterward professor of rhetoric. It was he, unless my memory fails me, who took our class out to survey a piece of land to the north or south-west of what was then the college grounds, including probably the place where he and Professor Smyth and Professor Packard afterward lived—the only thing connected with the mathematics which I now remember outside of the recitations and the preparation, such as they were, for the regular exercises.

"Of the books then used, the first, and that which went with us, I am not sure but through the whole course from the Freshman year to the Senior, was Webber's Mathematics. The only other book of pure mathematics was Playfair's edition of Euclid, in which we went through so much, I now forget how much, as we had time of the six books which comprised a large part of the work. Added to this, Enfield's Philosophy took in, with its natural science, not a little of mathematical illustration.

"Of the methods of instruction, I have already stated that the only exception I remember to simple recitation was a single slight piece of surveying. We were required to study the prescribed lesson in the book, then to repeat it, not of course word for word, but distinctly, to the professor or tutor at the recitation; that was all."

GEORGETOWN COLLEGE.*

Shortly after the close of the American Revolution the idea of establishing a college in Maryland, then the chief seat of the Catholic religion in this country, presented itself to the Rev. John Carroll, afterwards first Archbishop of Baltimore. Buildings were erected for this purpose in 1789, and a school first opened two years later. It rapidly grew into favor. Great attention was then paid to the classic languages, but only little to mathematics. Until 1806, when the college came into the hands of the Jesuits, the school was rather of preparatory grade. At this time a regular college course was arranged.

In 1807 Fr. James Wallace came to Georgetown. He had the classes here for two years. He was then sent to New York, where he taught in the "New York Literary Institution," an offshoot of Georgetown. While in New York he published a work on the Use of the Globes (New York, 1812). He returned to Georgetown in 1813 or 1814, and remained there until 1818, when he removed to Charleston, S. C. In 1821 his connection with the Society of Jesus was severed. After leaving Georgetown, he was for several years professor of mathematics in the South Carolina College. During his second stay at Georgetown he solved a problem proposed by the French Academy; as a reward they sent him many fine mathematical works. Professor Wallace was a man of ability, and a most patient and successful teacher.

Rev. Thomas C. Levins, born March 15, 1791, taught here from 1822 till 1825. He studied at Edinburgh, under Leslie, and then taught at Stonyhurst College, England. In 1825 he went to New York. Dr. Shea, in his Catholic Church in the United States (p. 403), states that Fr. Levins was one of the engineers of the Croton Aqueduct. He died in New York, May 6, 1843.

We have not been able to obtain more definite information on the early mathematical teaching at this college.

UNIVERSITY OF NORTH CAROLINA.

The first impulse towards the establishment of a university in North Carolina came, it seems, from the Scotch-Irish element occupying the midland belt of the State. "The early emigrants and settlers of this people brought their preachers, who also filled the office of teachers for the young. Tradition informs us that the most popular and best sustained of these nurseries of the young were located in the influential

* The above information is drawn mainly from a letter of Prof. J. F. Dawson, S. J., professor of physics and mechanics at Georgetown College.

counties of Iredell, Mecklenburgh, Guilford, and Orange. It was from these nurseries came the desire for higher education that formulated the article that decreed a State university. Doubtless the granting of a charter for William and Mary and for Harvard by the royal prerogative of the mother country, and the refusal of a like charter to Queen's College at Charlotte, in Mecklenburgh, during the colonial government, angered the *hornets*, fired the resentment of the Revolutionary patriots, and quickened their action in the blessings of liberty under the shield of the new-born Republic."*

The doors of the university first opened for the admission of students in 1795. It was organized after the model of Princeton College, which, in turn, was patterned after the Scottish universities. Shortly after the University of North Carolina had begun, Charles W. Harris, a graduate of Princeton College, was elected to the professorship of mathematics. He occupied this chair for only one year. It had been his original purpose to study law, and after one year's experience in teaching he resigned in order to enter the legal profession. He was regarded as a man of considerable ability, but he died at the age of 33.

He was succeeded by Rev. Joseph Caldwell, who was also a graduate of Princeton and a native of New Jersey. He had been one year tutor at his *alma mater*. This remarkable man gave for nearly forty years his best energy to the interests of the university. In 1804 he was elected president, which office he retained till his death in 1835, with the exception of four years, from 1812 to 1816, during which period he retired voluntarily to the professorship of mathematics so as to secure more time for the study of theology.

At first the faculty was very small. In 1814 it consisted of "President Caldwell, Professor Bingham, and Tutor Henderson. Their college titles were 'Old Joe,' 'Old Slick,' and 'Little Dick.' 'Old Joe,' however, was only thirty years of age, and possessed * * * a formidable share of youthful activity."†

It is not generally known that Dr. Caldwell, in August, 1832, completed the first college observatory built in the United States. "It was," says Professor Love, "a brick structure about 25 feet high, and contained a transit, an altitude and azimuth instrument, a portable telescope, an astronomical clock with mercurial pendulum, and other minor apparatus, all of which he bought in London in 1824 from the best makers. For want of means and interest, however, the observatory, after Dr. Caldwell's death in 1835, was permitted to go down, and

* Address by Paul C. Cameron in the inaugural proceedings at the University of North Carolina, June 3, 1885, p. 9. All the material for this sketch of mathematical teaching at that university has been furnished to the writer by Prof. James Lee Love, associate professor of mathematics at the University of North Carolina. Prof. Love has not only forwarded pamphlets, but has himself made careful investigation into the history of the institution, and kindly communicated his results to the writer.

† Fifty Years Since, by William Hooper, 1859, p. 10.

even the records of observations made there from 1832 to 1835 are not now known to exist."*

Professor Caldwell was a man of liberal and progressive views. He laid wisely the foundations of a great university in library and philosophical apparatus, as well as in the courses of study and in the men he gathered around him in his faculty. In remembrance of his long and untiring devotion to the institution a monument has been erected to him, by the alumni, in a grove surrounding the university.

When Caldwell went to Chapel Hill he found the college in a feeble state, nearly destitute of buildings, library, and apparatus; the students were very rough. We read of "unpleasant upheavals and volcanic eruptions" among them. Moreover, the bill of fare with which the minds of the students were obliged to content themselves was very meager. For admission in mathematics the elements of arithmetic were required from the beginning, in 1795, to 1835. In 1800 the requirement was "arithmetic as far as the rule of three;" in 1834, "arithmetic to square root." In our early arithmetics the rule of three was given for integers before fractions were touched upon, and we imagine that fractions were not required for admission, nor even any knowledge of integral arithmetic beyond the merest elements. The mathematical course offered in 1795 was as follows: (1) *Arithmetic* in a scientific manner; (2) *algebra*, and the application of algebra to geometry; (3) *Euclid's elements*; (4) *trigonometry* and its application to mensuration of heights and distances, of surfaces and solids, and to surveying and navigation; (5) *Conic sections*; (6) *the doctrine of the sphere and cylinder*; (7) *the projection of the sphere*; (8) *spherical trigonometry*; (9) *the doctrine of fluxions*; (10) *the doctrine of chances and annuities*. "The first *four* courses," says Professor Love, "were to be required for graduation. The remaining courses were to be taught *if requested, but they were not requested.*"

The text-books used prior to 1868 cannot now be entirely determined. The first algebra used was probably Thomas Simpson's. It was certainly studied in 1803 and in 1815, and, perhaps, as late as 1826. The first geometry studied was Robert Simson's Euclid. On the application of trigonometry to mensuration, Ewing's Synopsis was used first—certainly as early as 1798. About 1810 President Caldwell prepared a course in geometry, based on Simson's Euclid. This was used by the students in manuscript, copies having been made and handed down from class to class. Hutton's Geometry was introduced in 1816. In 1822 Dr. Caldwell published his geometry, under the title, "A Compendious System of Elementary Geometry." It was used for some years. Bound with this book in one volume was a treatise on trigonometry. The plane trigonometry was prepared by himself; the spherical was Robert Simson's. No record has been found as to the trigonometry used prior to 1822, though Simson's was probably the one. It does not appear that

* See also an article by Professor Love in the Nation for August 16, 1888.

the study and use of logarithms was introduced until 1811. Natural philosophy and astronomy were taught from the beginning. Ferguson's text was the one first used. In natural philosophy Nicholson's was used down to 1809, then Helsham's until 1816.

Dr. Caldwell was professor of mathematics from 1796 to 1817, but his activity extended in many other directions. He "taught mathematics, natural and moral philosophy, and did all the preaching." An interesting, though one-sided, picture of him as a teacher of geometry (about the year 1810) is given by William Hooper, one of the alumni: "There being but three teachers in college (president, professor of languages, and tutor), the Seniors and Juniors had but one recitation per day. The Juniors had their first taste of geometry, in a little elementary treatise, drawn up by Dr. Caldwell, in manuscript, and not then finished. Copies were to be had only by transcribing, and in process of time, they, of course, were swarming with errors. But this was a decided advantage to the Junior, who stuck to his text, without minding his diagram. For, if he happened to say the angle at A was equal to the angle at B, when in fact the diagram showed no angle at B at all, but one at C, if Dr. Caldwell corrected him, he had it always in his power to say, "Well, that was what I thought myself, but it ain't so in the book, and I thought you knew better than I." We may well suppose that the doctor was completely silenced by this unexpected application of the *argumentum ad hominem*. * * * The Junior having safely got through with his mathematical recitation at eleven o'clock, was free till the next day at the same hour."* It will be remembered, that the blackboard—that simple machine which doubles the teaching power of an instructor in geometry—was then unknown in America.

Fluxions were not taught at that time. William Hooper says in his humorous way, "As for chemistry and differential and integral calculus, and all that, we never heard of such hard things. They had not then crossed the Roanoke, nor did they appear among us till they were brought in by the northern barbarians about the year 1818."† These northern barbarians were Elisha Mitchell and Denison Olmsted. The latter introduced chemistry, mineralogy, and geology into the university. Dr. Mitchell was a New Englander. He graduated at Yale in the class of 1813 with Olmsted. He began teaching immediately after graduation, and in 1816 was appointed tutor at Yale. At the University of North Carolina he held the chair of mathematics from 1817 to 1825, and performed his duties with energy and success. When Dr. Olmsted was called to Yale, he assumed the vacant chair of chemistry, which position he filled with great credit until his death in 1857. He lost his life by falling over a precipice, in the darkness, while engaged in the scientific exploration of Mitchell's Peak in western North Carolina.

In 1818, after the arrival of Mitchell, spherical trigonometry, conic sections, and fluxions were introduced into the course of study leading

*Fifty Years Since, p. 23. † Page 17 of his address.

to the degree of A. B. The course was as follows: *Freshman year*, arithmetic completed, algebra begun; *Sophomore year*, algebra completed, geometry; *Junior year*, plane trigonometry, logarithms, mensuration, navigation, spherical trigonometry, conic sections, fluxions; *Senior year*, astronomy, natural philosophy. It will be noticed that the course began now in the Freshman instead of the Sophomore year, as formerly. If it was faithfully carried out, then it must have been very creditable to the institution at that time. It remained nearly unchanged for seventeen years. As regards the text-books, it is probable that Simpson's Algebra was used by Mitchell; also Hutton's, and since 1822, Caldwell's Geometry and Trigonometry, and Vince's Conic Sections. In 1823, Day's work on mensuration was taught. No record has been found as to what text-books were used when fluxions were first introduced. It is possible, however, that Vince's and Hutton's were the ones. In astronomy Nicholson's was used for a long time. Cavallo's Natural Philosophy and Wood's Mechanics were used, the latter since 1822.

Mr. Paul C. Cameron gives an interesting reminiscence of B. F. Moore, a once prominent lawyer. "Often has he entertained me," says Cameron, "with the lights and shades of his college life; how grandly he marched through the recitations in the languages taught in the first and second years of his college life; how deep and suddenly he went *under* when he struck the mathematical course of the Junior year; how he wrote to his father and appealed to him to take him home and place him behind the plow. His father *refuses*, and tells him to make known his difficulties to his professor. He hands his father's letter to Dr. Mitchell, who invites him to his study and gives him instruction by the use of his knife and a piece of white pine, cutting for him blocks of mathematical figures, to be used in the demonstrations of his propositions. Turning the light on him in this way, he was enabled to continue his course with satisfaction."

UNIVERSITY OF SOUTH CAROLINA.*

The South Carolina College threw open its doors for students in January, 1805. The first mathematical teacher at the college was Elisha Hammond of Massachusetts. He was a graduate of Dartmouth College, and when called to this position, was principal of Mt. Bethel Academy in Newberry, S. C. After one year's service he resigned and returned to the academy. Judge Evans, a student under him, says "His personal appearance and manners were very captivating, and his popularity for the period of his connection with the college was scarcely inferior to that of Dr. Maxey." Dr. Maxey was the president.

Rev. Joseph Caldwell, the father of the University of North Carolina, was then invited to the chair of mathematics, but he declined. Paul H. Perrault was elected to the place, but in 1811 he was removed for

* For the larger part of our information respecting this institution, we are indebted to Professor E. W. Davis, professor of mathematics and astronomy at the university.

"neglect of college duties." He is said to have been "well skilled in mathematics," but "wanting in that dignity which a Freshman would expect in a learned professor." After his separation from the college he was attached to the Army as a topographical engineer.

The mathematical professor for the next four years was George Blackburn. He was a graduate of Trinity College, Dublin. He taught in a military academy in Philadelphia; afterward he was teacher in Virginia, and was then called to the chair of mathematics and astronomy in William and Mary College. Thence he went to the South Carolina College. In 1812 he was employed by the State of South Carolina to run the boundary line between North and South Carolina. An old student says of him: "He was a man of quick and vigorous understanding, an able mathematician, and a most excellent instructor." Another: "Professor Blackburn was a first-rate mathematician; he taught mathematics as a science, and not as a matter of memory. From him I learned the demonstration of many difficult problems; and with his aid I understood much of that abstruse and difficult science as applied to natural philosophy and astronomy." He made students think. What detracted somewhat from his power as a teacher was his irritability.

In the better colleges of that day, the curriculum in mathematics embraced a short course on fluxions, or calculus. Though the plans of study included then about all the subjects pursued in the average American college of to-day, these subjects were not taught with the same thoroughness. Moreover, we are now teaching at least twice as much under each branch as was taught at the beginning of this century. In consequence of this, students of former times began the study of fluxions when, for lack of prepatory drill in lower branches, they were far less able to wrestle with the difficulties of the transcendental analysis than are our students of to-day. Professor Blackburn's teaching of the calculus, as narrated by M. La Borde, in his History of South Carolina College (p. 82), presents a picture of a Senior class vainly struggling with the intricacies of this subject. The class lost interest in the study and was very remiss in its attendance upon him, and those who did attend failed so completely in unraveling the mysteries of the transcendental analysis, as to force from the lips of the professor the remark, "that it might be that half of his class were very smart fellows, for he never saw them; but the half who attended his recitations were as laborious as oxen, but as stupid as asses." It need hardly be said that this was the cause of a students' rebellion.

After leaving the college, Professor Blackburn made latitude and longitude observations for the State map, under Governor Allston. Later he settled in Baltimore, where, with Dr. Jennings, he founded Asbury College. His last days were spent in Columbia, S. C.

From 1815 to 1820, Christian Hanckel, a Philadelphian and graduate of the University of Pennsylvania (class of 1810), was professor of mathematics. He took holy orders at St. Michael's, Charleston. His

main inducement to accept the chair was the chance to build up the Protestant Episcopal Church in Columbia. On leaving the college, he went to St. Paul's Church, Charleston.

The requirements for admission were, according to catalogue, at the beginning, "arithmetic, including proportion." This, most probably, did not include fractions. In 1836 the terms were "arithmetic, including fractions and the extraction of roots."

In the earliest course of mathematics at this college, the Freshmen took up arithmetic; the Sophomores, common and decimal fractions with extraction of roots; the Juniors, geometry, and theoretical and practical astronomy; the Seniors, exercises in higher mathematics as directed by the faculty. We are not certain that this curriculum embraced algebra. If taught then, it was a Senior study. Fractions were, it seems, not only not required for admission, but were not studied before the Sophomore year.

The course for the year 1811 was considerably stronger. The Freshmen were instructed in vulgar and decimal fractions, with extraction of roots; the Sophomores had lectures on algebra; the Juniors studied Hutton's course of mathematics; the Seniors had lectures by the "professor of mathematics, mechanical philosophy, and astronomy." From the anecdote told of Professor Blackburn, we know that at this time, or soon after, "calculus" (probably fluxions) was taught in the fourth year.

KENTUCKY UNIVERSITY.

About the year 1785 was opened in Lincoln County, Kentucky, a school called the Transylvania Seminary. Four years later it was moved to Lexington, Fayette County, where, in 1790, was held "the first public college commencement in the West of which we have any record." On January 1, 1799, the Transylvania Seminary and a similar school, called the Kentucky Academy, were united under the name of Transylvania University. The Transylvania University existed under this name until 1865, when it was merged in Kentucky University, and the consolidation has since been conducted under the name and charter of the latter.

Little has been done, in the past, to preserve the history of these institutions. Some of the records appear to have been lost, and those that are still extant give but little general information. The data on the very special subject of mathematical teaching are exceedingly meagre. The little information we are about to give was kindly furnished us by President Chas. Louis Loos, of Kentucky University.

The records of Transylvania University show that on September 16, 1799, "mathematics" was one of the subjects taught. On October 18, of the same year, the following books are mentioned in the mathematical course: *First year*, "Geography by Guthrie or Morse; algebra by Saunderson, Simson's Euclid, six books; trigonometry and mensuration, Gibson; Navigation, Patoun or Morse; Simson's conic sections."

Second year, "Natural philosophy and astronomy, Ferguson." These data are by no means destitute of interest. They show from what sources the young mathematician "in the West" drew his intellectual food, in early days. On October 26, 1799, Rev. James Blythe was elected professor of "science," which term was made to include mathematics. In 1803 the professor of science (J. Blythe) is called professor of mathematics and natural philosophy, and his duties were to teach "geography, arithmetic, algebra, geometry, surveying, navigation, conic sections, natural philosophy, and astronomy." In 1805 the course was the same as the one just given, except that geography, arithmetic, and surveying are not mentioned.

The entry in the records for March, 1816, gives the following course in mathematics:

Freshmen, first six books of Euclid, plane trigonometry, surveying, navigation, geography; *Juniors*, algebra as far as affected equations, spherical trigonometry, conic sections, natural philosophy, ancient geography; *Seniors*, astronomy." In 1817 Webber's mathematics is mentioned as a text-book.

THE UNITED STATES MILITARY ACADEMY.

The germinal idea of the United States Military Academy was put forth by George Washington, who felt, probably more than any one else, the necessity of having accomplished engineers in time of war. The Military Academy was established by Congress in 1802. The act was limited in its provisions and did not raise the academy above a military post, where the officers of engineers might give or receive instruction when not on other duty. The major of engineers was superintendent, the two captains were instructors, and the cadets were pupils.*

The major was Jonathan Williams; the two captains were William H. Barron and Jared Mansfield.

Major Williams, in a report to the Government in 1808, gives us some notion of the early instruction at the academy. He says that "The major occasionally read lectures on fortifications, gave practical lessons in the field, and taught the use of instruments generally. The two captains taught mathematics, the one in the line of geometrical, the other in that of algebraical demonstrations." Mansfield taught also natural philosophy. He had previously been teacher of mathematics and navigation at New Haven, and then at Philadelphia. He had published "essays" of some originality on mathematics and physics. They fell under the notice of Thomas Jefferson, and were the means that led to his appointment by the President as captain of engineers for the very purpose of becoming teacher at West Point. But after one year's teaching he was appointed by Jefferson, in 1805, to establish meridian lines and base lines in the North-West Territory for the pur-

* The U. S. Military Academy at West Point, by Edward D. Mansfield, LL. D.

pose of public surveys. His position remained vacant until his return, after the War of 1812.

In 1806 Alden Partridge became assistant in mathematics. He was a native of Vermont, had entered Dartmouth College in 1802, but before completing his course became cadet at West Point.

Captain Barron was relieved in 1807 by Ferdinand R. Hassler, who continued there until 1810, when he resigned. The following year he was called to the United States Coast Survey. Hassler was a Swiss. It was again the keen eye of President Jefferson that recognized the talent and secured the services of this foreigner, who had shortly before landed on our shores. Hassler's teaching power must have been hampered somewhat by his limited acquaintance with the English language. While at West Point he began writing his "Elements of Analytic Trigonometry," published by him in 1826. It was written in French and then translated for publication by Professor Renwick. From its preface we take the following: "It was the desire of introducing into the course of mathematics at West Point the most useful mode of instruction in this branch that led me to the preparation of this work as early as the year 1807." Hassler was, no doubt, the first one to teach *analytic* trigonometry in this country—the first one to discard the old "line-system."

About the same time Christian Zoeller, also a Swiss, was made instructor in drawing. He was "an amiable man of no high attainments."

Down to the year 1812 the academy was in a chaotic condition. There was no regular corps of instructors, and no regular classes. There had been no continuous study of any subject except mathematics. Referring to Hassler, Major Williams says in his report of 1808, "During the last year a citizen of eminent talents as a mathematician has been employed as principal teacher," and "being the only teacher designated by the law, he is the only one that, exclusive of the corps of engineers, can be said to belong to the institution." In conclusion the major says: "In short, the Military Academy as it now stands is like a foundling, barely existing among the mountains, nurtured at a distance out of sight, and almost unknown to its legitimate parents."

In vain did Jefferson in 1808 and Madison in 1810 recommend to Congress the enlargement of the academy. It was not until the nation was roused by the shock of war that Congress began to act. In 1812 Congress made liberal appropriations and passed an act reorganizing the institution. The next five years are the formation period of the academy. The first reform to be accomplished was the placing of the instruction on a higher level. The first academic faculty was constituted as follows: Col. Jared Mansfield, professor of natural and experimental philosophy; Andrew Ellicott, professor of mathematics; Alden Partridge, professor of engineering; Christian Zoeller, professor of drawing. We see from this that Mansfield held now the same place as in 1804, and Partridge was promoted from assistant to the rank of professor.

Mansfield and Ellicott had long been in the service of the General Government and of State governments in the capacity of surveyors and astronomers, and had established a wide reputation for both their practical and theoretical knowledge of mathematics. But now they were old men, and their ideas were somewhat old-fashioned. The workings of this faculty were not altogether harmonious. Partridge, being strong-willed and eccentric, wanted to have everything his own way. He was removed from his place. The appointment of Major Sylvanus Thayer, in 1817, to the superintendency of the academy marks a new era in its history.

Some notion of the instruction in mathematics at West Point between 1812 and 1817 may be obtained from the following extract from the curriculum which, in 1816, received the official approval of the Secretary of War.

"*Mathematics.*—A complete course of mathematics shall embrace the following branches, namely: The nature and construction of logarithms and the use of the tables; algebra, to include the solution of cubic equations, with all the preceding rules; geometry, to include plane and solid geometry, also ratios and proportions, and the construction of geometrical problems, application of algebra to geometry, practical geometry on the ground, mensuration of planes and solids; plane trigonometry, with its application to surveying and the mensuration of heights and distances; spherical trigonometry, with its application to the solution of spherical problems; the doctrine of infinite series; conic sections, with their application to projectiles; fluxions, to be taught at the option of the professor and student."

There was, however, no instruction in fluxions. E. D. Mansfield, in his historical sketch of the academy, does not include fluxions in the curriculum for 1816, but he remarks that calculus was added to the course a year or two later. The text-book then in use was Hutton's Mathematics.

Thus far the cadets were admitted to the academy without entrance examinations, and poor results were reached. Many cadets were unfit by prior study for the subjects they had to pursue. Rank and assignment to the various army corps were not made to depend upon merit.*

SELF-TAUGHT MATHEMATICIANS.

The foremost American mathematician of this time, like David Rittenhouse and Thomas Godfrey, had not enjoyed the privileges of a college education; like them, he was self-taught. We have reference to Nathaniel Bowditch.†

* The College Book, edited by Charles F. Richardson and Henry A. Clark, p. 216.

† This sketch is extracted from the Memoirs of Nathaniel Bowditch, by his son, Nathaniel I. Bowditch (Boston, 1839); from the Discourse on the Life and Character of Nathaniel Bowditch, by Alexander Young (Boston, 1838); from the eulogy by Professor Pickering (Boston, 1838), and from the eulogy by Judge Daniel A. White (Salem, 1838). A full list of Bowditch's mathematical papers may be found in the Mathematical Monthly, Vol. II.

It is instructive to study the history of his early life and to ascertain the influences under which his mind was formed. He was born at Salem, Mass., in 1773. His parents were poor, and he had often to content himself with a dinner consisting chiefly of potatoes, and at near approach of winter to continue wearing the thin garments of summer. After attending for a short time a dame's school near Salem, he entered Watson's school, which was the best school in Salem. It was wholly inadequate to furnish the ground-work and elements of a respectable education. He entered the school at the age of seven and remained there three years.

Bowditch early showed a great fondness for mathematics; but on account of his extreme youth his master refused to admit him to this study until he had procured from his father a special request to that effect. On one occasion he solved a problem in arithmetic which the instructor thought must be far above his comprehension. On being asked who had been doing the sum for him he answered, "Nobody—I did it myself." He was then accused of falsehood and treated with much severity.

When he was ten years old he left school to work in the shop of his father, who was a cooper. He received no regular instruction after leaving school, excepting a few lessons in book-keeping. He became soon after an apprentice to a ship-chandler, and afterward was clerk in a large mercantile establishment. It was during his apprenticeship that he disclosed that strong bent for mathematical studies. Every moment that he could snatch from the counter was given to the slate. When he was only fifteen years old he made an almanac for the year 1790, containing all the usual tables, calculations of the eclipses and other phenomena, and even the customary predictions of the weather.

When he was fourteen years old he one day got from an elder brother a vague account of a method of working out problems by *letters* instead of *figures*. This novelty excited his curiosity; he succeeded in borrowing an algebra, and "that night," says he, "I did not close my eyes." He read it, and read it again, and mastered its contents; and copied it out from beginning to end.

Subsequently he acquired access to an extensive scientific library of Dr. Richard Kirwan, an Irish scientist, which had been captured in the British channel by a privateer and sold to a society of gentlemen at Salem. This became the basis of the present Salem Atheneum. He found there the Philosophical Transactions of the Royal Society of London, from which he made full and minute abstracts of the mathematical papers contained in them. At this time he was too poor to buy books, and this was the only way in which he could manage to have them for convenient reference. The title page of one of these manuscript volumes states that it contains, with the next volume, "A complete collection of all the mathematical papers in the Philosophical Transactions; extracts from various encyclopedias; from the Memoirs of the Paris Academy;

a complete copy of Emerson's Mechanics; a copy of Hamilton's Conics; extracts from Gravesande's and Martin's Philosophical Treatises; from Bernoulli, etc., etc." What perseverance, what energy, what enthusiasm is displayed in this laborious work of copying!

Bowditch was very fond of books, but having no guide in the selection of them his reading was at first of the most miscellaneous character. Thus he read every article in Chambers' Encyclopædia from beginning to end. He secured a copy of Newton's Principia, but as it was published in Latin he began the study of that language that he might read that great work. By great perseverance he learned enough Latin to enable him to read any work of science in it. He afterward learned French for the purpose of having access to the treasures of French mathematical science, and at a late period of his life he acquired some knowledge of the German language. When twenty-one years of age he had read the immortal work of Newton, and there were few in his State who surpassed him in mathematical attainments.

Bowditch did not long remain in the situation of a merchant's clerk. His mathematical talent, in a town distinguished for enterprise, could not fail of being called into exercise in connection with the art of navigation. He became a practical navigator. Between 1795 and 1804 he was on five sea voyages, all under the command of Captain Henry Prince, of Salem.

The leisure of the long East India voyages, when the ship was lazily sweeping along under the steady impulse of the trade winds, afforded him fine opportunities for pursuing his mathematical studies, as well as for indulging his taste in general literature. The French mathematician Lacroix acknowledged to a young American that he was indebted to Mr. Bowditch for communicating many errors in his works, which he had discovered in these same long India voyages. It was his practice both when at home and when at sea to rise at a very early hour in the morning and pursue his studies. He was often seen on deck "walking rapidly and apparently in deep thought, when it was well understood by all on board that he was not to be disturbed, as we supposed he was solving some difficult problem, and when he darted below the conclusion was that he had got the idea; if he were in the fore part of the ship when the idea came to him, he would actually run to the cabin, and his countenance would give the expression that he had found a prize."

"He loved to study himself," says Captain Prince, "and he loved to see others study. He was always fond of teaching others. He would do anything if any one would show a disposition to learn. Hence," he adds, "all was harmony on board; all had a zeal for study; all were ambitious to learn." On one occasion two sailors were zealously disputing, in the hearing of the captain and supercargo, respecting sines and co-sines. As a result of his teaching, the whole crew, yea, even the negro cook, acquired the knowledge of how to compute a lunar observation. When the captain once arrived at Manila, he was asked how he

contrived to find his way, in the face of a northeast monsoon, by mere dead reckoning. He replied, that "he had a crew of twelve men every one of whom could take and work a lunar observation as well for all practical purposes as Sir Isaac Newton himself, were he alive." During this conversation Bowditch sat "as modest as a maid, saying not a word, but holding his slate pencil in his mouth;" while another person remarked, that "there was more knowledge of navigation on board that ship than there ever was in all the vessels that have floated in Manila Bay.

At that period the common treatise on navigation was the well known work of Hamilton Moore, which had occasioned many a shipwreck, but which Bowditch, like other navigators, was obliged to use. He found it abounding with blunders and overrun with typographical errors. Of these last errors many thousands of more or less importance were corrected in the early revisions of the work. Bowditch published several editions of Moore's works under that author's name, but the whole book at length underwent so many changes and radical improvements as to justify him to take it out on his own name. This is the origin of Bowditch's Practical Navigator, the best book on navigation then in existence. The following particulars regarding the publication of this work have been handed down to us:

The first American edition was printed in 1801, but not published until 1802. The publisher, Mr. Blunt, took the work and a copy of Hamilton Moore, with all the errors marked, to England, called on the publishers of Hamilton Moore, and sold the printed copy of Bowditch on condition that the American edition should not be sold until June, 1802, to give them an opportunity to get theirs into the English market at the same time. The London edition was revised and newly arranged by Thomas Kirby, teacher of mathematics and nautical astronomy. It was recommended as an improvement on Bowditch, but it contained many errors. This gave occasion to a British writer, Andrew Mackay, who published a rival work on navigation, to make Dr. Bowditch's supposed inaccuracies a particular object of attack.* This charge was emphatically repelled by Bowditch in the new edition of 1807.

From Harvard College Bowditch received the highest encouragement to pursue his scientific studies. In July, 1802, when his ship was windbound in Boston, he went to attend the commencement exercises at Harvard; and among the honorary degrees conferred, he thought he heard his own name announced as a master of arts; but it was not till congratulated by a friend that he became satisfied that his senses had not deceived him. He always spoke of this as one of the proudest days of his life, and amid all subsequent distinctions conferred upon him from foreign countries, he recurred to this with greatest pleasure.

* Memoirs of American Academy of Arts and Science, Vol. II, 1846, Eulogy on Bowditch, note C.

On quitting the sea, in 1803, he was appointed president of an insurance company in Salem, the duties of which he continued to discharge for twenty years, when he accepted the position of actuary of the Massachusetts Hospital Life Insurance Company in Boston. For many years he discharged the duties of this office with the greatest fidelity and skill.

He was several times solicited to accept positions in various literary institutions. In 1806 he was chosen to fill the Hollis Professorship of Mathematics at Harvard. He received from Thomas Jefferson the offer of the professorship of mathematics at the University of Virginia. Jefferson said in his letter: "We are satisfied we can get from no country a professor of higher qualifications than yourself for our mathematical department." In 1820 he was asked to permit his name to be presented to the President of the United States to fill a vacant chair at the U. S. Military Academy at West Point. Bowditch could not be persuaded to accept any of these positions.

The work for which Bowditch was for a long time exclusively known was his Practical Navigator. This gave him a wide-spread popularity among sea faring people everywhere. Bowditch himself did not consider this work as one which would advance his scientific reputation. What established his celebrity as a man of science was not his Practical Navigator, but his translation, with a commentary, of the epoch-making work of Laplace, called the Mécanique Céleste.

Later on we shall speak of this translation at length. Bowditch contributed many articles to the American edition of Rees's Cyclopædia.

The question may be asked, how should Bowditch be ranked as a mathematician? In answer to this we may say, that he is acknowledged by all as having stood at the head of scientific men of this country, and to have contributed more to his country's reputation than any contemporary scientist. But a giant in Liliput is not necessarily a giant in another country. Though a man of great energy and intellectual powers, he can not be pronounced a first-class mathematician. He was a man of learning, but not a man of genius in the sense that Newton, Leibnitz, Gauss, Abel, Pascal, and Archimedes were men of genius. The estimate that Bowditch made of his own capacities and gifts was, in our opinion, accurate, fair, and just. He did not overrate his talents, nor did he, with assumed humility, purposely underrate his powers. He is reported as having once said, "People are very kind and polite, in mentioning me in the same breath with Laplace, and blending my name with his. But they mistake both me and him; we are very different men. I trust I understand his works, and can supply his deficiencies, and record the successive advances of the science, and perhaps append some improvements. But Laplace was a genius, a discoverer, an inventor; and yet I hope I know as much about mathematics as Playfair!"

The career of Bowditch furnishes us with an excellent illustration of how much may be accomplished through indefatigable energy and per-

servance by a mind which, though naturally far above the average mind, is, nevertheless, lacking the powers of real genius.

A mathematician of considerable local reputation was Enoch Lewis (1776–1856). He was a native of Pennsylvania and a Quaker. In 1799 he became teacher of mathematics at the West Town Boarding School, established by the Society of Friends. He was the author of treatises on arithmetic, algebra, and trigonometry.

Under the tuition of Enoch Lewis, for six months, at the Friends' Boarding School at West Town, was John Gummere, who was then about twenty years old. Excepting in reading, writing, and arithmetic, he had received no instruction whatever up to that time. After teaching elementary schools for six years, he determined in 1814 to open a boarding school in Burlington. The following story characterizes the young man.*

He determined to give courses of lectures in natural philosophy and chemistry, and proposed to his brother, who had joined him in the school, that he should take the latter. The brother replied that he had never opened a book on chemistry. "Neither have I," said John, "on natural philosophy." It was then objected that they could not obtain the appropriate instruments and apparatus in this country. "But we can get them," he said, "from London." It was suggested that they might fail in so making themselves masters of their respective subjects as to pursue them advantageously. "But we shall not fail," said he; "only determine and the thing is half done." An order was sent to London for apparatus, both philosophical and chemical, a better supply of which was provided for his institution (at an expense of several thousand dollars) than was to be found in any private institution in this country.

Gummere acquired considerable reputation as a teacher and writer. He was for over forty years teacher in Friends' schools in Pennsylvania and New Jersey. He once declined the proffered chair of mathematics at the University of Pennsylvania. He contributed astronomical papers to the American Philosophical Society. The most celebrated of his works was his Surveying (1814), which went through a large number of editions. It was more extensively known and more highly prized than any other work on surveying. His treatise on theoretical and practical astronomy was once used as a text-book at West Point and other leading scientific institutions. In preparing it he had greatly profited by French models.

Mention should be made here of the mathematical studies of Walter Folger, a lawyer, of Nantucket. They will throw light upon the kind of instruction which was then being given at our American ports, in night schools for navigators. After attending common schools, Folger studied land-surveying, in which, without the least assistance, he became exceedingly skillful. "In the winter of 1782–83 he attended an

* Memorials of the Life and Character of John Gummere, by William J. Allison.

evening school in which he studied navigation, and readily acquainted himself with these branches. Nothing of a mathematical character seemed ever to present any difficulties to his mind. He mastered algebra and fluxions without assistance, and while in his teens he read Euclid as he would read a narrative, no problem arresting his progress: and yet, so little did he know of language, or of anything appertaining to it, that he had reached the years of manhood, as he often confessed, before he knew the definition of the word *grammar*."*

"His father finally succeeded in obtaining for him a work of navigation, to which, for the first time, was appended Dr. Maskelyne's method of obtaining the longitude at sea by means of lunar distances. This delighted him, and at the age of eighteen, [when] prostrated with sickness, he familiarized himself with the problem, and the engagement so diverted his mind from his infirmities that he speedily regained his strength. He immediately applied all his influence to the encouragement of the use of this method among his fellow-townsmen, then universally engaged in the prosecution of whaling voyages. To numbers he gave personal instruction, and the first American ship-master who determined his longitude by lunar observations is said to have been one of his pupils." A similar school was held in Philadelphia by Robert Patterson.

Surveying of Government Lands.

In a new and growing country like ours it was only natural that the art of surveying should have been early cultivated. But to a surveyor some knowledge of the rudiments of geometry and trigonometry was indispensable. As early as 1761 there was published, or reprinted, in Philadelphia a work entitled, Subtential Plane Trigonometry, by Thomas Abel, presumably an English teacher. In 1785 there was reprinted in Philadelpia an edition of Robert Gibson's Practical Surveying, which first appeared in London in 1767. This enjoyed an extended circulation. In 1799 appeared in Wilmington the first popular American treatise on surveying, by Zachariah Jess, a teacher and practical surveyor, of Delaware. In the preface to Gummere's Treatise on Surveying (1814) we read: "The works of Gibson and Jess are the only ones at present in general use. The former, though much the better of the two, is deficient in many respects." In 1796 was published in New York, The Art of Surveying Made Easy, by John Love, and at Litchfield, An Accurate System of Surveying, by Samuel Moore. In 1806 Rev. Abel Flint published his Geometry and Trigonometry, with a Treatise on Surveying. Flint graduated at Yale in 1785, was tutor at Brown till 1790, afterward studied theology, and then became pastor at Hartford, Conn.

The publication of Gibson's Surveying in 1785 was very timely, for it was in this very year that Congress passed an ordinance specifying

* "A Brief Memoir of the late Walter Folger, of Nantucket," by William Mitchell, in the American Journal of Science and Arts, second series, Vol. IX, No. 27, May, 1850.

that surveyors, as they were respectively qualified, should proceed to divide the western territory into townships of 6 miles square by lines running due north and south, and others crossing these at right angles as near as may be. Each township should be subdivided into lots of one mile square. This system was not universally approved, for it tended to delay the sale of public lands till they could be correctly measured. In the Madison Papers (Vol II, p. 640) we read that the Eastern States favored the plan adopted, while the Southern were "biased in favor of indiscriminate location." Kentucky and Tennessee adhered to the old plan of indiscriminate location. This occasioned so much litigation in those States that it has been said that as much money was annually expended there in land-title litigation as would defray their taxes for the support of the severest war. Lands surveyed by the United States, on the other hand, were comparatively without any legal difficulty. In fact, one great object of the Government system was the removal of all temptation to incur the curse pronounced by Moses on him "who removeth his neighbor's landmark." The corners of each section were carefully located by marked trees, whose species, diameter, distance, and bearing were entered upon the field-notes. If the marked tree at any one corner were destroyed, then its location could be determined from the other corners. Though a great improvement on previous modes of surveying, it is inaccurate and rude indeed as compared with the refined triangulation surveys now carried on by the United States Coast and Geodetic Survey.

Most conspicuous in the execution of the early Government surveys were Andrew Ellicott and Jared Mansfield. Ellicott was engaged in a large number of surveys. At various times he was appointed commissioner for marking the boundaries of Virginia, Pennsylvania, and New York; in 1789 he was selected by Washington to survey the land lying between Pennsylvania and Lake Erie; in 1790 he was employed, with his brother Joseph, in surveying and laying out the city of Washington; in 1792 he was made Surveyor-General of the United States; in 1796 he was appointed United States Commissioner, under the treaty of San Lorenzo el Real, to determine the boundary between the United States and the Spanish possessions on the south. It is stated that he sent observations to Delambre, of France, remarking that they were made by a "self-taught astronomer, and the only practical one now in the United States." This was after the death of David Rittenhouse.

More prominently connected with the survey of the North-West Territory than Ellicott was Jared Mansfield. He was a graduate of Yale College. In 1801 (?) he published Essays, Mathematical and Physical. From the perusal of his works alone the illustrious Thomas Jefferson was induced to bring him into public life. In 1803 he was appointed surveyor-general of the North-West Territory. His first work was to determine astronomically certain lines of latitude and the principal meridians on which the surveys were to proceed. To carry out this

work astronomical instruments were needed. President Jefferson ordered the purchase from London of a transit instrument, a telescope, an astronomical clock, and a sextant. The first principal meridian began at the mouth of the Great Miami; the second at a point 5 miles south-west of the confluence of Little Blue River with the Ohio; the third at the confluence of the Ohio and the Mississippi Rivers; the fourth at the junction of the Illinois and Mississippi; the fifth at the mouth of the Arkansas River. A large number of other meridians, or "base-lines," have since been established.*

In view of the fact that our Government has had, all in all, nearly 3,000,000 square miles of land to sell or to otherwise dispose of, and that the sale had always to be preceded by a survey, it must be evident that there was a demand for surveyors. They could earn a comparatively easy subsistence while a student of pure mathematics might have gone a begging for a living. About 1816 a friend of Comte in this country warned that French mathematician and philosopher against the purely practical spirit that prevailed in this new country, and against coming here, by saying: "If Lagrange were to come to the United States he could only earn his livelihood by turning surveyor."

Mathematical Journals.

The number of mathematical journals published in this country since the beginning of this century is much greater than one might suppose. A full historical sketch of these periodicals has been given by Dr. David S. Hart in the Analyst (Vol. II, pp. 131-8, 1875), and we shall make free use of his valuable article.

The oldest mathematical journal in America was the Mathematical Correspondent. It was established by gentlemen in New York and other cities, who had long felt the want of a periodical which should do for America what the Ladies' Diary had done for England. George Baron was editor-in-chief. It was to be issued quarterly. The first number was issued in New York City on May 1, 1804. Only eight numbers ever appeared. An essay in this magazine on Diophantine analysis, by Robert Adrain, was the first attempt to introduce the study of this subject in America.

The main cause of the discontinuance of the journal lies in the prejudice which the editors, who were of Hibernian descent, entertained against American authors. A contributor, who called himself "A Rabbit," was permitted by the editors to sneer at several works written by American authors, such as Shepherd, Pike, Walsh, and others. The editors themselves also spoke in the most contemptuous manner of Col. Jared Mansfield, the superintendent of the Military Academy at West Point. Baron advertised on the cover of No. 2 of the Correspondent a

* For further information on the early surveys, see Niles's Register, Vol. XII, pp. 97, 406; Vol. XVI, p. 362.

lecture delivered by him in New York, which contains, as he says, "a complete refutation of the false and spurious principles ignorantly imposed on the public in the new American Practical Navigator, written by N. Bowditch." The sub-editors endorsed the above. But some of these attacks, especially "A Rabbit's," seem to have created trouble, and on p. 154 the editor says: "'A Rabbit' will not in any future number be permitted to propose questions concerning the blunders of stupid Shepherds; we had rather soar aloft with the eagle than waddle in the mud with the goose." For some hidden reason, Baron resigned the editorship. Many of the subscribers neglected to pay, and the paper soon died out.

The next periodical was the Analyst, or Mathematical Museum, edited by Robert Adrain. The first number was published in Philadelphia in 1808. Five numbers only appeared. We have spoken of this periodical at some length when we wrote about Robert Adrain. It contained the valuable original work of Adrain on the Law of Probability of Errors. Besides the editor, N. Bowditch, Alexander M. Fisher, and Melatiah Nash were among the contributors to the Analyst.

In 1818, William Marrat became editor of the Scientific Journal, which was published at Perth Amboy, N. J., in monthly numbers. Nine numbers are all that are known to have appeared. The cause of the discontinuance seems to have been the departure of Mr. Marrat for England.

In 1825 Robert Adrain started in New York a second periodical, the Mathematical Diary, which was published quarterly during the first two years and annually during the last two. The last number contains an excellent likeness of Lagrange, and a sketch of his life. After the first year the editorship of the journal passed into the hands of James Ryan, the author of several mathematical works. In the preface to the first number of the Mathematical Diary, Robert Adrain said: "The principal object of the present little work is to excite the genius and industry of those who have a taste for mathematical studies by affording them an opportunity of laying their speculations before the public in an advantageous manner. * * * It is well known to mathematicians that nothing contributes more to the development of mathematical genius than the efforts made by the student to discover the solutions of new and interesting questions." These words may have been prompted by his own experience. We have already pointed out how the Analyst, which was edited by him seventeen years previously, was the medium of publishing the first proofs of the all important Law of the Facility of Error in Observations.

Nearly all the more prominent mathematicians of America were contributors to the Diary. Among them were Robert Adrain, N. Bowditch, Theodore Strong, Eugene Nulty, Benjamin Peirce, Benjamin Hallowell, William Lenhart, M. O'Shannessy, Henry J. Anderson, and others.

In 1832 the publication was suspended on account of an unfortunate

quarrel among the mathematicians. Mr. Samuel Ward, a then recent graduate of Columbia College, had in part the management of the last number, in which he inserted a dialogue, written by himself, exhibiting in a ridiculous light Henry J. Anderson, then professor of mathematics at Columbia College. High words passed between the parties and their friends, which resulted in the complete breaking up of the Mathematical Diary. Samuel Ward was afterward editor of Young's Algebra. In later years he followed wholly different pursuits. He became known in Washington as the "king of the lobby," and as the giver of the best dinners of any man in America.

According to Dr. Hart, a journal called the Mathematical Companion, was started by John D. Williams in 1828, and continued for four years. The periodical, says Dr. Hart, was evidently gotten up as a rival of the Mathematical Diary. The writer has never seen a copy of this periodical. There is one in the Harvard library.* Mr. Williams had many opponents, and a bitter contest was carried on between the two parties. He finally issued his fourteen famous "challenge problems," directed against all the mathematicians in America, excepting only Dr. Bowditch, Professor Strong, and Eugene Nulty. Six of these are impossible. All the others have been solved by several persons.†

The periodicals which we have named were devoted entirely to mathematics. In addition to these there were publications which were given

* Dr. Artemas Martin sends us the full title of the journal, as found in Bolton's Catalogue of Scientific and Technical Periodicals, 1665 to 1882, published by the Smithsonian Institution, p. 360—"The Mathematical Companion, containing new researches and improvements in the mathematics, with collections of questions proposed and resolved by ingenious correspondents. Edited by Williams; 1 vol., 18 mo., New York, 1828-31."

† In the Educational Notes and Queries, edited by W. D. Henkle, Vol. II, No. 11, January, 1876, will be found a copy of a communication to a newspaper made by John D. Williams in 1832, containing the "fourteen challenge problems," and beginning as follows:

"Messrs. Editors.—It is this day six months since, under the signature of *Diophantus*, I proposed through the medium of your paper to the mathematicians of America, a collection of problems in Diophantine analysis. No correct solution having as yet been received to the whole of them, I take this opportunity to fulfill my pledge to furnish such, and inclosed they will come to your hands. I now desire to re-propose them for the ensuing six months; and shall except from my challenge the Hon. Nathaniel Bowditch, LL. D.,. etc., of Boston, Mass.; Mr. Eugene Nulty, of Philadelphia; and Prof. Theodore Strong, of Rutgers College, New Brunswick, N. J., only. The list of gentlemen challenged stands then as follows: Prof. Robert Adrain, University of Pennsylvania; Henry J. Anderson, Columbia College, N. Y.; Benjamin Peirce, Harvard University, Cambridge, Mass.; Mr. J. Ingersoll Bowditch, Boston, Mass.; Marcus Catlin, Hamilton College, Clinton, N. Y.; M. Floy, jr., New York; C. Gill, Sawpitts Academy, N. Y.; L. L. Inconnew, Cincinnati, Ohio; Benjamin Hallowell, Alexandria, Va.; Samuel Ward, 3rd New York—it being presumed that there are none in the United States with the exception of the above list would think of attempting their solution." Then follow the fourteen questions. All problems being in Diophantine analysis would tend to show that this subject was then a comparatively favorite study.

to science, or to useful information in general, but which gave part of their space to a "mathematical department." Foremost among these was the Ladies and Gentlemen's Diary, or United States Almanac, etc., edited by Melatiah Nash, for the years 1820, '21, '22. It contained much valuable information in astronomy and philosophy, enigmas, charades, queries, and mathematical problems, to be answered in the succeeding numbers. Other almanacs which generally contained mathematical problems were Thomas's Almanac, published at Worcester, Mass., which existed for more than one hundred years; the Maine Farmer's Almanac; two publications, each called the "Farmers' Almanac;" the Knickerbocker Almanac; the Anti-Masonic Almanac, commenced in 1828 at Rochester, N. Y. Other journals having a mathematical department were the American Monthly Magazine, commenced in New York in the year 1817; the Portico, which was started in Baltimore in 1816 and continued two or three years.

The mathematical journals spoken of were all of the most elementary kind, and, excepting No. IV of the Analyst, which contained Adrain's investigations on least squares, added nothing to the stock of mathematical science. These journals had an educational rather than scientific value. The proposal and solution of problems was the main work done by their contributors. Now, it will certainly be admitted that solving problems is one of the lowest forms of mathematical work. The existence of mathematical journals shows that since the beginning of this century there always were some persons interested in mathematics, but the number was so small that mathematical journals never were a financial success. All the early mathematical periodicals had merely an ephemeral existence.

III.

THE INFLUX OF FRENCH MATHEMATICS.

During the latter part of the eighteenth century we see the French people rising with fearful unanimity, destroying their old institutions, and upon their ruins planting a new order of things. With this period begins the interest in popular education in France. A new impetus was given also to higher scientific education, which continued to be far in advance of that of the rest of Europe.

In 1794 was opened in Paris the Polytechnic School and in the following year the Schools of Application. The Polytechnic School gained a world-wide celebrity. The professors at this institution were men whose names are household words wherever science has a votary. Lagrange, Lacroix, and Poisson laid the basis to its course in analytical mathematics; Laplace, Ampère, and others to that of analytical mechanics and astronomy. Descriptive geometry and its applications had for their first teachers the founder of this science, the illustrious Monge and his celebrated pupils, Hachette and Arago.

The success of the Polytechnic School was phenomenal. It was the nurse of giants. Among its pupils were Arago, Biot, Bourdon, Cauchy, Chasles, Duhamel, Dupin, Gay-Lussac, Le Verrier, Poncelet, Regnault. The Polytechnic School is of special interest to those who live in America, because the U. S. Military Academy at West Point was a germ from it.

Compared with the French mathematicians who flourished at the beginning of this century the contemporary American professors were mere Liliputians. The masterpieces of French scholars were unknown in America. What little mathematical knowledge existed here came to us through English channels. For that reason that epoch was called the period of the influx of English mathematics. As compared with colonial times, considerable attention was paid to mathematical studies during that period. But there was still a great dearth in original thinkers on mathematics among us. The genius of our people was exercised in different fields, and so the little science we had was borrowed from others.

But the time came when French writers were at last beginning to make their influence felt among us. We recognized their superiority over the English and profited by it. Mathematical studies received a new impetus. But even then ours was not the glory of the sun, but

only of the moon. The new period produced among us only one mathematician displaying real genius for original research.

It is naturally humiliating to an American when a foreign mathematician like Todhunter, well known for the fairness and candor of his views, pronounces a judgment on Americans like the following: "I have no wish to depreciate their labors; I know that they possess able mathematicians, and that in the department of astronomy they have produced meritorious works; but I maintain that as against us their utmost distinction almost vanishes. And yet, with their great population, their abundant wealth, their attention to education, their freedom from civil and religious disabilities, and their success in literature, we might well expect the most conspicuous eminence in mathematics."*

No thinking American will pronounce this estimate of American mathematicians as entirely unsound; it is, in fact, quite correct. The reasons for this want of productiveness certainly do not lie in any lack of power in the American mind. They will be found rather (1) in the want of interest in and appreciation of abstract scientific work on the part of the American people, and (2) in the bad methods of mathematical instruction in our elementary and higher institutions of learning. There has been no incentive in this country for any large body of men to direct their life-work, day by day, in the line of mathematical investigation. In former years our professors in colleges were, with few exceptions, over-worked in the recitation room; their routine work absorbed all their energies, thereby rendering their minds unfit for original research. Again, every teacher had a stomach; his wife and children had stomachs; the human being must be fed; a livelihood must be earned; the professor's salary was low; not unfrequently he had to add to his duties as instructor in college those of a preacher or private teacher, in order to make his living. Such conditions were not favorable for the growth of science.

But, in spite of all difficulties, there was much progress. The improvements in mathematical text-books and reforms in mathematical instruction were due to French influences. French authors displaced the English in many of our best institutions. It is somewhat of a misfortune, however, that we failed to gather in the full fruits of the French intellect. We followed in the path of French writers whose works had ceased to be the embodiment of the later results of French science; many of the works which we adopted were beginning to be "behind the times," when introduced in America. We used works of Bezout, Lacroix, and Bourdon. But Bezout flourished before the French Revolution, and Lacroix wrote most, if not all, of his books before the beginning of this century. In 1821 Cauchy published in Paris his *Cours d'Analyse*. If thoughtful attention and study had been given by our American text-book writers to this volume, then many a lax, loose, and unscientific method of treating mathematical subjects might have been

* The *Conflict of Studies* and Other Essays, by I. Todhunter. London, 1873, p. 180.

corrected at the outset. The wretched treatment of infinite series, as found in all our text-books, excepting the most recent, might have been rejected from the very beginning.

In thinking of the influx of French mathematics, we must guard against the impression that French authors and methods entirely displaced the English. English books continued to be used in some of our schools. Many an old English notion has remained with us to the present day. We still have the English weights and measures. The old line-system in trigonometry, which we got from the English, but which they long since rejected, has until very recently been finding favor among many of our teachers.

There have been improvements in the methods of instruction, but not so extensive as might be wished. Traditional methods have long had almost full sway. The mathematical teaching has been bad. One of the most baneful delusions by which the minds, not only of students, but even of many teachers of mathematics in our classical colleges, have been afflicted is, that mathematics can be mastered by the favored few, but lies beyond the grasp and power of the ordinary mind. This chimera has worked an untold amount of mischief in mathematical education. The students entered upon their studies with the feeling that there was no use trying to learn mathematics, and the teacher felt that there was no use trying to teach it. This humiliating opinion of the powers of the average human mind is one of the most unfortunate delusions which have ever misled the minds of American students and educators. It has prevailed among us from the earliest times. In the latter part of the last century, the notion was general among us that girls could not be taught fractions in arithmetic, and that lady teachers were unfit, for want of mental capacity, to give instruction in arithmetic. Warren Burton says that a school-mistress "would as soon have expected to teach the Arabic language as the numerical science." But this delusion has now vanished. The best instruction in elementary arithmetic is now given by lady teachers. Among the contributors to the American Journal of Mathematics there are two ladies. In the same way the delusion will soon vanish that the average college student is not able to grasp the more advanced branches of exact science. The trouble has been, all along, not so much in the lack of ability in students, as in the wretched character of the mathematical instruction. Such is the opinion of Professor Olney, one of the most efficient drill-masters and teachers of mathematics that this country has produced. In the preface to his General Geometry and Calculus he says: "Nor is it impracticable for the majority of students to become intelligent in these subjects. They do not lie beyond the reach of good common minds, nor require peculiar mental characteristics for their mastery. The difficulty hitherto has been in the methods of presentation, in the limited and totally inadequate amount of time assigned them, and more than all in the preconceived notion of their abstruseness."

One of the causes of the bad instruction in our colleges has been the system of tutorships. Fortunately, this relic of scholasticism is now rapidly disappearing. Young students who needed a skilled teacher of long experience to guide them and to awaken in them a spirit of free inquiry were intrusted to inexperienced youths who had just graduated from college, and who had themselves never felt the glow of the spirit of independent inquiry. Students did not find their mathematics interesting, nor did they understand it well. Their hatred of mathematics had its cause in these two facts, which stand in the closest possible connection with each other. "We might say, either that the study failed of being understood, because it was uninteresting, or that it awakened no interest, because it was not well understood. Both these statements were true."* Professor Eddy truly says that very few students "do really become in any true sense masters of the mathematical subjects which they study, or indeed have sufficient practice in the principles which they attempt to learn, to be capable of judging whether they have been so mastered as to accomplish the ends which should be sought in mathematical training." The great desideratum in our preparatory schools and colleges has been less memorizing, less cramming, more thorough training in the fundamental branches, more object teaching, more drill, more frequent and well-guided original inquiries, greater freedom from formalism, a stronger spirit of free inquiry.

Says Professor Eddy: "When, as often happens, our college graduates go abroad for post-graduate study in departments requiring previous mathematical training, what do they find their requirements in this direction to amount to? I think I may say that a large proportion of them find themselves almost hopelessly lacking in the essentials of such training, and not at all fitted to make proper improvement of the advantages of which they have sought to avail themselves. Our young men are unequal to the mathematical studies which those of the same age, but of European academic training, successfully carry. Now, where does the difficulty lie? Not in any inferior talent for these studies, as I have the best of reasons for believing, but from a lack of opportunity for obtaining a comprehension of the infinitesimal calculus, in which they usually find themselves almost wholly wanting." Nor are they always able to manipulate, with any degree of ease, the more complicated expressions of ordinary algebra. They have been taught by a "daily lecture instead of a daily drill," a method of teaching which is like "explaining tactics instead of practicing them." Or, whenever text-books were used, "the recitations were mere hearings of lessons, without comment or collateral instruction."†

Professor Eddy's reminiscences of his own study of mathematics in college are not pleasant. Nor is his experience exceptional. On the con-

* "College Mathematics," by Henry T. Eddy, in the Proceedings of the American Association for the Advancement of Science, Vol. XXXIII, 1884.

† Harvard Reminiscences, by A. P. Peabody, p. 201.

trary it has been the rule rather than the exception in our classical colleges. In reply to a request made by the writer to give his recollections of the mathematical teaching at one of our oldest classical colleges, a now prominent professor of mathematics replied that he did not think he had "any such recollections" as he "should care to put in print." Another one gives his reminiscences, but marks his letter "personal and private."

If our classical colleges had caught something of the spirit that must have prevailed at the Polytechnic School in Paris in the days of Lagrange, Laplace, Lacroix, Ampère, when it produced such thinkers as Arago, Cauchy, Le Verrier, then the list of our prominent mathematicians and astronomers would doubtless have been doubled or tripled. We got from the French some of their old text-books, but we failed to catch their love of scientific study and inquiry.

On a previous page it has been stated that Americans had come to recognize the superiority of French mathematicians over the English. It should have been added that we did not see this superiority until it was pointed out to us by the English themselves. The influx of French mathematics into the United States was preceded by an influx of French mathematics into England. In Britain there were men who had come to deplore the very small progress that science was making there, as compared with its racing progress on the continent. In 1813 the "Analytical Society" was formed at Cambridge. This was a small club established by Peacock, John Herschel, Babbage, and a few other students at Cambridge, to promote, as it was humorously expressed, the principles of pure "D-ism," that is, of the Leibnitzian notation in the calculus, against those of "dot-age," or of the Newtonian notation. This struggle ended in the introduction into Cambridge of the Continental notation ($\frac{dy}{dx}$) to the exclusion of the fluxional notation ($\dot{y}$). This was a great step in advance, not on account of any great superiority of the Leibnitzian over the Newtonian notation, but because the adoption of the former opened up to English students the vast storehouses of Continental discoveries.

The movement against the fluxional notation began in this country almost ten years later than it did in England, and proceeded more quietly. John Farrar, of Harvard, translated from the French the Differential and Integral Calculus of Bezout, which employed the Continental notation, in 1824. Professor Fisher, of Yale, who died in 1822, published mathematical articles in Silliman's Journal, employing the new notation. At an earlier date than this there were men connected with West Point who had been trained in the Continental system. Thus, F. R. Hassler, educated at the University of Bern, was teacher of mathematics at West Point from 1808 to 1810. Probably neither calculus nor fluxions were taught there during that time, for, as late as 1816, we read in the West Point curriculum that *fluxions* were "to be taught at the

option of professor and student." In 1817, Crozet, trained at the Polytechnic School in Paris, became teacher of engineering at the Military Academy. In this country he, sometimes at least, used the Newtonian notation. He did so, for instance, in the solution, in French, of a problem which he published in the Portico, of Baltimore, in 1817. The Leibnitzian notation must have been introduced at the Military Academy very soon after the year 1817.

Robert Adrain used the English notation in his earlier writings. In the Portico, Vol. III, he does so, but in Nash's Ladies and Gentlemen's Diary, No. II, published in New York in 1820, he employs the notation dx. We are told that while he was at Columbia College, between 1813 and 1826, he wrote a manuscript treatise on the Differential and Integral Calculus. We know also that he was a diligent student of the works of Lagrange and Laplace, which contained the notation of Leibnitz throughout. The first article in the Memoirs of the American Academy of Arts and Sciences, which contains the "d-istic" notation, was published in 1818 by F. T. Schubert. It is well known that Bowditch began the translation of the Mécanique Céleste of Laplace as early as 1814. At that time he was, therefore, thoroughly conversant with pure "D-ism." He had been converted to the new "ism" on the long sea voyages, from 1795 to 1804, when he studied Lacroix's Calculus. In general, it may be stated that the change of notation took place in the United States about the close of the first quarter of this century.

The publication of Bowditch's Laplace, begun in 1829, gave a powerful stimulus to the study of French mathematics and to the general advancement of mathematical learning in America. Says Edward Everett: "This may be considered as opening a new era in the history of American science."

This may be a convenient place to consider that work at some length. As it originally appeared in France, the Mécanique Céleste was described by the Edinburgh Review, one of the leading scientific journals in Great Britain, as being of so abstruse and profound a character that there were scarcely a dozen men in all that country capable of reading it with any tolerable facility. These remarks created great curiosity in Bowditch to explore the work. He began translating it in 1814, and pursued it with such ardor and persistence that he accomplished it in only two years.

In order to state briefly the object of the work of La Place, we quote from his preface to it as follows:

"Toward the end of the seventeenth century, Newton published his discovery of universal gravitation. Mathematicians have since that epoch succeeded in reducing to this great law of nature all the known phenomena of the system of the world, and have thus given to the theories of the heavenly bodies and to astronomical tables an unexpected degree of precision. My object is to present a connected view of these theories which are now scattered in a great number of works. The whole of the results of gravitation upon the equilibrium and motions

of the fluid and solid bodies which compose the solar system and the similar systems existing in the immensity of space, constitute the object of *Celestial Mechanics*, or the application of the principles of mechanics to the motions and figures of the heavenly bodies. Astronomy, considered in the most general manner, is a great problem of mechanics in which the elements of the motions are the arbitrary constant quantities. The solution of this problem depends, at the same time, upon the accuracy of the observations and upon the perfection of the analysis. It is very important to reject every empirical process, and to complete the analysis, so that it shall not be necessary to derive from observations any but indispensable data. The intention of this work is to obtain, as much as may be in my power, this interesting result."

Though the translation was completed as early as 1817, the publication did not begin until 1829. In 1817 the income of Bowditch was so small that he could not afford to have the translation published. The American Academy of Arts and Sciences offered to publish the work at their own expense. He was also solicited to publish it by subscription. But his independence of spirit induced him to decline these proposals. He was aware that the work would find but few readers, and he did not wish any one to feel compelled or to be induced to subscribe for it, lest he should have it in his power to say, "I patronized Mr. Bowditch by buying his book, which I can not read." Later on he was able to commence the publication at his own expense.

The objects which Bowditch endeavored to accomplish by his translation and commentary, as stated by his biographers, were as follows:

(1) To supply those steps in the demonstration which could not be discovered without much study, and which had rendered the original work so difficult. The difficulty arose not merely from the intrinsic complexity of the subject and the medium of proof by the higher branches of mathematics, but chiefly from the circumstance that the author, taking it for granted that the subject would be as plain and easy to others as to himself, very often omits the intermediate steps and connecting links in his demonstrations. He jumps over the interval and grasps the conclusion by intuition. Bowditch used to say, "I never come across one of Laplace's '*Thus it plainly appears*' without feeling sure that I have hours of hard work before me to fill up the chasm and find out and show *how* it plainly appears."*

(2) The second great object of the translation was to continue the original work to the present time, so as to include the many improvements and discoveries in mathematical science that had been made during the twenty-five years succeeding the first publication. It is gratifying to know that the most eminent of contemporary mathemati-

* "The Mécanique Céleste is by no means easy reading. Biot, who assisted Laplace in revising it for the press, says that Laplace himself was frequently unable to recover the details in the chain of reasoning, and if satisfied that the conclusions were correct he was content to insert the constantly recurring formula, 'Il est aisé à voir.'" W. W. R. BALL'S *Short History of Mathematics*, p. 357.

cians pronounced his commentary a success, and agreed that Bowditch had attained the end he had in view, namely, to bring the work up with the times. Says Lacroix, July 5, 1836: "I am more and more astonished at a task so laborious and extensive. I perceive that you do not confine yourself to the mere text of your author and to the elucidations which it requires, but you subjoin the parallel passages and subsequent remarks of those geometers who have treated of the same subjects; so that your work will embrace the actual state of the science at the time of its publication." Legendre, July 2, 1832, says: "Your work is not merely a translation with a commentary; I regard it as a new edition, augmented and improved, and such a one as might have come from the hands of the author himself, if he had consulted his true interest, that is, if he had been solicitously studious of being clear." Mr. Babbage, of England, August 5, 1832, says: "It is a proud circumstance for America, that she has preceded her parent country in such an undertaking; and we in England must be content that our language is made the vehicle of the sublimest portion of human knowledge, and be grateful to you for rendering it more accessible." Similar testimony was given by Bessel and Encke in Germany; Puissant in France; Sir John Herschel, Airy, Francis Baily in England, and Cacciatore in Italy.

Bowditch once remarked that however flattering the testimony from foreigners might be, yet the most grateful tribute of commendation he had ever received was contained in a letter from a backwoodsman of the West, who wrote to him to point out an error in his translation of the Mécanique Céleste. "It is an actual error," said he, "which had escaped my own observation. The simple fact that my work had reached the hands of one on the outer verge of civilization who could understand and estimate it was more gratifying to my feelings than the eulogies of men of science and the commendatory votes of Academies." In America, many college professors were enabled by means of the translation and commentary to read and understand the Mécanique Céleste, who would otherwise have looked upon this work as a sealed book.

During the first thirty-five or forty years of this century but little was accomplished in this country in the line of astronomical observations. More was done in that respect during the days of David Rittenhouse than in the early part of this century. But, all at once, a great impetus was given to this kind of scientific work. In 1830 was erected the Yale College Observatory; in 1831 the observatory at the University of North Carolina; in 1836 the Williams College Observatory; in 1838 the Hudson Observatory, Ohio; in 1840 the Philadelphia High School Observatory and the West Point Observatory; in 1842 the National Observatory at Washington. Since then a large number of other observatories with excellent instruments have been built.

A plan for a National Observatory was submitted to the Government by Mr. Hassler, in his project for the survey of the Atlantic coast, as early as 1807. The proposition met with no favor. For many years

Congress opposed every such scheme. John Quincy Adams, in his annual message of 1825, strongly urged this subject upon the attention of Congress. In one place he said, "It is with no feeling of pride, as an American, that the remark may be made that, on the comparatively small territorial surface of Europe there are existing upward of one hundred and thirty of these light houses of the skies; while throughout the whole American hemisphere there is not one." President Adams's appeal was received with a general torrent of ridicule. "The proposition," says Loomis,* "to establish a light-house in the skies became a common by-word of reproach." It was not till 1842 that an appropriation was passed for an observatory, under the disguised name of a "Depot of Charts and Instruments."

It need hardly be said that in later years the U. S. Government has been very liberal in the encouragement of science.

Elementary Schools.

The beginning of this period is marked by a great revival of elementary education. Pestalozzian ideas had gained a foothold in England, and were now commencing to force their way into the western continent. In 1806 F. J. N. Neef, once an assistant to Pestalozzi, came to this country, and began teaching and disseminating the ideas of the Swiss reformer. The first fruit of Pestalozzian ideas in the teaching of arithmetic among us was Warren Colburn's Intellectual Arithmetic upon the Inductive Method of Instruction, known as the "First Lessons."

Warren Colburn worked, while a boy, at the machinist's trade. He then entered Harvard and graduated in 1820, having "mastered calculus and read a large part of Laplace." He then taught a select school in Boston. At this time he began preparing his little book. Of special interest is the following statement of Mr. Batchelder, of Cambridge, which shows how the First Lessons were prepared: "I remember once, in conversing with him with respect to his arithmetic, he remarked that the pupils who were under his tuition made his arithmetic for him; that he had only to give attention to the questions they asked and the proper answers and explanations to be given, in order to anticipate the doubts and difficulties that would arise in the minds of the pupils." He had read Pestalozzi, most probably, while in college. A manuscript copy of his First Lessons was furnished by Colburn to his friend George B. Emerson for use in a school for girls, and the former received valuable suggestions from the latter. The success of the book was almost immediate. No school-book had ever had such sale among us as this. Over three and one-half million copies were used in this country, and it was translated into several European languages.

Colburn's First Lessons embodied what was then a new idea among us. Instead of introducing the young pupil to the science of numbers, as did

* Recent Progress in Astronomy, especially in the United States, by Elias Loomis. New York, 1856, p. 205.

old Dilworth, by the question, "What is arithmetic?" and the answer, "Arithmetic is the art or science of computing by numbers, either whole or in fractions," he was initiated into this science by the following simple question: "How many thumbs have you on your right hand? How many on your left? How many on both together?" The idea was to begin with the concrete and known, instead of the abstract and unknown, and then to proceed gradually and by successive steps to subjects more difficult. In the publication of this book, the study of arithmetic in the schools of this country received its best impulse. "It led to the adoption of methods of teaching that have lifted the mind from the slavery of dull routine to the freedom of independent thought." (Edward Brooks.)

Colburn's First Lessons was followed in 1826 by his Arithmetic upon the Inductive Method of Instruction, being a Sequel to Intellectual Arithmetic. This was considered by its author to be superior to the First Lessons, but it did not meet with so great success. In 1825 he published his Algebra upon the Inductive Method of Instruction. Mr. Colburn did not long engage in teaching. Three years after graduation from college he was appointed superintendent of a manufacturing company at Waltham, and, soon after, of one at Lowell, Mass. He possessed great mechanical genius and administrative ability.

Though the First Lessons met with ready appreciation in New England, it must not be imagined that there was no opposition to it. Old notions could not be laid aside at once, and even where the new ideas had gained entrance, new books could not always be had readily. Now-a days we are apt to forget the difficulty and expense of transportation during the times preceding our railroad era. Says J. Stockton, in the preface to his Western Calculator (fourth edition, 1823, Pittsburg, Pa.), "to furnish our numerous schools, in the western (!) country, with a plain and practical treatise of arithmetic, compiled and printed among ourselves, thereby saving a heavy annual expense in the purchase of such books, east of the mountains, *and likewise the carriage thereof*, have been the motives which induced the compiler to undertake this work."

In spite of all obstacles Colburn's books gained ground steadily. Other books were written upon the same idea by different teachers. Old books underwent revision, so as to embody the new methods in part. Thus, the celebrated School-masters' Arithmetic of Daniel Adams, first published in 1801, was made to undergo a radical change. The old work was "synthetic." "If that be a fault of the work," says the author, "it is a fault of the times in which it appeared. The analytic or inductive method of Pestalozzi * * * is among the improvements of later years. It has been applied to arithmetic with great ingenuity by Mr. Colburn in our own country." "Instructors of academies and common schools have been so long attached to the old synthetic method of instruction, that, unhappily, many are still (1829) strongly opposed to the introduction of the valuable works of Colburn." "This [Adams's] work combines the new and the old."

The great success of Colburn's book did not prevent the appearance of arithmetical works that were quite as worthless as any of earlier years. There appeared others, on the other hand, which possessed no little merit and became very popular. As examples of the latter we would mention the arithmetics of the two brothers, Benjamin D. and Frederick Emerson, both of whom were well-known teachers in Boston.

The arithmetics of later days are combinations of the old, as found in our early arithmetics, and the new as found in the works of Colburn. For example: Our old arithmetics generally rejected reasoning, but gave rules; Colburn's books reject rules, but encourage reasoning. The better class of our later arithmetics contain rules, but, at the same time, give demonstrations and encourage students to think.

About the year 1825 or 1830, the French notation of numbers began rapidly to displace the English. Large numbers came to be marked off in periods of three digits instead of six. The earliest book in which we have noticed the adoption of the French notation is Robert Patterson's edition of Dilworth's School-master's Assistant, Philadelphia, 1805; the latest in which we have seen the English notation used is M. Gibson's revised edition of Abijah and Josiah Fowler's Youth's Assistant, Jonesborough, Tenn., 1850. Some of our recent books explain both, but use the French.

It is a rather curious fact that the process of *cancellation* did not come to be generally used in our arithmetics before about 1850. In 1840 C. Tracy published an arithmetic in which cancelling was freely used, a feature which was then "entirely peculiar to this treatise," and which distinguished it "from all others." John L. Talbott's Practical Arithmetic (Cincinnati, 1853) gives the "cancelling system," but only in the appendix, and remarks in the preface to it, "In Europe this system has been very generally adopted in the higher schools, and in this country it is fast becoming known—and, so far as it is known, it supersedes the usual modes of operation." Charles Davies takes pains to remark on the title-page of his University Arithmetic (1857) that "the most improved methods of analysis and *cancellation*" have been employed.

The order in which the various arithmetical subjects have come to be taught has been generally improved upon. Federal money and compound interest no longer precede common and decimal fractions, but come after them. Fractions have been moved much further toward the front part of our books. The placing of fractions toward the end of arithmetics had been due to the fact that the majority of pupils in olden times did not pursue mathematics long enough to master fractions, and were thus put through a course in arithmetic with only integral numbers. Those who *did* study fractions were made to learn the rules of interest and proportion over again "in vulgar fractions," and then again "in decimal fractions." Some of the old topics, such as single and double position, have since been quite generally dropped, but we think that there is still room for improvement in that respect. Nothing would

be lost and much gained if alligation, square and cube root, mensuration, and some of the more difficult applications of percentage should be dropped from our arithmetics. At least one new subject has been quite generally and, we think, appropriately introduced into our books—the metric system.

Before the time of Colburn, mental arithmetic was quite unknown in our schools. Since then mental and written arithmetic have not always been so closely united as they should be. The methods used in the two were frequently quite diverse. Too often they were taught almost like distinct sciences, so that a pupil might be quite proficient in the one without knowing anything of the other.

Grube's method of teaching numbers to children has been in use among us, especially in the East, but has never been generally adopted. It is such a refined method that few teachers possess the skill to apply it readily. The method has a desirable tendency to train ready and rapid calculators, and has much to commend itself to teachers.*

Since the beginning of this century arithmetic has come to be regarded as the most important, because the most practical, science in our elementary schools. Every farmer wished his sons to be good calculators; every business man desired to be "quick at figures;" hence its value was high in the estimation of all. Bookmakers were quick to profit by this sentiment. They began to multiply the number of text-books in the course until there were two books in mental arithmetic, and three in written, in several of the series in general use. As a rule, the examples in our arithmetics have not been well graded; difficult examples have been introduced so early in the course as to embarrass and discourage even the best students. Many examples were regular puzzles, not only to young boys and girls, but to almost any one not trained in algebra. There are numerous problems that should never have found a place in our arithmetics. We could quote from arithmetics dozens and dozens of such problems, but we shall give only one.

The 137th problem of the miscellaneous questions in the third part of Emerson's North American Arithmetic, published in 1835, is as follows:

> If 12 oxen eat up 3¼ acres of grass in 4 weeks, and 21 oxen eat up 10 acres in 9 weeks, how many oxen will eat up 24 acres in 18 weeks; the grass being at first equal on every acre, and growing uniformly.

The idea of placing a problem of such difficulty in a book for boys and girls! The history of this problem in this country shows very plainly that it is beyond the power, not only of pupils, but even of teachers of arithmetic. Many teachers whose minds had been trained by the study of algebra and geometry and, perhaps, even higher branches of mathematics, wrestled with it in vain. There existed so much uncertainty regarding its true solution that a premium of fifty

* For further information on Grube's method, see Prof. T. H. Safford's monograph on Mathematical Teaching, pp. 19.

dollars was offered in June, 1835, for the most "lucid analytical solution" of this question. A committee was appointed, with P. Mackintosh as chairman, to examine the solutions presented and award the prize. The committee reported 112 solutions received, of which only 48 gave the true answer, and awarded the prize to Mr. James Robinson,* principal of the department of arithmetic, Bowdoin School, Boston.*

Think of it! Out of 112 of, presumably, the best arithmeticians in the country, only 48 got correct results; and yet this problem was intended to be solved by boys and girls.

But the history of our problem is not yet complete. Nearly twenty-five years later a revision of Mr. Robinson's solution was submitted to the National Teachers' Association, at Washington, by the Hon. Finley Bigger, then Register of the U. S. Treasury; it was referred to the Mathematical Monthly for publication, and was printed in Vol. II, No. 3, December, 1859, pp. 82–85. Mr. Bigger assumed, "for the purpose of elucidation," that the question was susceptible of two constructions, and obtained two answers in addition to the true one. The editor of the Monthly appended an algebraic solution, and showed that there was only one answer that would satisfy all the conditions of the problem, and that Mr. Bigger was wrong in his conclusions.

There is no ambiguity in the problem. Twenty-three years later, Dr. Artemas Martin published several solutions of the problem in the Mathematical Magazine. Dr. Martin does not consider Mr. Robinson's solution very "lucid," and pronounces it liable to at least one other objection—it makes "mincemeat" of the oxen, inasmuch as fractions of oxen occur throughout the analysis of the question.

There is another curious fact connected with the history of this problem. Neither Mr. Emerson, nor the committee, nor Mr. Robinson, nor Mr. Bigger, nor the National Teachers' Association, nor the Mathematical Monthly, alludes to the fact that the question is taken from the *Arithmetica Universalis* of Sir Isaac Newton, published in 1704, which contains a "lucid analytical solution." Mr. Emerson's statement of the problem differs from that of Newton in this, that, owing to a misprint, the fraction $\frac{1}{2}$ instead of $\frac{1}{3}$ is given by the former in the number of acres contained in the first pasture, which mistake produces the absurd result of $37\frac{13}{175}$ oxen, instead of 36. The above question goes by the name of the "pasturage problem."

There exists a general feeling among mathematicians and educators that the teaching of arithmetic has been overdone in our schools. Parents have desired their older boys to be good mathematicians. But they failed to perceive the truth that the best review of arithmetic consists in the study of algebra; they looked upon algebra as utterly destitute of value. In consequence the boys have been made to waste

* Hendricks's Analyst, Vol. III, p. 75; also the Mathematical Magazine, edited by Dr. Artemas Martin, Vol. I, pp. 17 and 43.

their time at the study of circulating decimals, difficult problems in stocks and exchange, in general average, in alligation medial and alligation alternate, in square and cube root, and in combinations and permutations. From the manner in which these subjects have been treated in our arithmetics, a student derives very little mental training from them. The presentation of duodecimals is not only unphilosophical, but decidedly absurd.

Protests have been made from time to time against the over-study of arithmetic. Thus in 1866 the Superintendent of Public Instruction of California said in his Report (p. 119): "The crack classes are the arithmetic classes, and the merits of a whole school not unfrequently rise or fall with the exploits of the first class in arithmetic on 'examination day.' Arithmetic is well enough in its place, but the sky is not a blackboard, nor are mountains all made of chalk. Children have faculties other than that of calculation, and they need to be exercised on appropriate subjects." This doubtless voices the sentiments of many thinking teachers. Five years ago the writer heard Prof. Simon Newcomb, in a lecture at the Johns Hopkins University, protest against existing practices in the teaching of arithmetic.

Says Prof. T. H. Safford,* of Williams College: "The mathematics have their (disciplinary) value, and a very high one it is; but the lower mathematics, especially arithmetic, have been overdone in a certain direction; I mean that of riddles, puzzles, brain-spinning, as the Germans call it. While our boys and girls are given problems to solve which quite exceed their thinking powers—I don't suppose I could ever have gone successfully through Greenleaf's National Arithmetic till I had graduated from college—their minds are quite undeveloped in the power of observation, and they are often imperfectly trained in the four ground rules, especially in decimal fractions."*

A very remarkable and encouraging step toward reform was taken in 1887 by the Boston School Board. It passed the following orders concerning the study of arithmetic:†

"1. Home lessons in arithmetic should be given out only in exceptional cases.

"2. The mensuration of the trapezoid and of the trapezium, of the prism, pyramid, cone, and sphere; compound interest, cube root and its applications; equation of payments, exchange, similar surfaces, metric system, compound proportion, and compound partnership, should not be included in the required course.

"3. All exercises in fractions, commission, discount, and proportion, should be confined to small numbers, and to simple subjects and processes, the main purpose throughout being to secure thoroughness, accuracy, and a reasonable degree of facility in plain ordinary ciphering.

* The Development of Astronomy in the United States, 1888, p. 27.

† *The Academy*, January, 1888, article: "Arithmetic in Boston Schools," by General Francis A. Walker, President of the Massachusetts Institute of Technology.

"4. In 'practical problems,' and in examples illustrative of arithmetical principles, all exercises are to be avoided in which a fairly intelligent and attentive child of the age concerned would find any considerable difficulty in making the statement which is preliminary to the performance of the properly arithmetical operations. When arithmetical work is put into the form of practical or illustrative problems, it must be for the purpose of interesting and aiding the child in the performance of the arithmetical operations, and with a view to their common utility.

"5. In oral arithmetic no racing should be permitted; but the dictation should be of moderate rapidity.

"6. The average time devoted to arithmetic throughout the primary and grammar school course should be three and a half hours a week; and in the third primary grade not more than two hours, and in the first and second primary grades not more than three and a half hours each per week."

The considerations which led the School Board to introduce these changes are admirably set forth by General Francis A. Walker. The regulation regarding home lessons in arithmetic may be a good one under the conditions existing in Boston at the time of its adoption, but can hardly be recommended for general adoption. It sounds somewhat arbitrary. The reasons which led to its adoption are, (1) a tendency among grammar school teachers to unduly magnify the importance of arithmetic; (2) the injustice done as between pupil and pupil by giving home lessons, since the facilities for study at home are so very different among pupils; (3) the absence of the teacher prevents any authoritative interposition to put a stop to excessive, and therefore damaging, study over problems in the lesson. "In the old flogging days of the Army and Navy," says General Walker, "it was always required that the surgeon should stand by, to feel the pulse of the poor wretch under the lash, to watch the signs of approaching nervous collapse, and, in his discretion, to forbid the punishment to proceed further. But in the case of our young children who are assigned home lessons in arithmetic, no such humane provision exists. Were the work being done in the open school room, the severest master would, when he saw that the child did not understand the problem, could not do the work, and that it was only becoming more excited and fatigued by repeated attempts, interpose either to give assistance or to put a stop to the exercise. In the case of home lessons, however, the ambitious and sensitive child finds no relief. The work may go on long after the child should have been in bed until a state is reached where further persistence is not only in the highest degree injurious, but has no longer any possible relation to success."

"Regarding the remaining five orders, considered as a body," says General Walker, "it may be said that the committee, in framing them.

were actuated by the belief that both loss of time and misdirection of effort, with even some positively injurious consequences, were involved in the teaching of arithmetic, as carried on in some of the Boston schools. And here let me say, to prevent misapprehension, that the committee at no time intended to reflect on the schools of our own city as compared with those of neighboring cities and towns. Personally, I believe that the teaching of arithmetic has been more humane and rational of late years in the schools of Boston than in those of most New England towns and cities. What, then, are the faults complained of?

"*First*—That the amount of time devoted to this study is in excess of what can fairly be allotted to it, in the face of the demands of other and equally important branches of study.

"*Secondly*—That the study of arithmetic is very largely pursued by methods supposed to conduce to general mental training, which, in a great degree, sacrifice that facility and accuracy in numerical computations so essential in the after-life of the pupil, whether as a student in the higher schools or as a bread-winner.

"*Thirdly*—That, as arithmetic is taught in many, perhaps in most schools, the possible advantages of this branch of study, even as a means of general mental training and of the development of the reasoning powers, are, whether by fault of the text-book or of the individual teacher or of the standards adopted for examination, largely sacrificed through making the exercises of undue difficulty and complexity, which not only destroys their disciplinary value but becomes a means of positive injury."

The whole paper of General Walker is well worth reading. In one respect, however, we can not endorse the action of the Board. It seems to us that the metric system should be retained, even if the tables of apothecaries' weights and fluid measure, and of the mariner's measure, had to be omitted to make room for it. The memorizing of the tables in the metric system is not difficult. Moreover, what problems offer better opportunities for a good, thorough course in the use of decimal fractions than those involving meters and decimeters.

But there is still another reason for urging the spread of a knowledge of the metric system in elementary schools. If the masses have once acquired sufficient knowledge and familiarity with it as to see its transcending superiority over the old traditional tables of weights and measures now in use, then we may look forward more hopefully to the early approach of the time when the French weights and measures will be declared the only legal ones in the United States.

European nations that are generally regarded as being much more conservative than our own, have introduced them, to the exclusion of older ones. Even the miniature republic of Switzerland has, within the last ten years, adopted the metric system. The change was brought about without serious inconvenience.

UNITED STATES MILITARY ACADEMY.*

In 1817 began a new epoch in the history of the United States Military Academy. At this time Maj. Sylvanus Thayer became superintendent, and under him the Academy entered upon a career of unusual prosperity. Thayer was a native of Massachusetts, graduated at Dartmouth College, and then entered the Military Academy as a cadet in 1807. He was appointed lieutenant in the corps of engineers in 1808. At the close of the War of 1812 he was sent abroad by the Government to look into the military systems of Europe, particularly of France. After his return the Academy was reorganized according to French ideals, but without discarding entirely English teachings. Prof. Charles Davies says that in the construction of the course of study at West Point, "the beautiful theories of the French were happily combined with the practical methods of the English systems, and the same has since been done, essentially, in the schools of England and France." Maj. E. C. Boynton, in his History of West Point, summarizes the services of Major Thayer in the following manner: "The division of classes into sections, the transfers between the latter, the weekly rendering of class reports, showing the daily progress, the system and scale of daily marks, the establishment of relative class rank among the members, the publication of the Annual Register, the introduction of the Board of Visitors, the check-book system, the prepondering influence of the blackboard, and the essential parts of the regulations for the Military Academy as they stand to this day, are some of the evidence of the indefatigable efforts of Major Thayer to insure method, order, and prosperity to the institution. It was through the agency of Major Thayer that Prof. Claude Crozet, the parent of descriptive geometry in America, and one of the first successful instructors in higher mathematics, permanent fortifications, and topographical curves, became attached to the Academy." Crozet had been a French officer under Napoleon, and a pupil at the Polytechnic School in Paris.

Thayer was superintendent at West Point from 1817 to 1833. The great reputation which the Academy obtained was chiefly due to his efforts. His discipline was very strict. The last years of his administration were years of trial to him. It is said that his discipline was counted too stern, and that he was not sustained, as he should have been, at the War Department. Difficulties arose between him and the President of the United States, resulting in his leaving the Academy. General Francis H. Smith says of him:† "Colonel Thayer held the reins with a firm hand during his entire administration, and if, at times, he transcended the limits of legitimate authority, no private pique or personal interest swayed his judgment. He was animated by the single desire to give

* For Official Registers of the Military Academy and for valuable information regarding it, we are indebted to the kindness of W. C. Brown, First Lieutenant First Cavalry, Adjutant.

† West Point Fifty Years Ago, New York, 1879, p. 6.

efficiency to his discipline, and to train every graduate upon the highest model of the true soldier."

Andrew Ellicott was professor of mathematics from 1813 to 1820. The following description of him applies to the time preceding the arrival of Thayer. Says E. D. Mansfield: "There are some who will recollect Professor Ellicott sitting at his desk at the end of a long room, in the second story of what was called the Mess Hall, teaching geometry and algebra, looking and acting precisely like the old-fashioned school-master, of whom it was written,

> "'And still they gazed, and still the wonder grew
> That one small head could carry all he knew.'

"In the other end of the room, or in the next room, was his acting assistant, Stephen H. Long. * * * The text-book used was Hutton's Mathematics, and at that time the best to be had. * * * It was a good text-book then, for there were no cadets trained to pursue deeper or more analytical works."

As already stated, Superintendent Thayer caused the classes to be divided into sections. From the reminiscences given by John H. B. Latrobe, who entered the Academy as a cadet in 1818, we see that the various sections received their mathematical instruction from assistants, and that the professor of mathematics occasionally visited the sections. Mr. Latrobe says:* "I do not remember upon what principle our class of one hundred and seventeen members was divided into four sections; I recollect, however, that I was put into the first section. * * * Our recitation room was next the guard room, on the first floor of the North Barracks. Here, on a rostrum, between two windows, sat Assistant Professor S. Stanhope Smith, and here, with the first volume of Hutton's Mathematics in hand, I began my West Point education. * * *

"I am not sure that we had desks, but rather think that we were seated on benches against the wall, with a blackboard to supply the place of pen and ink and slates, although I am not certain about the slates. Generally we had the section room to ourselves. Sometimes, however, Mr. Ellicott would pay us a visit and ask a few questions, ending with giving us a sum in algebra, to explain what was meant by 'an infinite series,' which was the name he went by in the corps."

"I have," continues Latrobe (p. 29), "no other recollection of him as an instructor, except once when, while learning surveying, we were chaining a line from a point in front of his house to an angle of Fort Clinton, and back again. Our accuracy quite astonished the good old professor, to whom we did not admit that it was owing to our having used the same holes that the pins had made in going and returning."

Professor Ellicott died at West Point and was buried in the cemetery there. "My last visit to it as a cadet," says Latrobe, "was when I

* Report Association of Graduates of the U. S. Military Academy, 1887, p. 8.

was on the escort that fired the vollies over the grave of Andrew Ellicott, the professor of mathematics who lies buried there."

Of Hutton's Mathematics Latrobe says: "I have often heard those who have been more recently educated at West Point speak disparagingly of the Huttonian day, as though *any one* could have graduated then." That this was not the case becomes evident when he says, "that the first sifting in June, 1819, of my one hundred and seventeen comrades of the year before, reduced the number to fifty-nine, the next sifting to forty-eight, and the number that got through the meshes of the sieve was but forty. Of the others, some resigned, some were 'turned back' to go over the year's course a second time, and some were found to be deficient altogether. These last were called, in the parlance of the cadets, 'Uncle Sam's bad bargains.'"

Jared Mansfield, the professor of natural and experimental philosophy, outlived Ellicott by ten years. Both were veteran surveyors and mathematicians. Mansfield retired from his chair in 1828. Mr. Latrobe says that Colonel Mansfield, "although a most competent instructor, was very near-sighted, and I am not prepared to say that this defect was not sometimes taken advantage of." Professor Church (class of 1828) says of him: "Professor Mansfield at my time was very old, yet quite enthusiastic in his branch of study, generally a mere listener to demonstrations, complimentary to a good one, but coldly silent to a bad one."

The great impulse to the study of mathematics at West Point was, however, due to younger men. One of these was Claude Crozet. After graduating at the Polytechnic School in Paris, he had been artillery officer under Napoleon. From 1816 to 1817 he was assistant professor of engineering at the Academy, and from 1817 to 1823 full professor. E. D. Mansfield has given us some interesting recollections of Crozet's earliest teaching at West Point. The Junior class of 1817–18 was the first class which commenced thoroughly the severe and complete course of studies at the Academy. Of Professor Crozet, Mansfield says that he was to teach engineering, but when he met the class he found that he would have to teach mathematics first, as not one of them had had the necessary preliminary training in pure mathematics for a course in engineering. "The surprise of the French engineer, instructed in the Polytechnique, may well be imagined when he commenced giving his class certain problems and instructions which not one of them could comprehend and perform."

Among the preliminary studies we find that descriptive geometry was included. "We doubt," says E. D. Mansfield, "whether at that time more than a dozen or two professors of science in this country knew there was such a thing; *certainly* they never taught it, and equally certain there was no text-book in the English language." This science, founded by Monge, was then scarcely thirty years old. Crozet meant to begin by teaching this branch, but a new difficulty arose. Just then he had no text-book on the subject, and geometry

could not be taught orally. What was to be done? "It was here at this precise time that Crozet, by aid of the carpenter and painter, introduced the blackboard and chalk. To him, as far as we know, is due the introduction of this simple machine. He found it in the Polytechnique of France." (E. D. Mansfield).

Crozet was, however, not the first one to use the blackboard in this country. Of Rev. Samuel J. May, of Boston, it is said that, "to the work of teaching a public school he then brought one acquisition which was novel in that day, and which it has taken a half century to introduce into elementary schools, private and public—a knowledge of the uses of the blackboard, *which he had seen for the first time in* 1813 *in the mathematical school kept by Rev. Francis Xavier Brosius*, a Catholic priest of France, who had one suspended on the wall with lumps of chalk on a ledge below and cloth hanging on either side."* One thing is certain: The blackboard was introduced in this country by Frenchmen. Its importance in the school room can hardly be overestimated. Simple and inexpensive as it is, its introduction into our colleges was not instantaneous. For geometrical teaching large tablets with printed diagrams were used in our best colleges long after Crozet had taught its use at West Point.

Crozet, says E. D. Mansfield, did not more than half understand English. "With extreme difficulty he makes himself understood and with extreme difficulty his class comprehend that two planes at right angles with one another are to be understood on the same surface of the blackboard, on which are represented two different projections of the same subject." The first problems were drawn and demonstrated on the blackboard by the professor; afterward they were drawn and demonstrated by the pupils, and then carefully copied into accurate drawings.

In 1821 Crozet published his Treatise on Descriptive Geometry, for the use of cadets of the U. S. Military Academy (New York). The first 87 pages were given to the elementary principles, and the next 63 pages to the application of descriptive geometry to spherics and conic sections. This is, according to our information, the first English work of any importance on descriptive geometry, and the first work published in this country which exhibits to the student that gem of geometry—Pascal's Theorem.

Crozet has been called the father of descriptive geometry in this country. He taught this as preparatory to engineering. It may justly be said, also, that the course of military science was greatly developed by him.

Mr. Latrobe favors us with the following recollections of him: "There are persons whose appearance is never effaced from the memory. Of this class was the professor of the art of engineering, Col. Claude Crozet, a tall, somewhat heavily-built man, of dark complexion, black

* "American Educational Biography," Barnard's Journal, Vol. XVI, p. 141, 1866.

hair and eyebrows, deep-set eyes, remarkable for their keen and bright expression, a firm mouth and square chin, a rapid speech and strong French accent. I can, even after the lapse of between sixty and seventy years, fancy that I see the man before me. He had been an engineer under Napoleon at the battle of Wagram and elsewhere, and the anecdotes with which he illustrated his teaching were far more interesting than the 'Science of War and Fortification,' which was the name of our text-book at the time. When he left the Academy he became chief engineer of the State of Virginia, which is indebted to him for the system that made her mountain roads the best, then, in America. Perhaps my recollection of Colonel Crozet is strengthened by my having seen him long after I ceased to be his pupil."

Ellicott was succeeded in the professorship of mathematics by David B. Douglass. He held it till 1823, when he was transferred to the department of engineering, where he taught till 1831. Professor Church (class of 1828) says of him: "Professor Douglass, of engineering, had the reputation of being an able engineer and a fine scholar, yet he was by no means a clear demonstrator. His style was diffuse and there was a great want of logical sequence in his language. Most of the course of engineering was given to the class by him from the blackboard." He was afterward the chief engineer of the Croton waterworks.

One of the text-books mentioned by E. D. Mansfield as having been used was the Mechanics of Dr. Gregory ("Old Greg."), who was professor at the Royal Military Academy at Woolwich. His works are collections of rules rather than expositions of principles, and are wanting in analysis. Gregory is at his best when he descends to the minutiæ of practice. For several years no adequate text-book was found for civil engineering. In 1823 Major O'Conner translated a Treatise on the Science of War by De Vernon, which had been prepared in 1805 by the order of the French Government, and was the text-book in the polytechnic school. This translation was used at the Academy for several years. "It was a miserable translation," says General Francis H. Smith, "but it was the best that could be had, and each member of the first class was required to take a copy, costing some $20."

After being vacated by Douglass, the chair of mathematics was taken by one whose name became known to nearly every school-boy in our land—Charles Davies. He was a native of Connecticut, graduated at the Academy in 1815, and then was made assistant professor of mathematics. He held the full professorship for fourteen years, until 1837. He earned for himself a wide reputation, not as an original investigator in mathematics, but as a teacher and as a compiler of popular text-books. He was always described by his pupils as an excellent instructor. Professor Church (class of 1828) says of him: "Professor Davies was then young, enthusiastic, a clear and logical demonstrator, and an admirable teacher. He had at once imbibed the spirit and fully sympathized in

the desires of the superintendent, and labored earnestly to carry them out in building up a logical system of instruction and recitation, which required not only a thorough understanding of the details of and reasons for everything proposed, but a clear, concise, and complete examination of it."

When Church was a cadet, according to his own statement, the methods of instruction were entirely new, and text-books very imperfect. The professors and teachers had themselves to learn the true use of the blackboard, and the strict and detailed manner of demonstration. In algebra the best text-book that could be obtained was a poor translation of Lacroix. In geometry we had a translation of Legendre; in trigonometry, a translation of Lacroix; in descriptive geometry, a small work by Crozet, containing only the elements without application to the intersection of surfaces or to warped surfaces. These, with the whole of shades, shadows, and perspective, stone-cutting, and problems in engineering, were given by lecture to the class. Notes were taken by the cadets, the drawings made in our rooms before the next morning, then presented for examination, and at once recited upon previous to the following lecture. The sections in mathematics, philosophy, and engineering were of twenty cadets each, and were kept in three hours daily. Biot's work on analytical geometry was used, and Lacroix's calculus.

Those who have toiled over Davies' text-books may enjoy the following reminiscences of him: "Don't you remember," says General F. H. Smith (class of 1833), "when muttering out an imperfect answer to one of his questions, how he would lean forward with one of his significant smiles and say, 'How's that, Mr. Bliss?' But I will not now dwell upon his long and faithful career in the department of mathematics. The results of his labors are to be seen in the distinguished career of his pupils and in his series of mathematical text-books, which are as household words everywhere in the United States."

When John H. B. Latrobe was a cadet, Davies was as yet only assistant professor. Latrobe speaks of him as follows: "My next professor of mathematics, in my second year's course, was one that I have no difficulty in describing and whom I can never forget, Charles Davies. Personally and mentally he was a remarkable man. Of the middle size, with a bright, intelligent face, characterized by projecting upper teeth, which procured for him the name of 'Tush' among the cadets, his whole figure was the embodiment of nervous energy and unyielding will. His fearless activity at a fire which happened in a room in the South Barracks, in 1819, added the name of 'Rush' to the other. He was a kindly natured man, too, and the patient perseverance that he devoted to the instruction of his class was not the least remarkable feature of his character. It was with Professor Davies that I began the study of descriptive geometry, for which no books in English had then been published. He had no assistance beyond the blackboard and his own intimate knowledge of the subject and faculty of oral explanation. For-

tunately this was exceptionally great, and even then there was no little amount of actual labor requisite to enable the pupil to understand the difference betweeen the horizontal and vertical planes and the uses to be made of them. It is to Professor Davies that I have always attributed in a great measure my subsequent successes at West Point, and hence this especial notice of him as a tribute to his memory. A much more enduring tribute is that awarded by the countless beneficiaries, the colleges, schools, and individuals who have profited by his numerous publications in connection with mathematical science."

Professor Davies taught for many years before he conceived the idea of issuing a series of text-books. Some of his books—as his Legendre and Bourdon—were adaptations from French works, modified to supply the wants of our schools; others were prepared on his own plan. While connected with the Academy as professor he published his Descriptive Geometry, 1826 (a more extensive work than Crozet's); Brewster's Translation of Legendre, 1828; Shades, Shadows, and Perspective, 1832; Bourdon's Algebra, 1834; Analytical Geometry, 1836; Differential and Integral Calculus, 1836; a Mental and Practical Arithmetic. Overwork in the preparation of these text-books caused bronchial affection, which forced him to resign his professorship in 1837. He visited Europe and soon after his return occupied the professorship of mathematics at Trinity College, Hartford, Conn., but ill-health again induced him to exchange the position for that of paymaster in the Army and treasurer at West Point. These offices he resigned in 1845. In 1848 he became professor of mathematics and philosophy in the University of New York, but in the following year he retired to Fishkill Landing, on the Hudson, that he might have leisure to complete his series of text-books. After teaching in the normal school at Albany, he was made professor of higher mathematics at Columbia College, in 1857.

In 1839 appeared his Elementary Algebra; in 1840, his Elementary Geometry and Trigonometry; in 1846, his University Arithmetic; in 1850, his Logic of Mathematics; in 1852, his Practical Mathematics; in 1855, jointly with William G. Peck, a Mathematical Dictionary.

Davies' series constituted a connected mathematical course, from primary arithmetic up to calculus. His books were, as a rule, perspicuous, clear, and logically arranged. They were not too difficult for the ordinary student, and contained elements of great popularity. The original editions would be found quite inadequate for the wants of schools of the present day. "The first translations of Bourdon and Legendre were imperfect" (Prof. C. W. Sears, class of 1837). Davies himself greatly modified some of his text-books in later editions. In his revisions he was greatly aided by his son-in-law, Prof. William G. Peck. The most recent revisions are those made by Prof. J. Howard Van Amringe, of Columbia College.

Brewster's Legendre underwent some changes in the hands of Davies. In the original work, as also in the translation of Brewster and Farrar,

each proposition was enunciated with reference to and by aid of the particular diagram used for the demonstration. But Davies gave the propositions without reference to particular figures and, to that extent, returned to the method of Euclid. In later editions Davies did not use Brewster's translation, but took the original and translated and adapted it to the courses in American schools. In trigonometry he was wedded to the line system.

The reasoning sometimes employed by Professor Davies in his books has been found to be open to objection. This is certainly true of his treatment of infinite series. In his Legendre the treatment of the *circle* is not such as will carry conviction to the young mind. Thus, he says in one edition, that "the circle is but a regular polygon with an infinite number of sides."* A trained mathematician who feels that he can give more rigorous proofs by sounder methods, whenever he may wish to do so, will employ this idea of the circle, and of curves in general, with profit and satisfaction. After much study he may even arrive at the conviction that the method of limits and that of infinitesimals are essentially alike. But it is the experience of the majority of our teachers that the infinitesimal method and the treatment of the circle as a polygon appear to beginners as enigmatical and obscure. Of our more recent geometries, the best and the most popular have abandoned those methods.

Nor is Davies' explanation of a limit and of the first differential co-efficient satisfactory. Listen to the testimony of one of his pupils:† "I had not been a teacher of the calculus long * * * before I discovered that I had almost everything to learn respecting it as a rational system of thought. Difficulties were continually suggested in the course of my reflection on this subject about which I had been taught nothing, and consequently knew nothing. I found, in short, that I had only been taught to work the calculus by certain rules, without knowing the real reasons or principles of those rules; pretty much as an engineer, who knows nothing about the mechanism or principle of an engine, is shown how to work it by a few superficial and unexplained rules."

It is our opinion that under Professors Davies and Church the philosophy of mathematics was neglected at West Point. If this criticism be true of West Point, which was for several decennia unquestionably the most influential mathematical school in the United States, how much more must it be true of the thousands of institutions throughout the country which came under its influence? If this stricture were not correct, then such a book as Bledsoe's Philosophy of Mathematics would never have been written; there would have been no occasion for it.

Of Davies' assistants, we shall mention Lieutenant Ross. General F. H. Smith says: "There was associated with Professor Davies, * * *

* Davies' Legendre, 1856, Book V, Scholium to Proposition XII.

† Prof. A. T. Bledsoe, Philosophy of Mathematics, 1867, p. 214, note.

as his chief assistant in mathematics, having charge of the fourth class, Lieut. Edward C. Ross, of the class of 1821. He was the best teacher of mathematics I ever knew, and it is singular, too, that he had no faculty of demonstration. He gave to our class many extra discussions in the difficult points in algebra, particularly on what he called the final equations, for he was not pleased with Farrar's translation of La Croix, our text-book in algebra, and he was preparing his translation of Bourdon. In putting upon the blackboards these extra demonstrations every line appeared as if it had been printed, so neat was he in the use of his chalk pencil. But when he commenced to explain he would twist and wriggle about from one side of the board to the other, pulling his long whiskers, and spitting out, in inordinate volumes, his tobacco juice. The class was as ignorant when he closed as when he began. We copied, word for word, what was written, well knowing that on the next day the first five would be called upon to make the discussion. We read to him what we had placed on the board. Then commenced his power as a teacher. In a series of orderly questions he would bring out the points of the discussion, step by step, sometimes occupying half an hour with each cadet, and when the three hours of recitation were over we knew the subject thoroughly. He was an expert in his power of questioning a class. He did this without note or book, and gave such earnestness and vividness to his examinations that he kept his class up to the highest pitch of interest all the time."

General Smith gives us a description also of Courtenay. "Edward H. Courtenay, who graduated at the head of Ross's class, was our professor of natural and experimental philosophy, fifty years ago. There never was a clearer minded—a more faithful teacher—or a more modest one than Professor Courtenay. Well do I remember the hesitating manner with which he would correct the grossest error on the part of a member of his section—*I hardly think so.* He resigned his professorship in 1834, and after holding many offices of high dignity, as professor and civil engineer, he was elected professor of mathematics in the University of Virginia in 1842."

Courtenay was instructor at West Point from 1821 to 1834, excepting the four years from 1824 to 1828. During his first years of teaching he was assistant professor of natural and experimental philosophy, and then assistant professor of engineering. After the resignation of Jared Mansfield he was appointed professor of natural and experimental philosophy, and acted in that capacity for five years. In 1833 he translated from the French of M. Boucharlat an elementary treatise on mechanics, and made additions and emendations.

The chair of military and civil engineering, made vacant by the resignation of Professor Douglass, was filled by the appointment of Lieut. Dennis H. Mahan. Mahan graduated at West Point in 1824, holding the first place in his class of thirty-one members. "After remaining at the Academy as an instructor for two years, he was ordered to Europe

to study public engineering works and military institutions. By special favor of the French Ministry of War, Lieutenant Mahan was allowed to join the Military School of Application for Engineers and Artillerists at Metz, where he remained for more than a year, under the instruction of men whose names were then, and are now, widely known in science."* When, after his return, he entered upon the duties of his department at West Point, he supplemented the meagre volume of O'Connor with extensive notes. These notes developed into his well-known treatises on Civil Engineering and Field Fortifications.

Such is the brief record of the professional career of Professor Mahan; but it fails to convey any adequate idea of the influence which he exerted upon engineering science in this country. To appreciate this, it must be remembered that for many of those forty-one years (during which he was professor) West Point was our only school of mathematical and physical science where the rigid requirements and high standard now deemed essential were even attempted. Every officer of the present corps of engineers who has served long enough to win reputation in the performance of the civil duties assigned to that corps, and many of the eminent civil engineers of the country as well, now gratefully remember how, before those old blackboards in that unpretending recitation room at West Point, they learned from Professor Mahan, with the rudiments of their profession, a high-toned discipline and the fundamental truth that without precision of ideas, rigid analysis, and hard work there can be no such thing as success.

"But if civil engineering owes much to our late colleague, military engineering and the science of war owe more. For many years, and up to the day of his death, he was in that branch of the profession confessedly the highest scholastic authority in America."*

The death of Mahan was pathetic. In his last years he often had fits of melancholy, and, in an instant of acute insanity, he plunged from a steamer into the Hudson and drowned.

Professor Davies was succeeded in the chair of mathematics by Prof. Albert E. Church. Church was a native of Connecticut, graduated at West Point in 1828, served as assistant professor from 1828 to 1831, also from 1833 to 1837. He was then acting professor of mathematics for about a year, and in 1838 he became full professor, retaining the chair till his death in 1878. He published four works which have been used considerably in American colleges. His Differential and Integral Calculus, 1842, was more extensive than that of Davies. In the new edition of 1851 a chapter on the Elements of the Calculus of Variations was inserted. In 1851 appeared his Analytical Geometry, in which he followed somewhat the work of Biot on this subject. In 1857 his Plane and Spherical Trigonometry was published. In 1865 appeared his Elements of Descriptive Geometry, in the preparation of which he was

* Biographical Memoirs of the National Academy of Sciences, Vol. II, 1886, p. 32.

aided by the French works of Leroy and Oliver, and by the elaborate American work of Warren. Later editions give also the application of the subject to shades, shadows, and perspective. The Descriptive Geometry met with larger sales than any of his other works.

As a teacher, Professor Church is spoken of by General F. H. Smith as follows: "Prof. Albert E. Church was an assistant professor of mathematics when my class entered in 1829. He occasionally heard my section in the third class course and exhibited then the clearness and perspicuity which marked his long career as a professor of mathematics."

Prof. Arthur S. Hardy (class of 1869) gives the following reminiscences of the mathematical teaching in his day:*

"The class was divided into sections of from ten to fifteen. The alphabetical arrangement, first adopted, became in a few weeks a classification by scholarship—transfers up and down being made weekly. The descent was easy, but it was hard to rise a section. The last section was called *Les Immortels* (lazy mortals?). In each section each studies recited daily. The sections were taught by army officers detailed at Professor Church's request. The latter had no section, but generally visited each daily. Each recitation was one hour and a half long. Professor Church's visits were dreaded. He usually asked questions. His questioning was searching. He was a stickler for form—it was not enough to *mean* right.

"Personally he did not inspire me; he had no magnetism—was dry as dust, as his text-books are. He delivered one lecture on the calculus. I never got a glimpse of the philosophy of mathematics—of its history, methods of growth. The calculus was a machine, where results were indisputable, but its mechanism a mystery. I do not think he had a great mathematical mind. It was geometrical, rather than an analytic one. A problem, which he and Professor Bartlett once attacked together, the latter solved by a few symbols on a piece of paper, while the former drew a diagram with his cane on the gravel—to Bartlett's disgust, who despised geometry. Church's text-books are French adaptations, minus the luminousness and finish of form of French text-books.

"The only instance of Church's being disconcerted was on being told by a cadet that the reason for + becoming — in passing through zero was that the cross-piece got knocked off in going through. You can imagine that the would-be wit was placed in arrest.

"The mathematical recitation at West Point was a drill-room. In my judgment its result was a soldier who knew the manœuvres, but it did not give an independent, self-reliant grasp of methods of research. In descriptive geometry, the Academy had a magnificent collection of models, but they were shown us *after* the study was finished—in other words, mental discipline was the object—practical helps and ends were secondary. Great changes have been made since."

William H. C. Bartlett (class of 1826) was assistant professor for

*Letter to the writer, November 12, 1888.

several years at West Point. He was permanently appointed professor of natural and experimental philosophy in 1836. In 1871 he was retired from military service at his own request, and shortly after he accepted the place of actuary for the Mutual Life Insurance Company of New York.

The need of an astronomical observatory being felt at West Point, Professor Bartlett went abroad in 1840 to order instruments and visit observatories. On his return it was necessary to provide room for the instruments in the new library building of the school, on account of the great prejudice existing in Congress against a separate observatory.*

Bartlett published treatises on Optics, 1839; Acoustics; Synthetic Mechanics, 1850; Analytical Mechanics, 1853; Spherical Astronomy, 1855. He contributed also to Silliman's Journal. His Analytical Mechanics is the first American work of its kind which starts out with, and evolves everything from, that precious intellectual acquisition of the nineteenth century—the laws of the indestructibility of matter and energy. Dr. R. S. McCulloch (who, by the way, rewrote Bartlett's Mechanics without allowing his own name to appear anywhere in the revised edition) says:† "More than thirty years ago, at West Point, Professor Bartlett, in his treatise on Analytical Mechanics, still used there as a text-book, had deduced the whole science from one single equation, or formula, well known to every cadet as his equation *A*; and he thus expressed and discussed fully what now is generally called the Law of the Conservation of Energy."

Bartlett's successor was Prof. Peter S. Michie, the present incumbent in the chair. Michie graduated in 1863, and has been instructor there since 1867. He has published Wave Motion, Relating to Sound and Light, 1882; Hydrostatics; and Analytical Mechanics, 1886. The first edition of the last treatise was never published; the second edition, 1887, differs considerably from the first. It is on the plan of Bartlett's book on the same subject, but it is confined to mechanics of solids. It contains also a good introduction to graphical statics, a subject which, in recent years, has come to be studied in this country. The first to place a treatise on graphical statics in the hands of American engineers was A. Jay Du Bois,‡ professor at Lehigh University, Pa. This subject owes its development chiefly to Culmann, who, in 1866, published in Zurich his *Graphische Statik*. In technical schools in Europe this method has been favorably received. In this country original contributions of great value have been made to this subject by Prof. Henry T. Eddy, of the University of Cincinnati.§

* The Development of Astronomy in the United States, by Prof. T. H. Safford, 1888, p. 19.

† Papers read before the New Orleans Academy of Sciences, 1886-87, Vol. I., No. 1, p. 120.

‡ The Elements of Graphical Statics and their Application to Framed Structures, New York, 1875.

§ Van Nostrand's Engineering Magazine, 1878. Article: "A New General Method in Graphical Statics."

In 1841 Professor Church was aided in his department by five assistants. This number has been increased since, and is now nine. These assistants have been, we believe, always selected from young graduates of the Academy. The course of study in pure and applied mathematics was, in 1841, as follows: *Fourth Class* (first year), Davies' Bourdon, Legendre, and Descriptive Geometry; *Third Class*, Davies' works on Shades and Shadows, Spherical Projections and Warped Surfaces, Surveying, Analytical Geometry, and Calculus; *Second Class*, Courtenay's Boucharlat's Traité de Mécanique, Roget's Electricity, Magnetism, Electro-Magnetism and Electro-Dynamics, Bartlett's Optics, Gummere's Astronomy; *First Class*, Mahan's Treatises on Field Fortification, Lithographic Notes on Permanent Fortification, Attack and Defence, Mines and other Accessories, Composition of Armies, Strategy, Course of Civil Engineering, Lithographic Notes on Architecture, Stone Cutting, Mechanics (studied by the first section only).

As Church's and Bartlett's text-books came from the press they were introduced in place of earlier ones. Thus, Davies' Geometry, Calculus, Descriptive Geometry, and Trigonometry, Gummere's Astronomy, and Courtenay's Boucharlat's Mechanics were gradually displaced by new books. But some of Davies' books have been retained to the present day. We may here state that the power of selecting text-books does not lie with each individual professor, but with the Academic Board.

After the death of Mahan, in 1871, the chair of military and civil engineering was given to Junius B. Wheeler, of the class of 1855. He retired in 1884, and was succeeded by James Mercur. Professor Wheeler gradually substituted books of his own in place of Mahan's treatises.

Professor Church's successor is Prof. Edgar W. Bass, of the class of 1868. By him more attention is given to the philosophical exposition of fundamental principles than was given by his predecessors. Davies' and Church's text-books are still used, but they are much modified by copious notes by Professor Bass. In calculus the notation of Leibnitz has always been used, but now the Modern is also given. At present the calculus is based upon the Newtonian conception of rates, but his notation is not used. In 1879 determinants and least squares were introduced into the course of study, Peck's Determinants and Chauvenet's Least Squares being the text-books used.

The present mode of instruction in mathematics involves recitations by cadets at the blackboard, lectures and explanation of the text, numerous applications of each principle, and written recitations by the students. The sections number from nine to twelve students, for one and a half hour's instruction. Three hours are allotted for the study of each mathematical lesson. Recitations are daily, Sundays excepted.

The course of study for 1888 is as follows: *Fourth class*, Davies' elements of Algebra, Legendre's Geometry, Ludlow's Elements of Trigonometry, Davies' Surveying, Church's Analytical Geometry; *Third class*,

Church's Analytical Geometry, Descriptive Geometry with its applications to Spherical Projections, Bass's Introduction to the Differential Calculus, Church's Calculus, Church's Shades, Shadows, and Perspective, Chauvenet's Treatise on the method of Least Squares; *Second class*, Michie's Mechanics, Bartlett's Astronomy, Michie's Elements of Wave Motion relating to Sound and Light; *First class*, Wheeler's Civil Engineering, Field Fortifications, Mercur's Mahan's Permanent Fortifications (edition of 1887); Wheeler's Military Engineering (Siege Operations and Military Mining), Elements of the Art and Science of War, and Mahan's Stereotomy. For reference, is used a book called Royal Engineers' Aide-Mémoire, Parts I and II.

It may be stated, in conclusion, that the U. S. Military Academy has contributed to the educational force of the country no less than thirty-five presidents of universities or colleges, twenty-seven principals of academies and schools, eleven regents and chancellors of educational institutions and one hundred and nineteen professors and teachers, making a total of one hundred and ninety-two instructors of youth distributed throughout the country.*

HARVARD COLLEGE.

In 1807 John Farrar succeeded Samuel Webber in the chair of mathematics and natural philosophy. Farrar was a native of Massachusetts. After graduating at Harvard he studied theology at Andover, but having been appointed tutor of Greek, in 1805, he never entered the ministerial office. He retained his chair till 1836, when he resigned in consequence of a painful illness that finally caused his death. He was a most amiable, social, and excellent man, and endeared to his friends. By the students he was familiarly called "Jack Farrar."

Prof. Andrew P. Peabody gives the following reminiscences of him:† "He delivered, when I was in college, a lecture every week to the Junior class on natural philosophy, and one to the Senior class on astronomy. His were the only exercises at which there was no need of a roll-call. No student was willingly absent. The professor had no notes, and commenced his lecture in a conversational tone and manner, very much as if he were explaining his subject to a single learner. But whatever the subject, he very soon rose from prosaic details to general laws and principles, which he seemed ever to approach with blended enthusiasm and reverence, as if he were investigating and expounding divine mysteries. His face glowed with inspiration of his theme. His voice, which was unmanageable as he grew warm, broke into a shrill falsetto; and with the first high treble notes the class began to listen with breathless stillness, so that a pin-fall could, I doubt not, have been heard through the room. This high key once reached there was no return to the lower

*Annual Report of the Board of Visitors to the U. S. Military Academy made to the Secretary of War, for the year 1886.

†Harvard Reminiscences, by Andrew P. Peabody, Boston, 1888, p. 70.

notes, nor any intermission in the outflow and quickening rush of lofty thought and profound feeling, till the bell announced the close of the hour, and he piled up all the meaning that he could stow into a parting sentence, which was at once the climax of the lecture, and the climax of an ascending scale of vocal utterance higher, I think, than is within the range of an ordinary soprano singer. I still remember portions of his lectures, and they now seem to me no less impressive than they did in my boyhood."*

Josiah Quincy † says in his diary, which he kept while a student at college, that by the prolixity of Professor Everett in his lectures, "we gained a miss from Farrar for the fourth time this term. This was much to the gratification of the class, who in general hate his branch, though they like him."

Professor Farrar did not distinguish himself by original research in mathematics, but he was prominent and among the first to introduce important reforms in the mathematical teaching in American colleges. He was the first American to abandon English authors and to place translations of Continental works on mathematics in the hands of students in the New World.

In 1818 appeared Farrar's Introduction to the Elements of Algebra, selected from the Algebra of Euler. Notwithstanding the transcending genius of Euler as a mathematician and the high estimation he was held on the Continent, his algebra was scarcely to be met with previous to this time, either in America or England. It was written by the author after he became blind, and was dictated to a young man entirely without education, who by this means became an expert algebraist.‡ Farrar's Euler was a very elementary book, and was intended for students preparing to enter college. It differed from the English works in this, that it taught pupils to reason, instead of to memorize without understanding.

In the same year appeared also Farrar's translation of the Algebra of Lacroix, which was first published in France about twenty years previously. Lacroix was one of the most celebrated and successful teachers and writers of mathematical text-books in France. Farrar translated also Lacroix's Arithmetic, but this does not appear to have been

* Professor Peabody continues his reminiscences as follows: "I recall distinctly a lecture in which he exhibited, in its various aspects, the idea that in mathematical science, and in it alone, man sees things precisely as God sees them, handles the very scale and compasses with which the Creator planned and built the universe; another in which he represented the law of gravitation as coincident with, and demonstrative of, the divine omnipresence; another, in which he made us almost hear the music of the spheres, as he described the grand procession, in infinite space and in immeasurable orbits, of our own system and the (so called) fixed stars. His lectures were poems, and hardly poems in prose; for his language was unconsciously rythmical, and his utterances were like a temple chant."

† Figures of the Past, p. 23.

‡ North American Review, 1818.

received as favorably in this country as the other works of the Harvard professor.

In 1819 was published at Cambridge the Geometry of that famous French mathematician, Legendre. A similar translation was made in England by David Brewster. Legendre has been the greatest modern rival of Euclid. In France, in most schools in America, and in some English institutions, the venerable and hoary-headed Euclid was made to withdraw and make room for Legendre.

If the question be asked, what is the difference between the geometries of Euclid and Legendre, we would answer that the main object of Legendre was to make geometry easier and more palatable to students. This he succeeded in doing, but at a sacrifice of scientific rigor. The following are the principal points of difference between Euclid and Legendre: (1) Legendre treats the theory of parallels differently; (2) Legendre does not give anything on proportion, but refers the student to algebra or arithmetic. The objection to this procedure is that in arithmetic and algebra, the properties of proportion are unfolded with regard to numbers, but *not* with regard to magnitude in general. From a scientific point of view this is a serious objection, especially if we remember that in geometry incommensurable quantities arise quite as frequently as commensurable quantities do. Euclid's treatment of proportion displays wonderful skill and rigor, but is very difficult and abstract for students beginning the study of geometry; (3) Euclid never supposes a line to be drawn until he has first demonstrated the possibility and shown the manner of drawing it. Legendre is not so scrupulous, but makes use of what are called "hypothetical constructions." (4) Legendre introduces new matter, especially in solid geometry, changes the order of propositions, and gives new definitions (as, for instance, his definition of a straight line).

In 1820 Farrar published his translation of Lacroix's Trigonometry. The original gave the centesimal division of the circle, but in the translation the sexagesimal notation was introduced. This trigonometry adopted the "line system." Bound together with this book was the "Application of Algebra to Geometry." This was selected from the Algebra of Bezout. Regarding this selection Professor Farrar says: "It was the intention of the compiler to have made use of the more improved treatise of Lacroix or that of Biot upon this subject; but as analytical geometry has hitherto made no part of the mathematics taught in the public seminaries of the United States, and as only a small portion of time is allotted to such studies, and this is in many instances at an age not sufficiently mature for inquiries of an abstract nature, it was thought best to make the experiment with a treatise distinguished for its simplicity and plainness." *

The next book in the "Cambridge Course of Mathematics," as Farrar's works were called, was an Elementary Treatise on the Application

* See advertisement to the Treatise.

of Trigonometry (1828), in the preparation of which were used Cagnoli's and Bonnycastle's Trigonometries, Delambre's Astronomy, Bezout's Navigation, and Puissant and Malortie's Topography.

In 1824 were published the First Principles of the Differential and Integral Calculus, "taken chiefly from the mathematics of Bezout." This is the first text-book published in America on the calculus and employing the notation of Leibnitz. It is based on the infinitesimal method. Bezout flourished in France before the Revolution. His works were, therefore, at this time, rather old, but his calculus was selected in preference to others "on account of the plain and perspicuous manner for which the author is so well known, as also on account of its brevity and adaptation in other respects to the wants of those who have but little time to devote to such studies." *

The introduction is taken from Carnot's *Réflexions*, and gives the explanation by the "compensation of errors."

The translation of Bezout's calculus is only in part the work of Professor Farrar. After having begun it, he was obliged to go to the Azores, on account of the health of his wife, and the translation was completed by George B. Emerson. He had it printed with his introduction and notes, so that when Professor Farrar returned he found it ready for use in the college. †

Farrar's translations and selections from French authors were at once adopted as text-books in some of our best institutions. Several books in the series were used at the U. S. Military Academy and at the University of Virginia.

The professor of mathematics and natural philosophy was always assisted by tutors. They generally taught the pure mathematics to the lower classes. In 1825 there were three. One of them, James Hayward, had been tutor for five years, and had striven to reform the teaching of elementary geometry. He was made professor in 1826, but a year later he severed his connection with the college and engaged in civil engineering, in which he became a high authority. The original survey of the Boston and Providence Railway was made by him. Among the other tutors of note who served during the time of Professor Farrar were Thomas Sherwin, A. P. Peabody, and Benjamin Peirce. Dr. Peabody is now Plummer professor of Christian morals, emeritus. Among Harvard men of Farrar's time are also Charles Henry Davis, who afterward served on the Coast Survey and established the American Ephemeris, and Sears Cook Walker, who, later, became a noted astronomer.

We now proceed to inquire into the terms for admission and the courses of study. Since 1816 the whole arithmetic has been required for admission to Harvard College. In 1819 a trifling amount of algebra was added. The catalogue of 1825 specifies the requirements as follows:

*Advertisement to the translation.

†Barnard's Journal, 1878, "Schools as They Should Be," by George B. Emerson.

"Fundamental rules of arithmetic; vulgar and decimal fractions; proportion, simple and compound; single and double fellowship; alligation, medial and alternate; and algebra to the end of simple equations, comprehending also the doctrine of roots and powers, and arithmetical and geometrical progression." The books used in the examination were the Cambridge editions of Lacroix's Arithmetic and Euler's Algebra. In 1841 Euler's Algebra *or* the First Lessons in Algebra were required. No other changes were made until 1843. The catalogue for that year mentions for admission Davies' First Lessons in Algebra to "Extraction of Square Root;" and "An Introduction to Geometry from the most approved Prussian text-books, to VII.—Of Proportions." No other subjects were added until 1866–67, though there were some changes in the text-books. In 1850 Davies' and Hill's Arithmetics are mentioned; in 1853 Davies' and Chase's Arithmetics; in 1859 Davies', Chase's, or Eaton's Arithmetics, Euler's Algebra, or Davies' First Lessons, or Sherwin's Common School Algebra, and the Introduction to Geometry; in 1865 Chase's Arithmetic, Sherwin's Algebra, Hill's Second Book in Geometry, Parts I and II, or "An Introduction to Geometry as the Science of Form" as far as p. 130.

In addition to these statements taken from catalogues it will be interesting to add the following account, given by Prof. William F. Allen, of the class of 1851:* "The requirements for admission were not much above a common school. That is, I got my arithmetic and algebra in a country district school (well taught). Geometry I picked up for myself in a very small quantity. I remember at the entrance examination I was asked what an angle was. I thought I knew, but I think I convinced the examiner that I didn't; however, I got in clear."

During the first ten or eleven years of his teaching Professor Farrar used the books of Samuel Webber. A second edition of Webber's Mathematics appeared during Professor Farrar's incumbency. In 1818 the course of study in mathematics was as follows:† *Freshmen*, algebra and geometry, during the first and second term and three weeks of the first term. *Sophomores*, algebra, trigonometry and its applications to heights and distances, and navigation during the third term. *Juniors*, natural philosophy and astronomy (Enfield's), mensuration of superficies and solids, and surveying during the third term. In place of Hebrew, on the written request of their parents or guardians, students were permitted to attend to mathematics with the private instructor, or Greek, or Latin, or French; *Seniors*, conic sections and spherical geometry during the first term and half of the second. We are informed, moreover, that for the attendance on the private instuctor in mathematics, which was optional, there was a separate charge, at the rate of $7.50 per quarter.

There was a public examination of each class in the third term and a

* Letter to the writer, November 6, 1888. † North American Review, March, 1818.

public exhibition of performances in composition, elocution, and in the mathematical sciences, three times a year. Prizes were also given. The Bowdoin prize dissertation was read in the Chapel in the third term. Of these prizes, the first premium was given in 1815 to Jared Sparks, of the Senior class, for a dissertation "On the character of Sir Isaac Newton, and the influences and importance of his discoveries." The title of this essay would show that Sparks had, very probably, studied fluxions, though this branch was not included in the cirriculum for 1818, given above. Fluxions never had been a regular study, obligatory upon all the students, though provisions seem to have been made for those wishing to prosecute it.

During the twenty-nine years that Farrar was professor, from 1807 to 1836, 275 theses were written by students on mathematical subjects at Harvard, and deposited in the library of the college. Of these about one-fourth contain solutions of fluxional problems (or problems involving the differential and integral calculus); a little less than one-half are on the calculation and projection of eclipses; the remaining on algebra, mechanics, surveying, etc. Many of these papers are interesting memorials of men since become in different degrees famous. Thus George Bancroft wrote, in 1817, a thesis, "Invenire Motum Verum Modorum Lunæ;" George B. Emerson, on "Fluxional Solutions of Problems in Harmonicks" (1817); Warren Colburn, on "Calculation of the Orbit of the Comet of 1819;" Sears Cook Walker, in 1825, on "The Transit of Venus in 1882," and "The Effect of Parallax upon the Transit in 1882;" Benjamin Peirce, in 1828, on "Solutions of Questions * * * from the Mathematical Diary, etc.;" Wendell Phillips, in 1831, on "Some Beautiful Results to which we are Led by the Differential Calculus in the Development of Functions."*

The catalogue for 1820 shows that Webber's Mathematics and Euclid's Geometry had been discarded. Farrar's new books came now to be used. The Freshmen studied Legendre's Geometry and Lacroix's Algebra. Analytic methods began to acquire a foothold. Conic sections were displaced by analytic geometry, which, with trigonometry, was begun in the Sophomore and concluded in the Junior year. The Catalogues from 1821 to 1824, inclusive, do not give the course of study. In 1824 the Juniors studied, during the second term, differential calculus from Bezout's work, unless they exercised their privilege of electing modern languages in place of mathematics. The catalogue of 1830 shows some slight changes in the course. The Freshmen studied Legendre's Plane Geometry, algebra, solid geometry; the Sophomores, trigonometry and its applications, topography, and *calculus;* the Juniors, natural philosophy and mechanics in the second term, and electricity and magnetism in the third term; the Seniors, optics and natural philosophy.

The following remarks by Dr. Peabody applying to this period are

* "Mathematical Theses of Junior and Senior Classes, 1782–1839, by Henry C. Badger," Bibliographical Contributions of the Library of Harvard College, No. 32.

instructive: "The chief labor and the crowning honor of successful scholarship were in mathematics and the classics. The mathematical course extended through the entire four years, embracing the differential calculus, the mathematical treatment of all departments of physical science then studied, and a thoroughly mathematical treatise on astronomy. (Gummere's, afterward replaced by Farrar's almost purely descriptive treatise."*)

The year 1832 marks an epoch in the history of mathematical teaching at Harvard. It was then that Benjamin Peirce became professor of mathematics and natural philosophy.

While there had been men in this country who had cultivated mathematics with ardor, they had seldom possessed the talent and aspirations for original research in this science. We have had many who were called "mathematicians," but if this name be used in the highest sense, and be conferred upon only such persons as have been able to discover mathematical truths not previously known to man, then it can fall upon very few Americans. The mere ability of mastering the contents of even difficult mathematical books, or of compiling good school-books in this science, does not make him a mathematician worthy of standing by the side of Legendre, the Bernoullis, Wallis, Abel, Tartaglia, or Pythagoras—to say nothing of such master minds as Archimedes, Leibnitz, and Newton. But at last we have come to a name which we may pronounce with pride as being that of an American *mathematician*. We need not hesitate to rank along with the names of Wallis and John Bernoulli that of Benjamin Peirce.

It has been said that a young boy detected an error in the solution given to a problem by Nathaniel Bowditch. "Bring me the boy who corrects my mathematics," said Bowditch, and Benjamin Peirce, thirty years later, dedicated one of his great works "To the cherished and revered memory of my master in science, Nathaniel Bowditch, the father of American Geometry." The title of "father of American Geometry," which Peirce confers upon his beloved master, has been bestowed by foreign mathematicians upon Peirce himself. Sir William Thomson referred, in an address before Section A of the British Association, to Peirce as, "the founder of high mathematics in America." On a similar occasion Arthur Cayley spoke of him as the "father of American mathematics."

Benjamin Peirce was born at Salem in 1809. He entered Harvard College at the age of sixteen, and devoted himself chiefly to mathematics, carrying the study far beyond the limits of the college course. Thus he attended lectures on higher mathematics by Francis Grund. While an under-graduate he was a pupil of Nathaniel Bowditch, who perceived the genius of the young man and predicted his future greatness. Bowditch directed him in the development of his scientific

* Harvard Reminiscences, by A. P. Peabody, p. 203.

powers, and gave him valuable instruction in geometry and analysis. When Bowditch was publishing his translation and commentary of the Mécanique Céleste, Peirce helped in reading the proof-sheets, and thereby contributed greatly toward rendering it free from errors. This critical reading of that great work of Laplace must have been an education to him in itself. Indeed, a great part of Peirce's scientific labors was in the field of analytic mechanics.

Dr. Peabody gives the following reminiscences of Peirce:* "While Benjamin Peirce the younger was still an under-graduate * * * it was said that in the class-room he not infrequently gave demonstrations that were not in the text-book, but were more direct, summary, or purely scientific than those in the lessons of the day. College classes were then farther apart than they are now; but even in our Senior year we listened, not without wonder, to the reports that came up to our elevated platform of this wonderful Freshman, who was going to carry off the highest mathematical honors of the university. On graduating, he went to Northampton as a teacher in Mr. Bancroft's Round Hill School, and returned to Cambridge in 1831 as tutor. The next year the absence of Professor Farrar in Europe left him at the head of the mathematical department (which he retained till his death), the following year receiving the appointment of professor; while Mr. Farrar on his return was still unable to take charge of class instruction."

In 1842 Peirce was appointed Perkins professor of astronomy and mathematics. This position he held until his death, in October, 1880. Tutor Henry Flint is the only person ever connected with the college for a longer time.

We shall first speak of the mathematical text-books written by Peirce, then of his record as a teacher, and, lastly, of his original researches.†

As soon as he entered upon his career as teacher of mathematics at Harvard he began the preparation of mathematical text-books. In 1835 appeared his Elementary Treatise on Plane Trigonometry, and in 1836 his Elementary Treatise on Spherical Trigonometry. The two were published in a single volume in later editions. In 1836 appeared also his Elementary Treatise on Sound; in 1837, his Elementary Treatise on Plane and Solid Geometry and his Elementary Treatise on Algebra; during the period 1841–46 he wrote and published in two volumes his Elementary Treatise on Curves, Functions, and Forces; in 1855, was published his Analytical Mechanics.

Rev. Thomas Hill, ex-President of Harvard and an early pupil of Peirce, speaks of these books as follows: "They were so full of novelties that they never became widely popular, except, perhaps, the Trigonometry; but they had a permanent influence upon mathematical teaching in this country; most of their novelties have now become commonplaces in all text-books. The introduction of infinitesimals or of limits

* Harvard Reminiscences, p. 181.

† We shall draw freely from the Memorial Collection, by Moses King, 1881.

into elementary books; the recognition of direction as a fundamental idea; the use of Hassler's definition of sine as an arithmetical quotient, free from entangling alliance with the size of the triangle; the similar deliverance of the expression of derivative functions and differential co-efficients from the superfluous introduction of infinitesimals; the fearless and avowed introduction of new axioms, when confinement to Euclid's made a demonstration long and tedious—in one or two of these points European writers moved simultaneously with Peirce, but in all he was an independent inventor, and nearly all are now generally adopted."

The ratio system in trigonometry was used before this by Hassler in his masterly, but ill-appreciated, work on Analytic Trigonometry, and also by Charles Bonnycastle in his Inductive Geometry. But this system met with no favor among teachers. The most popular works on trigonometry, such as the works of Davies and Loomis, as also those of Smyth, Hackley, Robinson, Brooks, and Olney, adhered to the old and obsolete "line system," and it was not till within comparatively recent years that the "ratio system" came to be generally adopted. The old "line system" was brought to America from England, but the English discarded it earlier than we did. In 1849 De Morgan wrote that the old method of defining trigonometric terms was universal in England until *very lately*.

The final victory of the system in this country is due chiefly to the efforts of Peirce, Chauvenet, and their followers. It is significant that Loomis, in a late edition of his trigonometry, has been driven by the demands of the times to abandon the old system.

The advisability of using infinitesimals and the idea of direction in elementary text-books will be discussed in another place.

About the beginning of the second quarter of this century considerable dissatisfaction came to exist among the public about the college system as it was then conducted in this country. The people demanded a change from the old scholastic methods. Then for the first time arose the now familiar cry against forcing the ancient languages upon all students entering college. It was demanded that greater prominence be given to modern languages, to English literature, to practical mechanics, and that the student should have some freedom in the selection of his studies. Though some few modifications were made here and there in the college courses, the "New Education" did not secure a firm hold upon our colleges until the third quarter of the present century.

In these reforms Harvard has always taken a prominent part. The elective system there has been traced back to 1824, when Juniors could choose a substitute for 38 lessons in Hebrew, and Seniors had the choice between chemistry and fluxions. Benjamin Peirce was an enthusiastic advocate of the elective system.

We now proceed to give the courses in mathematics during the early

part of Peirce's connection with the college. His own text-books were adopted as soon as they came from the press. In 1836 and 1837 the Freshmen used Walker's Geometry, Smyth's Algebra, Peirce's Plane and Spherical Trigonometry; the Sophomores, Farrar's books on Analytical Geometry, Calculus, and Natural Philosophy; the Juniors continued the Natural Philosophy.

In the catalogue for 1838 we notice important changes. The Freshmen studied Peirce's Geometry and Algebra; the Sophomore class was divided into three sections, of which the first pursued practical mathematics, including mensuration, dialing, construction of charts, surveying, the use of globes and instruments in surveying, during the first term; and during the second term the general principles of civil engineering, nautical astronomy, and the use of the quadrant. This section was evidently intended to meet the demands of the time for practical knowledge, without having first laid a broad and secure theoretical foundation. But little could be accomplished in civil engineering without a knowledge of calculus. The second section reviewed arithmetic, geometry, and algebra; then took up conic sections, fluxions, and the mathematical theory of mechanics. The third section, intended for students of mathematical talents and taste, pursued analytic geometry, theory of numbers and functions, differential and integral calculus, and mechanics.

But this arrangement did not prove satisfactory. The facts are that Professor Peirce's text-books were found very difficult, and Peirce himself was not a good teacher, except for boys of mathematical genius. Peirce was anxious to introduce the elective system, so that students without mathematical ability would not be forced to pursue mathematics beyond their elements. In May, 1838, a vote was passed, permitting students to discontinue their mathematics at the end of the Freshman year if they chose to. The catalogue for 1839 announced that "every student who has completed during the Freshman year the studies of geometry, and algebra, plain trigonometry with its applications to heights and distances, to navigation, to surveying, and that of spherical trigonometry, and who has passed a satisfactory examination in each to the acceptance of the mathematical department and a committee of the overseers—may discontinue the study of mathematics at the end of the Freshman year, at the written request of his parent."

Referring to these changes the president said, the following year, that the liberty to discontinue mathematics at the end of the year had been found highly acceptable to both students and parents and had, thus far, been attended by no ill consequences; that elections in the secondary course had had a tendency to encourage those capable of profiting by the study of that branch; that those possessing mathematical talent were stimulated; that of fifty-five, only eight continued mathematics; and that the head of the department considered the voluntary system superior. The difficulties in the mathematical course for the Sopho-

mores seemed to be removed. But how about the Freshmen? Mathematical studies were not popular with them; they complained of overwork. In 1839 the committee on studies reported that "the mathematical studies of the Freshman class are so extensive as to encroach materially upon the time and attention due to other branches," and proposed to remove the time when mathematical studies may be discontinued, from the end of the Freshman to the first term of the Sophomore year.

The catalogue for 1838–39 gives no mathematics for the Junior and Senior years. The following year Peirce's Treatise on Sound was studied by the Juniors. In 1841 an extended mathematical course was offered in the Junior and Senior years. The Juniors were to study Peirce's Treatise on Sound and the Calculus of Variations and Residuals; the Seniors, Poisson's Mécanique Analytique and Celestial Mechanics. The number of students venturing to enter such difficult but enchanted fields of study were but few. In 1843 there were only two sections instead of three as before. One was called the course in Practical Mathematics, comprising Peirce's Plane and Spherical Trigonometry; the other was called the course in Theoretical Mathematics, in which Peirce's Algebra was concluded, and his Curves, Functions, and Forces, studied as far as "Quadratic Loci." These two courses continued through the Junior and Senior years. The studies offered varied somewhat from year to year.

In obedience to the practical demands of the times, the Lawrence Scientific School was opened in 1842 as a branch of Harvard. It began as a school of chemistry. But by the year 1847 the plan of this school was broadened so as to embrace other sciences. "There shall be established in the University an advanced school for instruction in theoretical and practical science and in other usual branches of academic learning." Instruction was to be given by Professor Horsford in chemistry, by Professor Agassiz in zoölogy and geology, by Professor Lovering in experimental philosophy, by William Bond in practical astronomy, and by Professor Peirce in higher mathematics, especially in analytical and celestial mechanics. The course offered by Professor Peirce to students in this school, in 1848, was as follows:

COURSE IN MATHEMATICS AND ASTRONOMY.

I.—CURVES AND FUNCTIONS.

Regular course.	*Parallel course.*
PEIRCE. Curves and Functions.	BIOT. Géométrie Analytique.
LACROIX. Calcul Différential et Intégral.	CAUCHY. Cours d'Analyse de l'École Royale Polytechnique.
CAUCHY. Les Applications du Calcul Infinitésimal à la Géométrie.	HAMILTON's researches respecting quaternions. (Transactions of the Royal Irish Academy, Vol. XXI.)
MONGE. Application de l'Analyse á la Géométrie.	

II.—ANALYTICAL AND CELESTIAL MECHANICS.

Regular course.	*Parellel course.*
LAPLACE. Mécanique Céleste, translated, with a Commentary, by Dr. Bowditch. Vol. I.	POISSON. Mécanique Analytique.
BOWDITCH. On the Computation of the Orbits of a Planet or Comet; Appendix to Vol. III of his translation.	LAGRANGE. Mécanique Analytique.
AIRY. Figure of the Earth, from the *Encyclopædia Metropolitana.*	HAMILTON. General Method in Dynamics, from the London Philosophical Transactions for 1834 and 1835.
AIRY. Tides, from the *Encyclopædia Metropolitana.*	GAUSS. Theoria Motus Corporum Coelestium.
	BESSEL. Untersuchungen.
	LEVERRIER. Développements sur Plusieurs Points de la Théorie des Perturbations des Planètes.
	LEVERRIER. Les Variations Séculaires des Élémens des Orbites, pour les Sept Planètes Principales.
	LEVERRIER. Théorie des Mouvements de Mercure.
	LEVERRIER. Recherches sur les Mouvments de la Planète Herschel.
	ADAMS. Explanation of the Observed Irregularities in the Motion of Uranus, on the Hypothesis of Disturbances caused by a more distant Planet.

III.—MECHANICAL THEORY OF LIGHT.

Regular course.	*Parallel course.*
AIRY. Mathematical Essays.	CAUCHY. Exercices d'Analyse et de Physique Mathématiques.
MACCULLAGH, On the Laws of Crystalline Reflection and Refraction. (Transactions of the Royal Irish Academy, Vol. XVIII.)	NEUMANN. Theoretische Untersuchung der Gesetze, nach Welchen das Licht reflectirt und gebrochen wird. (Transactions of Berlin Academy for 1835.)

Such a course of studies had never before been open to American students in any American college. Such a course, or any other equally advanced, was never presented in any other American institution before the arrival at the Johns Hopkins University of Professor Sylvester. It must be admitted that the great mass of Harvard students never studied more mathematics than was absolutely required for their degree, but now and then Peirce had a pupil who liked mathematics, understood the greatness of his teaching, and appreciated and loved his character. Peirce was the center of an influence which led to the starting of many a since distinguished scientific career. Prof. T. H. Safford, one of his favorite pupils, says: "Among distinguished scholars of the years which I remember, were Prof. G. P. Bond, afterward of the Observatory; Dr. B. A. Gould, celebrated as an astronomer; Rev. Thomas Hill, for a while president of Harvard; Prof. J. D. Runkle, of the Institute of Technology, Boston; Prof. J. E. Oliver, of Cornell Univer-

sity; Prof. A. Hall and Prof. S. Newcomb, of the U. S. Navy; Mr. W. P. G. Bartlett, since deceased; Mr. G. W. Hill, of the Nautical Almanac Office; Mr. Chauncey Wright, known as a philosopher; Prof. James M. Peirce and President Eliot, of Harvard; Prof. C. M. Woodward, of St. Louis; Rev. G. W. Searle, of St. Paul's (R. C.) Church, New York City; Prof. W. Watson, formerly of the Institute of Technology; Professor Byerly, now at Harvard."*

Of Peirce as a teacher, Dr. A. P. Peabody gives us an interesting account.† It refers to the first year that Peirce was professor at Harvard.

"For the academic year 1832–33, I, as tutor, divided the mathematical instruction with Mr. Peirce. * * * He took to himself the instruction of the Freshmen. The instruction of the other three classes we shared, each of us taking two of the four sections into which the class was divided, and interchanging our sections every fortnight. * * * In one respect I was Mr. Peirce's superior, solely because I was so very far his inferior. I am certain I was the better instructor of the two. The course in the Sophomore and Junior years, embracing a treatise on the Differential Calculus, with references to the calculus in the text-books on mechanics and other branches of mixed mathematics, was hardly within the unaided grasp of some of our best scholars; and, though no student dared to go to the tutor's room by daylight, it was no uncommon thing for one to come furtively in the evening to ask his teacher's aid in some difficult problem or demonstration. For this purpose resort was had to me more frequently than to my colleague, and often by students who for the fortnight belonged to one of his sections. The reason was obvious. No one was more cordially ready than he to give such help as he could; but his intuition of the whole ground was so keen and comprehensive that he could not take cognizance of the slow and tentative processes of mind by which an ordinary learner was compelled to make his step-by-step progress. In his explanations he would take giant strides; and his frequent, 'you see,' indicated what he saw clearly, but that of which his pupils could get hardly a glimpse. * * *

"Our year's work was on the whole satisfactory, and yet I think that we were both convinced that the differential calculus ought not to have been a part of a prescribed course. There was a great deal of faltering and floundering, even among else good scholars. * * * Our examinations were *viva voce*, in the presence of a committee of reputed experts in each several department. We shrank from the verdict of our special committee in no part of our work except the calculus. As the day approached for the examination in that branch we were solicitous that Robert Treat Paine, who was on the committee, should not be present; for we supposed him to be the only member of the committee who was conversant with the calculus. He did not come, and we were

*Letter to the writer, November 6, 1888. †Harvard Reminiscences, 1888, p. 182.

glad. * * * If there were defects and shortcomings, there was certainly no one present who could detect them."

Peirce had no success in teaching mathematics to students not mathematically inclined. Repeated and loud complaints were made at Harvard that the mathematical teaching was poor. The majority of students disliked the study and dropped it as soon as possible. Says Prof. William F. Allen (class of 1851) in a letter to the writer:

"I am no mathematician, but that I am so little of one is due to the wretched instruction at Harvard. Professor Peirce was admirable for students with mathematical minds, but had no capacity with others. He took only elective classes, and of course I didn't elect. Only two did in our class of about sixty, and I believe they soon dropped it. In my Freshman year I had very good instruction from Mr. Child, now the professor of English literature, and editor of ballads. I had algebra and geometry with him, and did fairly well. In the Sophomore year (trigonometry and analytic geometry) we had a different instructor, and it was a mere farce. In analytic geometry I was taken up once in the course of the term, on rectangular co-ordinates in space, and I knew perfectly well (although I was never so told) that at examination I should be called up upon rectangular co-ordinates in space. (Written examinations had never been heard of.) When examination day came (a committee in attendance) the tutor was sick, and a shudder ran through the class. But he heroically pulled himself together and held his examination in person, and I was examined upon rectangular co-ordinates in space. The sum of my knowledge of analytical geometry at the present day is that there are such things (or were) as rectangular co-ordinates in space—and I suppose there must also be some out of space. * * * Peirce's text-books were used. His geometry I liked much, also the algebra, only that it was pretty hard. * * *

"I graduated in 1851, and I remember when I was in Germany two or three years later, I met a gentleman who had just returned from America—a young German *Gelehrte*—and he assured me that there was not one mathematician in the United States, and only one astronomer, Peirce. It was not an agreeable thing to be told, for a patriotic young American as I was then, but I suppose it was not far from the truth."

In January, 1848, Thomas Sherwin, by order of the committee for examination in mathematics reported that in 1847 there were present for examination but one Senior in Bowditch's Laplace, and only five Juniors in Curves and Functions. He went on to say that mathematics could be made attractive, that, hence, arose the inquiry, why this study was so very decidedly unpopular at the University, and why so general an opinion prevailed throughout the community, that the student stood less creditable in this branch than in others. The answer to this was that the text books were abstract and difficult, that few could comprehend them without much explanation, that Peirce's works were symmetrical and elegant, and could be perused with pleas-

ure by the adult mind, but that books for young students should be more simple. The report then says that there are mathematical works of no small merit, which embraced the same subjects as the text-books now used, which were much less difficult of comprehension, such as Bourdon's Algebra, J. R. Young's Treatises, and a recent edition of Hutton's Mathematics.

The majority report was followed by a minority report by Thomas Hill and J. Gill, which differed regarding the text-books at Harvard. "Your minority of the committee believe that these text-books, by their beauty and compactness of symbols, by their terseness and simplicity of style, by their vigor and originality of thought, and by their happy selection of lines of investigation, offer to the student a beautiful model of mathematical reasoning, and lead him by the most direct route to the higher regions of the calculus. For those students who intend to go farther than the every-day applications of trigonometry, this series of books is, in the judgment of the minority, by far the best series now in use."

While the good qualities of Peirce's text-books, as described by the minority, must be acknowledged, it is nevertheless true, that owing to their compactness and brevity, which characterize all the writings of Peirce, the books seemed obscure to beginners.* Still, however, they continued to be used at Harvard for many years longer.

Professor Peirce said in his report of November 6, 1849, on the teaching of higher mathematics in the college and the Lawrence Scientific School, that he had two pupils. One of these students was a member of the Lawrence Scientific School, and the other was the child, T. Henry Safford, who had attracted so much attention for his early development of mathematical ability. "These two students attended lectures on analytical mechanics, and young Safford showed himself perfectly competent to master the difficult subject of research, and once or twice surprised his teacher by the readiness with which he anticipated the object of some peculiar form of transformation. Up to this time Safford fully realizes his early promise of extraordinary powers as a geometer, but his friends cannot free themselves from apprehension, when they perceive that the growth of his body does not correspond to that of his intellect." He then states that with the mathematical pupil of the school the professor read also Lagrange's *Mécanique Analytique* and Laplace's *Théorie Analytique des Probabilités*.

*In another place, Rev. Thomas Hill speaks of these books as follows (memorial collection by Moses King): "His text-books were also complained of for their condensation, as being therefore obscure; but under competent teachers, the brevity was the cause of their superior lucidity. In the Waltham High School his books were used for many years, and the graduates attained thereby a clear and more useful, applicable knowledge of mathematics than was given at any other high school in this country; nor did they find any difficulty in mastering even the demonstration of Arbogast's Polynomial Theorem, as presented by Peirce. The latter half of the volume on the integral calculus, full of the marks of a great analytical genius, is the only part of all his text-books really too difficult for students of average ability."

As regards the number of students electing mathematics, the committee of overseers stated, in 1849, that as long as the choice is offered, the lighter labor will always be preferred, and that this tendency will probably get stronger. "Hebrew roots and polynomial roots will be neglected in a garden abounding with French bouquets and Italian music; and even now it can not surprise us that, while the Smith professor of French and Spanish language and literature, and instructor in Italian, is surrounded by a gay crowd of utilitarian admirers, the Perkins professor of mathematics and astronomy is working in his deep mines for one infant prodigy and one eminent Senior." Some Juniors studied analytical statics, and gave the best evidence of successful devotion to the subject.

The elective system was abandoned almost completely in 1850. Mathematical studies were elective only in the Junior and Senior years. In 1867 the elective system was again adopted at Harvard, and on the most liberal scale. Sophomore mathematics were again no longer "required." Peirce's books still held their ground. The only invaders were J. M. Peirce's Analytical Geometry, and, in 1865, Puckle's Analytic Geometry. In 1869 the committee of overseers reported that mathematics was a required study only for the Freshmen; that elective mathematics were taken this year by one hundred Sophomores, six Juniors, and eight Seniors; that the Sophomores and Juniors could elect either pure or applied. They also stated that "the number electing this department in the upper classes is never expected to be large, as the studies are advanced beyond what most students have either aptitude or occasion for."

We find that, "for the last few years of his life Professor Peirce had for his pupils only young men who were prepared for profounder study than ever entered into a required course, or a regularly planned curriculum; but he never before taught so efficiently, or with results so worthy of the mind and heart and soul, which he always put into his work." *

It will be instructive to listen to what former pupils of Peirce have to say of him. Prof. Truman Henry Safford, of Williams College, says in a letter to the writer: "I was a student at Harvard in the class of 1854. Prof. B. Peirce taught the Sophomores, I believe (I entered the Junior class), but not very well; he had hardly patience enough, I suppose. To the Juniors and Seniors he lectured on higher algebra, the calculus, and analytical mechanics. His lectures were substantially contained in his text-books—Algebra, Curves and Functions, and Analytic Mechanics. They were very interesting and inspiring to those who could follow them. There was but little practice; the examples in the book were generally worked out. In my class a number (twelve or so) took the first year's work; the second, which included integral calculus, complex numbers, and analytical mechanics, was taken by

* Harvard Reminiscences, by A. P. Peabody, p. 186.

four only. One of them was C. K. Lowell, afterwards a cavalry general in the Civil War, a nephew of Professor Lowell; another, George Putnam, Esq.; a third, W. C. Paine, afterwards a West Point scholar, where he was first in his class, and a lieutenant of engineers, but he resigned as a captain. The fourth was myself. I had heard some of Professor Peirce's lectures some years before, while a school-boy, but could not follow them so well."

For some years following 1838 Prof. Joseph Lovering taught classes in mathematics. Of him and Peirce, Edward E. Hale says:* "The classical men made us hate Latin and Greek; but the mathematical men (such men! Peirce and Lovering) made us love mathematics, and we shall always be grateful to them."

Says Thomas Wentworth Higginson: "As to mathematical instruction, this reform (elective system) was an especial benefit, for Professor Peirce's genius revelled in the new sensation of having voluntary pupils, and he gave a few of us his Curves and Functions as lectures, with running elucidations. Nothing could be more stimulating than to see our ardent instructor, suddenly seized with a new thought and forgetting our very existence, work away rapidly with the chalk upon a wholly new series of equations; and then, when he had forced himself into the utmost corner of the blackboard and could get no farther, to see him come back to earth with a sigh and proceed with his lecture. We did not know whither he was going, but that huddle of new equations seemed like a sudden outlet from this world, and a ladder to the stars. He gave a charm to the study of mathematics which for me has never waned, although the other pursuits of life soon drew me from that early love. This I have always regretted, and so did Peirce, who fancied that I had some faculty that way, and had me put, when but eighteen, on a committee to examine the mathematical classes of the college. Long after, when I was indicted for the attempted rescue of a fugitive slave, and the prison walls seemed impending, I met him in the street and told him that if I were imprisoned I should have time to read Laplace's *Mécanique Céleste*. 'In that case,' said the professor, who abhorred the abolitionists, 'I sincerely wish you may be.'"

Among the more prominent mathematical tutors of this period may be mentioned C. W. Eliot, now president of Harvard, and James Mills Peirce, a son of Benjamin Peirce. The latter graduated in 1853, was tutor from 1854 to 1858, and from 1860 to 1861, when he was made assistant professor of mathematics. In 1867 he became university professor of mathematics.

Benjamin Peirce presided for some years over a mathematical society. It comprised eight or ten men of some reputation in Boston and Cambridge, who met to discuss mathematical topics. Each member would present to the society such novelties as his inquiries into some particular branch had suggested, and "in the discussion which followed, it

* How I was Educated, Forum, I, April, 1886, p. 61.

would almost invariably appear that Peirce had, while the paper was being read, pushed out the author's methods to far wider results than the author had dreamed.* His mind moved with great rapidity, and it was with great difficulty that he brought himself to writing out even the briefest record of its excursions."

We now proceed to a brief account of Benjamin Peirce's original researches. Several original articles were contributed by him to the Mathematical Miscellany and to the Cambridge Miscellany. Peirce had planned an extended treatise on Physical and Celestial Mechanics, to be developed in four systems, of Analytical Mechanics, Celestial Mechanics, Potential Physics, and Analytic Morphology. Of these four, only one appeared, the system of Analytic Mechanics, in 1855. The substance of this was prepared as a part of a course of lectures for mathematical students at Harvard. The publication was undertaken at the request of some of his pupils, especially of J. D. Runkle. He consolidated the latest researches into a consistent and uniform treatise, and carried "back the fundamental principles of the science to a more profound and central origin." It was very far from being a mere compilation. In his books he supplanted many a traditional method in mathematics by concise and axiomatic definitions and demonstrations of his own invention. As an instance of this we mention his assumption as self-evident that a line which is wholly contained upon a limited surface, but which has neither beginning nor end on that surface, must be a curve re-entering upon itself. By this new axiom he reduces a demonstration which would otherwise occupy half a dozen pages to a few lines.†

Peirce's Analytical Mechanics was generally acknowledged at the time, even in Germany, to be the best of its kind.‡ An American student in Germany asked once an eminent German professor what book he would recommend on analytical mechanics. The reply was instantaneous, "There is nothing fresher and nothing more valuable than your own Peirce's recent quarto."

Benjamin Peirce was much interested in the comet of 1843, and in a few lectures he aroused by his great eloquence an interest in astronomy which led to the foundation of the observatory of Cambridge. His mathematical ability was first brought into general notice in connection with the discovery of Neptune. Messrs. Adams, of Cambridge, and Leverrier, of Paris, had calculated, from theory alone, where this planet ought to appear in the heavens, if visible, and Galle, of Berlin, discovered on September 23, 1846, the planet at the place indicated to him by Leverrier. Peirce began to study the planet's motion, and came to the conclusion that its discovery was a happy accident; not that Leverrier's calculations had not been exact, and wonderfully laborious,

* Nation, October 14, 1880.

† Rev. Thomas Hill, in the Memorial Collection, by Moses King.

‡ Nature. October 28, 1880.

and deserving of the highest honor, but because there were, in fact, two very different solutions of the perturbations of Uranus possible; Leverrier had correctly calculated one, but the actual planet in the sky represented the other, and the actual planet and Leverrier's ideal one lay in the same direction from the earth only in 1846.

Astronomers of to-day would hardly accept Peirce's conclusions. "His views came, probably, from a misapprehension of Leverrier's methods. There are two methods by which, in theory, the problem could be approached, that of *general* and that of *special* perturbations. Leverrier used the latter, while Peirce's criticisms seem directed against the former."*

On February 2, 1847, Mr. C. Walker, of Washington, discovered that a star observed by Lalande in May, 1795, must have been the planet Neptune. This observation afforded the means of an accurate determination of the orbit. Walker's orbit of Neptune furnished Peirce with materials for still more thorough investigation of the theory and re-determination of the perturbations. These perturbations enabled Walker to get an orbit more correct, which Peirce used again in his turn. Thus, eighteen months after the discovery of Neptune its orbit was calculated by American astronomers so accurately that the conformity between the predicted and observed places was far more close for Neptune than any other planet in the heavens.†

A few years later Peirce published his investigations on Saturn's rings. The younger Bond had seen the ring divide itself and re-unite, and had been led by this to deny the solidity of their structure. Peirce followed with a demonstration, on abstract grounds, of their non-solidity.‡ The same subject was afterward investigated again in England by James Clerk Maxwell.

Admiral C. H. Davis, a relative of Peirce, succeeded in persuading Congress to pay for the calculation of an American almanac for the sailors, so that we would not be dependent upon foreigners, which might be troublesome in case of war. The Nautical Almanac Office was established at first in Cambridge, under Davis's business management and Peirce's scientific control.§ One of the assistants in the office, appointed in 1849, was J. D. Runkle, then one of Peirce's pupils in the Lawrence Scientific School. He helped in the preparation of the American Ephemeris and Nautical Almanac, in which he continued to engage till 1884.

The publications of this office gained scientific recognition from the first. In 1852 were printed Peirce's Lunar Tables, to be used in making

* Prof. G. C. Comstock, Washburn Observatory, in a letter to the writer. Prof. C. A. Young claims that the discovery was not an accident (*General Astronomy*, p. 371).

† Proceedings American Association for Advancement of Science, Vol. VIII, 1854, address by B. A. Gould, jr., p. 18.

‡ Astronomical Journal (Gould's), Vol. II, p. 5.

§ Development of Astronomy in the United States, by T. H. Safford, 1888, p. 21.

computations for the Nautical Almanac. They were intended to serve only a temporary purpose until Hansen's long expected tables should make their appearance, but they continued to be used after that. He made very laborious and exact calculations of the occultations of the Pleiades, which furnished means of studying the form both of the earth and the moon.

From 1852 to 1867 Peirce had the direction of the longitude observations for the U. S. Coast Survey, and in 1867, after the death of Bache, he was appointed Superintendent of the U. S. Coast Survey, which office he held till 1874.

Benjamin Peirce was more celebrated, in his day, as a mathematical astronomer than as a cultivator of pure mathematics. His most important researches in pure mathematics were not placed in reach of the mathematical public until after his death. In our opinion, Peirce will be remembered by future generations for his investigations on Linear Associative Algebra, quite as well as for his other scientific achievements. He will be remembered as an algebraist as well as an astronomer. His thoughts were turned especially toward the logic of mathematics and the limits and extension of fundamental processes. He read several papers on algebra before the American Academy for the Advancement of Science. In 1870 one hundred lithographed copies of a memoir on Linear Associative Algebra, read before the National Academy of Sciences, were taken, for distribution among his friends. This memoir was at last published in the American Journal of Mathematics, Vol. IV, No. 2, with notes and addenda by C. S. Peirce, son of the author. Benjamin Peirce himself considered this memoir the best of his scientific efforts. The lithographed copies contain the following modest introductory remarks by the author, which are omitted in the American Journal of Mathematics:

"To my Friends:"

"This work has been the pleasantest mathematical effort of my life. In no other have I seemed to myself to have received so full a reward for my mental labor in the novelty and breadth of the results. I presume that to the uninitiated the formulas will appear cold and cheerless, but let it be remembered that, like other mathematical formulæ, they find their origin in the divine source of all geometry. Whether I shall have the satisfaction of taking part in their exposition, or whether that will remain for some more profound expositor, will be seen in the future."*

* Peirce distinguishes his algebras from each other by the number of their fundamental conceptions, or of the letters of their alphabet. Thus, an algebra which has only one letter in the alphabet is a *single* algebra; one that has two a *double* algebra, and so on. His investigation does not usually extend beyond the sextuple algebra. This classification he calls "cold and uninstructive, like the artificial Linnæan system of botany." "But it is useful in a preliminary investigation of algebras until a sufficient variety is obtained to afford the material for a natural classification." He *then begins his researches* with *single* algebra, then goes to double algebra, and so on,

Peirce's memoir is a wonderful volume. It is almost entitled to rank "as a *Principia* of the philosophical study of the laws of algebraical operation."

One of the pall-bearers at the funeral of the greatest American algebraist was Prof. J. J. Sylvester.

During the last ten years of his life Benjamin Peirce was relieved of much of the labor and responsibility falling upon the head of a department in a university by his son, Prof. James Mills Peirce. Though not the heir of his father's genius, Prof. J. M. Peirce is a thorough and able mathematician. He excels his father in being an excellent teacher. In 1857 he published an Analytic Geometry, which was used for some years as a text-book at Harvard. He has also published Three and Four Place Tables of Logarithmic and Trigonometric Functions, 1871; Elements of Logarithms, 1873, and Mathematical Tables, chiefly to Four Figures, 1st Series, 1879.

Connected with the mathematical department are, since 1870, Prof. C. J. White; since 1876, Prof. W. E. Byerly; since 1881, Prof. Benjamin O. Peirce, and Mr. George W. Sawin.

Professor Byerly published in 1880 his Elements of the Differential Calculus, and in 1882 his Elements of the Integral Calculus. Byerly's Calculus is a scholarly work. In the rigorous treatment and judicious selection of subjects and adaptability to class use it is, we believe, surpassed by no other American work. Professor Byerly uses the notation, $D_x y$, which was first employed in this country by Benjamin Peirce. In answer to a letter of inquiry regarding the history of this notation Professor Byerly says:* "It was certainly used with some

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up to sextuple, making nearly a hundred algebras, which he shows to be possible. Of all these, only three algebras had ever been heard of before. Of the two single algebras we have one—the common algebra, including arithmetic. Of the three double algebras we have one, the calculus of Leibnitz and Newton. Of over twenty quadruple algebras we have the quaternions of Hamilton.

Prof. Arthur Cayley, in his presidential address before the British Association, in 1883, speaking of Peirce's Linear Associative Algebra, said: "We here consider symbols A, B, etc., which are linear functions of a determinate number of letters or units, i, j, k, l, etc., with co-efficients which are ordinary analytical magnitudes, real or imaginary (viz, the co-efficients are in general of the form $x + iy$, where i is the beforementioned imaginary, or $\sqrt{-1}$). The letters i, j, etc., are such that every binary combination i^2, ij, ji, etc., (the ij being in general not equal to ji) is equal to a linear function of the letters, but under the restriction of satisfying the associative law, viz, for each combination of three letters $ij.k = i.jk$, so that there is a determinate and unique product of three or more letters; or, what is the same thing, the laws of combination of the units i, j, k, are defined by a multiplication table giving the values of i^2, ij, ji, etc.; the original units may be replaced by linear functions of these units, so as to give rise, for the units finally adopted, to a multiplication table of the most simple form; and it is very remarkable how frequently in these simplified forms we have nilpotent or idempotent symbols ($i^2=0$, or $i^2=i$, as the case may be), and symbols i, j, such that $ij=ji=0$; and, consequently, how simple are the forms of the multiplication tables which define the several systems, respectively."

* Letter to the writer, December 27, 1888.

freedom in England and on the Continent in the first half of this century. It is given in Barlow's Mathematical Dictionary, 1814, was used by Cauchy about 1830, by Tortolini in 1844, by Schlömilch in 1846, and by Boole and Carmichael somewhat later, and each of the authors I have mentioned uses the symbol as if it were a familiar one and without reference to its history."

Prof. Benjamin Osgood Peirce has published Elements of the Theory of the Newtonian Potential Function, 1886.

Since 1867 great changes have been made in the requirements for admission to Harvard and in the arrangement of the mathematical courses. Since that time the elective system has been in operation in full force. The terms for admission have been much increased.

From the selected sheets of the Harvard University Catalogue for 1888–89 we take the following regarding the requirements for admission, omitting whatever has no bearing on mathematics:

The examinations for admission embrace two classes of studies, *elementary* and *advanced*.

The elementary studies are not supposed to be equivalent to one another; Greek, Latin, and mathematics have much greater weight in the examinations than any of the rest.

The advanced studies are supposed to be equivalent in regard to time spent upon them at school, and will have the same weight in the examinations. Each of the advanced studies is taught in college in an elective course (or two half-courses) occupying three hours a week for a year; and the standard required at the entrance examinations is the same as in the corresponding college courses.

The elementary studies are prescribed for all candidates, except under the conditions named below (Paragraph I); and every candidate is further required to present himself for examination in not less than two of the advanced studies.

I. The advanced study numbered 6 together with one of the three numbered 7 (see below under "advanced studies in mathematics"), 8 (physics), and 9 (chemistry), may be substituted for either elementary Greek or elementary Latin.

Elementary Studies in Mathematics—

(*a*) Algebra, through quadratic equations. (The requirement in algebra embraces the following subjects: Factors, common divisors and multiples, fractions, ratios, and proportions; negative quantities and the interpretation of negative results; the doctrine of exponents; radicals and equations involving radicals; the binomial theorem for positive integral exponents and the extraction of roots; putting questions into equations, and the reduction of equations; the ordinary methods of elimination, and the solution of both numerical and literal equations of the first and second degrees, with one or more unknown quantities, and of problems leading to such equations.) (*b*) Plane geometry.

Advanced Studies in Mathematics—

6. *Mathematics.*—(*a*) Logarithms; plane trigonometry, with its applications to surveying and navigation. (*b*) Either solid geometry or the elements of analytic geometry.

7. *Mathematics.*—(*a*) Either the elements of analytic geometry or solid geometry. (*b*) Either elementary mechanics or advanced algebra.

The following books will serve to indicate the nature and amount of the requirements in logarithms and trigonometry, analytic geometry, and mechanics:

Logarithms and Trigonometry. Wheeler's Logarithms (Cambridge, Sever) or the unbracketed portions of Peirce's Elements of Logarithms (Boston, Ginn & Co.).

Wheeler's Plane Trigonometry (same publishers). Problems in Plane Trigonometry (Cambridge, Sever). Peirce's Mathematical Tables (Boston, Ginn & Co.).

Analytic Geometry. Briggs's Analytic Geometry (New York, Wiley & Co.).

Mechanics. Goodwin's Elementary Statics (London, Bell & Sons; Cambridge, Sever).

Advanced Algebra. Wentworth's College Algebra (Boston, Ginn & Co.), to article 498, omitting Chapters XIX, XX, XXIV, XXV, XXVII, XXVIII. The examination will be mainly occupied with the portions of algebra, as thus defined, which are not included in the elementary requirement in algebra; but elementary questions are not necessarily excluded.

All in all, there are nine "advanced" studies to choose from. Since one can enter the college after passing an examination on all the "elementary" studies, and on at least two of the "advanced" studies, it follows that the least amount of mathematics required for admission, as a regular student, is that stated above under the heading "Elementary Studies in Mathematics."

The following are the

Courses of instruction in mathematics.

(1888–89.)

A. Logarithms.—Plane Trigonometry, with its applications to Surveying and Navigation. *Half-course. Tu., Th., Sat., at* 11 (*first half-year*). Professor C. J. WHITE.

B. Analytic Geometry (elementary course). *Half-course. Tu., Th., Sat., at* 11 (*second half-year*). Professor C. J. WHITE.

C. Analytic Geometry (extended course). *Mon., Wed., Fri., at* 2. Professor BYERLY.

D. Algebra. *Half-course. Mon., Wed., Fri., at* 11 *and* 3 (*first half-year*). Mr. SAWIN.

G. Algebra (extended course). *Half-course. Tu., Th., Sat., at* 10 (*first half-year*). Mr. SAWIN.

E. Solid Geometry. *Half-course. Tu., Th., Sat., at* 10 (*second half-year*). Mr. SAWIN.

F. Elementary Mechanics. *Half-course. Mon., Wed., Fri., at* 12 (*second half-year*). Mr. SAWIN.

Not to be given after 1888–89.

Courses *A*, *B*, *E*, and *F* correspond to Advanced Mathematical Studies embraced, as optional studies, in the examination for admission to college.

1. Practical Applications of Plane Trigonometry.—Spherical Trigonometry.—Applications of Spherical Trigonometry to Astronomy and Navigation. *Wed., Fri., at* 3. Professor C. J. WHITE.

   Course 1 is open to Freshmen who have passed the examination in Plane Trigonometry.

2. Differential and Integral Calculus (First Course). *Mon., Wed., Fri., at* 11. Professor C. J. WHITE.

   Course 2 is open to those only who have taken Course *B* or Course *C*.

3. Analytic Geometry; higher course. *Mon., Wed., Fri., at* 10. Professor J. M. PEIRCE.

   Course 3 is intended for students who have taken Course *C;* but those who have taken Course *B* may elect it, if deemed qualified by the instructor.

4. The Elements of Mechanics. *Tu., Th., Sat., at* 9. Professor B. O. PEIRCE.

   Course 4 is intended for students who take or have taken Course 2.

   Candidates for Second-Year Honors may take Courses 2 and 3, or 2 and 4. Other courses may be accepted on special petition.

5. Differential and Integral Calculus (Second Course). *Mon., Wed., Fri., at* 11. Professor BYERLY.

[6. Quaternions and Theoretical Mechanics. *Mon., Wed., Fri., at* 12, Professor J. M. PEIRCE.]
Omitted in 1888–89.

[7. Higher Plane Curves. Professor J. M. PEIRCE.]
Omitted in 1888–89.

8. Analytic Mechanics. *Mon., Wed., Fri., at* 10. Professor BYERLY.

9. Quaternions and Theoretical Mechanics (Second Course). *Mon. at* 12, *Fri.* 11-1. Professor J. M. PEIRCE.

10. Trigonometric Series; Introduction to Spherical Harmonics.—Theory of the Potential. *Tu., Th., at* 12, *Wed. at* 10. Professors BYERLY and B. O. PEIRCE.

[11. Hydromechanics. Professor B. O. PEIRCE.]
Omitted in 1888–89.

13. The Theory of Functions. *Mon. at* 11, *Wed.* 11-1. Professor J. M. PEIRCE.

20. Special Advanced Study and Research.—The work of the following courses will consist in investigations and reading, to be carried on by the students in the courses, under the guidance of the instructors. Students will be expected to present their results from week to week in the form of lectures and theses.

(*a*) Questions in the Theory of Functions. *Wed.*, 3.30–5.30. Professor J. M. PEIRCE.

(*b*) Higher Algebra (First Course). Mr. SAWIN.

Some few studies in the college course are prescribed, but *all mathematical studies are elective.* No mathematics need therefore be studied in college. A student can, if he chooses, get the degree of bachelor of arts without having had more mathematics than plane geometry and algebra through quadratic equations—the minimum requirement for admission.

We conclude this article by quotations from a letter by Prof. L. M. Hoskins, of the University of Wisconsin, who, in the year 1884–85, was honored with a fellowship at Harvard, and studied higher mathematics there.

"There were two courses, in 'quaternions and theoretical mechanics,' given by Prof. J. M. Peirce, each three lectures weekly for the year. The first course gave the elements of quaternions and the dynamics of a particle, covering about the ground of Tait and Steele's dynamics of a particle, but treated by quaternion methods largely. The second course continued with higher applications. * * * A third course, on analytic mechanics, was offered by Prof. J. M. Peirce, consisting of lectures, following Benjamin Peirce's Analytic Mechanics, for the first half-year, and for the second Routh's Rigid Dynamics, the part relating to moving axes and relative motion, oscillations about equilibrium, oscillations about a given state of motion, motion of a rigid body under any forces—in short, the first five chapters of Volume II. * * * I attended also a course in "arbitrary functions," by Prof. W. E. Byerly. This covered most of the ground of Riemann's *Partielle Differentialgleichungen.* The main subject treated is the methods of solution of partial differential equations subject to given conditions, a class of problems constantly arising in physics. The course naturally includes the proof and discussion of Fourier's Theorem, and the treatment of the different kinds of spherical harmonics, since these are of great use in the

solution of certain classes of partial differential equations. On the whole, I found this as attractive a part of pure mathematics as I ever entered. * * *

"I may remark that the branches of higher mathematics to which most attention is paid at Harvard seem to be theoretical mechanics and quaternions. This is doubtless due to the influence of Benjamin Peirce, whose attainments in the former line are well known, and who was also among the first to recognize the high value of quaternion methods. * * * I am able to give both of them (Professors J. M. Peirce and Byerly) high praise as teachers of mathematics. Both are clear, logical lecturers, and popular with the students. * * * Of him [Prof. B.O. Peirce] I have little personal knowledge, but am sure no professor was held in higher estimation, both as to attainments and ability as a teacher. * * * In 1884–85 the number of graduate students taking mathematics was five. * * *

"A mathematical 'seminar' was maintained with a good deal of interest, with weekly meetings throughout the greater part of the year. These meetings were under the supervision of the mathematical faculty, and were rather informal in nature, though a formal programme was usually carried out, usually by volunteer lectures or solutions."

YALE COLLEGE.

The successor in the chair of mathematics and natural philosophy to the lamented Professor Fisher was Matthew R. Dutton. He was a graduate of the college and was professor from 1822 to 1825. He was the author of a work on conic sections and spherical trigonometry. This work was subsequently revised by Dwight, who "laid the students and teachers of that day under everlasting obligations by his simplification and abbreviation of those endless algebraic formulæ in Dutton's Conic Sections."

From 1825 to 1836 Denison Olmsted occupied the chair which had been made vacant by the death of Professor Dutton. He was born in Hartford, Conn., in 1791, graduated at Yale in 1813, became tutor there in 1815, was elected professor of chemistry at the University of North Carolina in 1815, and finally returned thence to assume the duties of the chair of mathematics and physics at Yale. He had made natural philosophy and chemistry his specialty, and possessed no special fitness for the teaching of mathematics.

Professor Olmsted was renowned as a teacher rather than an original investigator. His teaching power was indeed great, and he exerted a beneficial influence, not only in college, but also upon the education in the common schools of Connecticut. In 1831 appeared his Natural Philosophy, which superseded the antiquated work of Enfield. In the next year was written his School Philosophy, a more elementary work; and in 1839 his Astronomy. He wrote also the Rudiments of Natural Philosophy and Astronomy, which passed through some fifty editions.

His Natural Philosophy and Astronomy came to be almost universally used in our colleges. The Philosophy was later revised by Prof. E. S. Snell, of Amherst College, and, still later, by Prof. J. Ficklin, of the University of Missouri. In point of scientific accuracy Olmsted's works were sometimes rather defective. They were somewhat old-fashioned.

As early as 1830 a good telescope of moderate size was procured at Yale College. For want of an observatory it was difficult to make accurate observations with it. But it served, nevertheless, the excellent purpose of furnishing a means of observing the great November shower of meteors, which occurred not long afterward. These showers, Halley's Comet, and the telescope enabled Professor Olmsted to arouse a great deal of astronomical enthusiasm at Yale, and for a few years a number of students turned their attention to astronomy. Of the mathematicians and astronomers who graduated in those days are Stanley and Mason, long since deceased; and Loomis and Lyman, who are now aged professors at Yale. The ablest mathematician and astronomer which Yale has produced is William Chauvenet. As a teacher he was never connected with his *alma mater*, though a professorship was offered him twice.

In 1835 the chair of mathematics and natural philosophy was divided into two separate ones—Olmsted retaining that of natural philosophy, and Anthony D. Stanley being elected to that of mathematics.

One of the most prominent of early tutors in mathematics at Yale was F. A. P. Barnard, of the class of 1828, which was known as a "mathematical class," for the mathematical talent it embraced. While tutor at Yale he prepared an edition of Bridge's Conic Sections, in which the work was substantially rewritten and also considerably enlarged.

We proceed to examine the mathematical courses during the time of Professors Dutton and Olmsted.

In 1824, Arithmetic was the requirement in mathematics for admission; in 1833, "Barnard's or Adam's Arithmetic;" in 1845, Arithmetic and Day's Algebra to quadratics.

The mathematical course for 1824 was, for Freshmen, Day's Algebra during the first two terms, with no mathematics for the third term; for Sophomores, six books of Playfair's Euclid during the first and part of the second term, and Day's Mathematics (including plane trigonometry, logarithms, mensuration of surfaces and solids, isoperimetry, navigation and surveying) and Dutton's Conic Sections, during the rest of the year; for Juniors, Dutton's Spherical Trigonometry during the first term, and Enfield's Astronomy and Vince's Fluxions during the third term. The Seniors had no mathematics in their course. In 1825 the study of Euclid was begun at the close of the Freshman year. Vince's Fluxions still appeared in that year as a text-book.

The writer has not been able to see catalogues for the years 1826–32. In 1833, Olmsted's Natural Philosophy was in use. "Fluxions" were also named, but this meant then, most likely, the differential and in-

tegral calculus. In the Sophomore year, Bridge's Conic Sections (probably Barnard's edition) was used in place of Dutton's. No changes in the course were made for several years after.

The teaching of mathematics to the two lower classes in college was generally intrusted to young and inexperienced tutors, who had, as a rule, a very meagre acquaintance with the subjects which they were supposed to teach. It is therefore not surprising that poor results were generally reached, and that the study of mathematics was very unpopular. Especially unpopular was the study of conic sections. No efforts seem to have been made on the part of tutors to make this study more attractive and to show its usefulness, by pointing out its application in the study of physics and astronomy. Moreover, the old books on conic sections were as dry as dust. The dissatisfaction among students finally culminated, in 1830, in what is known as the "conic sections rebellion." Rebellions among students were then not unfrequent. Some years previously had taken place the "bread and butter rebellion," caused by the poor quality of board that the students were receiving. Neither their physical nor their intellectual food seems to have been palatable to them. The "conic sections rebellion" was a refusal, on the part of the Sophomores, to recite in the manner prescribed by the college rules. They petitioned that the method of recitation required by the college be changed, that they might "explain conic sections from the book, and not demonstrate them from the figure."* We should judge from this that the practice had hitherto prevailed of simply asking the student to explain certain parts of the subject, with the book open before him, without requiring him to go to the blackboard (if blackboards were used) to explain the lesson from his own figure independently of the book. We have not been able to ascertain at what time the blackboard was introduced into the mathematical class-room at Yale, but it is not unlikely that the above rebellion arose in the attempt, on the part of the faculty, to introduce such improved methods as the use of the blackboard would suggest. The new methods may have called for greater effort on the part of the student, and may thus have brought about the "rebellion."

The general impression prevailed at Yale, in those days, that the mathematical course there was a very difficult and thorough one. "This fancy certainly derived some support from comparison with the classical course, as *compared with which* the mathematical was undoubtedly a good one."†

Mr. Bristed, who entered Yale in 1835, says that in mathematics the classes studied books rather than subjects, and crammed from one day to another. "A great deal of the work," says he "of the second and third year consisted of long calculations of examples worked with logarithms, which consumed a great deal of time without giving any insight

* Yale College: a Sketch of its History, by William L. Kingsley, p. 137.

† Five Years in English Universities, by Charles Astor Bristed, 3d ed., 1873, p. 456.

into principles, and were equally distasteful to the good and the bad mathematicians." "They (the best mathematicians) complained that with the exception of two prizes for problems during the Freshman and Sophomore years, and an occasional 'original demonstration' in the recitation room, they had no chance of showing their superior ability and acquirements; that much of their time was lost in long arithmetical and logarithmical computations; that classical men were continually tempted to 'skin' (copy) the solutions of these examples, and thus put themselves unjustly on a level with them." The bad practice of giving long and tedious examples to work has been quite prevalent in our colleges until within recent years, especially in trigonometry. For ordinary class-work four-place logarithmic tables are sufficient, we should think. Prof. J. M. Peirce, of Harvard, has done much toward inaugurating a reform in this matter, by his publication of four-place tables. Such tables are of sufficient accuracy even in connection with the ordinary physical experiments which the student may make in the laboratory.

In 1836 Anthony D. Stanley became professor of mathematics. He held this place until his death in 1853. He was a native of Connecticut and graduated at Yale in 1830. Two years later he was appointed tutor and afterward professor. We are told that Professor Stanley took special interest in the theory of numbers, and that he had once an excellent occasion to show his skill. "In 1835 an anonymous writer in the Stamford Sentinel challenged the entire faculty of Yale to arrange the nine digits in such order that their square root could be extracted without a remainder. In a few days Mr. Stanley over the signature "X" gave the required solution, and added that the question admitted of more than one answer, and called upon the proposer to produce them. To this challenge his opponent made an evasive reply, in which he stated the number of solutions to be nine, but did not communicate any solution."* Stanley found twenty-eight different solutions, but even a larger number is possible.

It seems that when Stanley was appointed professor he did not immediately enter upon the discharge of the duties of the chair, but went to Europe two years and spent most of the time in Paris, where he attended lectures at the Sorbonne and College of France. In 1846 he published Tables of Logarithms, which were uncommonly accurate. In 1848 appeared his Elementary Treatise on Spherical Geometery and Trigonometry. In the preceding year he published an article in the American Journal of Science, "On the Variation of a Differential Coefficient of a Function of any Number of Variables." "In this memoir," says Professor Loomis, "he resolves a problem which had already occupied the attention of La Grange, Poisson, Ostrogradsky, and Pagani, the latter of whom was the only one who obtained a correct solution of it. Professor Stanley here gives a solution of the same problem more

* Yale College: a Sketch of its History, by William L. Kingsley.

simple and concise than Pagani's, and which was discovered before receiving the solution of that mathematician."* In 1849 he suffered from a severe cold, and he sought relief in Italy and Egypt. On his return he assisted in completing the revision of Day's Algebra, which he had begun before leaving. He died in 1853.

Somewhat later than Prof. Charles Davies, Prof. Elias Loomis began the publication of mathematical text-books, which, like Davies' works, became extremely popular throughout the United States. Professor Loomis has been connected with Yale College since 1860, but not as professor of mathematics. Indeed, his specialty has not been mathematics. His original contributions to science have been in other fields. At Yale he has been professor of natural philosophy and astronomy. His chief scientific work has been as a meteorologist and astronomical observer. In his younger days Professor Loomis was a man of wonderful activity, but now he is nearly four score years old and an invalid.

Professor Loomis was born in Connecticut in 1811; was graduated at Yale College in 1830. After graduating he occasionally contributed solutions to Ryan's Mathematical Diary. He was for a time tutor in Yale. Together with Professor Twining, of West Point, he made observations for determining the altitude of shooting-stars. These were, most likely, the first concerted observations of the kind made in America. He was the first one in America to discover Halley's Comet in 1835. The next year he was chosen professor of mathematics and natural philosophy at the Western Reserve College, with permission to spend a year in Europe. In Paris he attended lectures of Arago, Biot, Poisson, Dulong, Pouillet, and others. He returned with astronomical, physical, and meteorological instruments, and during the next season an astronomical observatory was erected at the Western Reserve College, in Ohio. Only three observatories existed in the United States before this, namely, at the University of North Carolina, at Yale, and at Williams College.

In 1844 Professor Loomis became professor in the University of the City of New York. "Having here no instruments for observation, he was induced to undertake the preparation of a text-book on algebra; especially designed for the use of his own classes. This book prepared the way for a second, and the second was followed by a third, until, ultimately, his text-books embraced the whole range of mathematics and natural philosophy, astronomy, and meteorology."* His principal mathematical and astronomical text-books are, Plane and Spherical Trigonometry, 1848; Analytical Geometry and Calculus, 1857; Elements of Algebra, 1851; Elements of Geometry and Conic Sections, 1851; Practical Astronomy, 1855; Elements of Arithmetic, 1863. His treatise on astronomy, now obsolescent, received in its day high commendation from leading astronomers. Some of his mathematical text-books were, at first, very thin, but were gradually enlarged in subse-

* Yale College; a Sketch of its History, by William L. Kingsley.

quent editions. Thus, his Analytical Geometry and Calculus were at first combined in one small volume, while, subsequently, the two subjects were published separately in volumes about as large, each, as the earlier combined volume.

The books of Loomis were written in a clear, simple style, and were well adapted for use in the class-room. There was nothing in them which any student of ordinary ability and application did not readily master. These characteristics made Loomis's works very popular. A student desiring to secure a somewhat extended knowledge of the various mathematical subjects would hardly have found Loomis's works to answer his purpose; nor would the works of Davies have given him better satisfaction. He would have found more of what he wanted in the books of Peirce and Chauvenet. Nor were Loomis's works always up with the times. The treatment of series is bad, both in his Algebra and in his Calculus. Again, take the following statement, for instance: "No general solution of an equation higher than the fourth degree has yet been discovered."* This piece of historical information is unsatisfactory; for, in the first place, M. Hermite has given a *transcendental* solution of the quintic and, in the second place, Abel and Wantzel have proved that an *algebraic* solution of equations higher than the fourth degree is impossible. Perhaps the best mathematical work, in point of accuracy, is his Elementary Geometry. It has been said of American writers that, while they have given up Euclid, they have modified Legendre's Geometry so as to make it resemble Euclid as much as possible. This applies to Loomis with greater force, perhaps, than to any other author. His trigonometry has been wedded to the old "line system," and it is only within the last two or three years that a divorce has been secured.

While Loomis has made no original contributions to pure mathematics, he has not been idle in other lines of research. He has contributed one hundred or more papers (chiefly on astronomical, meteorological, and physical subjects) to the American Philosophical Society, Connecticut Academy, Smithsonian Institution, American Journal of Science, and Gould's Astronomical Journal. Some of his papers have been reprinted in Europe. His Contributions to Meteorology was translated into French.

Professor Stanley's successor in the mathematical chair at Yale is Professor Hubert Anson Newton. He graduated at Yale in 1850, after which he studied higher mathematics. In 1852 he was made tutor, and when he entered upon that office in 1853 he was given charge of the entire mathematical department at once, owing to the illness of Professor Stanley. In 1855 he was elected full professor, with permission to spend one year abroad. In 1856 he began the active discharge of the duties of the chair, which he still holds. Professor Newton's publications have been restricted almost exclusively to scientific papers, which

* Treatise on Algebra, 1873, p. 334.

have appeared in the Memoirs of the National Academy of Sciences and in the American Journal of Science. He is best known to science for his observations on shooting-stars and star-showers. He wrote for the Encyclopædia Britannica the article on "Meteorites." His work in pure mathematics includes a paper "On the Construction of Certain Curves by Points," published in the Mathematical Monthly, and on "Certain Transcendental Curves."

Since 1871 Eugene L. Richards has been assistant professor of mathematics. He is the author of a Trigonometry.

In 1873 John E. Clark, who had been professor at the University of Michigan, was chosen professor of mathematics at Yale. Since 1881 Andrew W. Phillips has been assistant professor of mathematics; also William Beebe since 1882. Phillips and Beebe have written a novel and successful treatise on Graphic Algebra.

In 1871 J. Willard Gibbs was elected professor of mathematical physics. He graduated at Yale in 1858, and after graduation continued his mathematical and physical studies. He was tutor from 1863 to 1866. Afterward he went to Europe and spent three years in study at Paris, Berlin, and Heidelberg. Much of his time has been given to thermodynamics. He contributed in 1873 to the Connecticut Academy an article on Graphical Methods in Thermodynamics of Fluids. In the same year appeared A Method of Geometric Representation of the Thermodynamic Properties of Substances by Means of Surfaces.

But Professor Gibbs's studies have been carried on also in the field of pure mathematics. He has published a treatise on the Elements of Vector Analysis, which is a triple algebra, as distinguished from quaternions, a quadruple algebra. Vector analysis has been applied by Professor Gibbs to about the same kind of problems as quaternions. The advantage claimed for vector analysis over quaternions is that the former reaches solutions more simply and directly, and that its principles can be developed more concisely. In 1886 Professor Gibbs read an exceedingly interesting paper before the American Association for the Advancement of Science on Multiple Algebra, which contains an excellent sketch of the development of this science in the hands of Grassman, Hamilton, Hankel, Benjamin Peirce, Sylvester, Cayley, and others. As to the applications of multiple algebra, Professor Gibbs says:*

"Maxwell's Treatise on Electricity and Magnetism has done so much to familiarize students of physics with quaternion notations that it seems impossible that this subject should ever again be entirely divorced from the methods of multiple algebra.

"I wish that I could say as much of astronomy. It is, I think, to be regretted that the oldest of the scientific applications of mathematics, the most dignified, the most conservative, should keep so far aloof from the youngest of mathematical methods."

We now return to the courses of study at Yale College. The catalogue

* Proceedings American Association for the Advancement of Science, 1886, p. 62.

of 1845 shows that "Day's Algebra to quadratics" was added to "arithmetic" as a requirement for admission to college. In 1852 Thomson's was the arithmetic recommended. In 1855 the requirements were again increased by the addition of two books in Playfair's Euclid. In 1870 the terms were higher arithmetic, Loomis's Algebra to quadratics, and two books of Playfair's Euclid (or the first, third, and fourth books of Davies' Legendre, or of Loomis' Elements of Geometry); in 1885, arithmetic, algebra as far as logarithms in Loomis, first book in Euclid, and the first thirty-three exercises thereon in Todhunter's edition (or the first four books in another geometry); in 1887, higher arithmetic (including the metric system of weights and measures), algebra (Loomis as far as logarithms), plane geometry. All candidates for admission are examined on the same studies, no matter what courses they may wish to pursue in college. It is also worthy of remark that, since 1885, the use of Euclid as a text-book in geometry has been discontinued at Yale, and Princeton is now the only prominent college in the country which still adheres to Euclid.

We come now to the mathematical course in college. In 1848 it was as follows:

Freshmen, Day's Algebra, Playfair's Euclid; *Sophomores*, Day's Mathematics, Bridge's Conic Sections, and Stanley's Spherical Geometry and Trigonometry; *Juniors*, Olmsted's Natural Philosophy, Mechanics, Hydraulics, Hydrostatics, Olmsted's Astronomy, Analytical Geometry or Fluxions (optional).

Fluxions seem to have been optional all the time, though in previous catalogues they appear as a regular study. Analytical geometry was also optional for that year. In 1852 Loomis's Analytical Geometry and Calculus appear in the catalogue as Sophomore studies. In 1854 Bridge's Conic Sections *or* Analytical Geometry appear as part of the work of the Sophomore year; and Church's Differential Calculus in the Junior year. But analytical geometry and calculus were elective studies. "Those desirous of pursuing higher mathematics are allowed to choose *analytical geometry* in place of *regular mathematics* in the third term Sophomore, and calculus in the Junior for Greek and Latin."

In 1858 Loomis's Calculus is given in the Sophomore year, and Todhunter's in the Junior.

The course was as follows in 1870: *Freshmen*, Loomis's Algebra, Playfair's Euclid, Loomis's Conic Sections; *Sophomores*, Loomis's Trigonometry, Stanley's Spherical Geometry, Davies' Analytical Geometry; *Juniors*, Calculus, Loomis's Astronomy. The next year, Chauvenet's Geometry was used with Euclid in the Freshman class.

In 1885 the course was—*Freshmen*, Todhunter's Euclid (Books III and IV), Chauvenet's Geometry, Richard's Plane Trigonometry, Phillips and Beebe's Graphic Algebra; *Sophomores*, Loomis's Analytical Geometry (plane and solid), Dana's Mechanics; *Juniors*, Loomis's Astronomy (required), Calculus, Geodesy, Descriptive Geometry (all three *elective); Seniors*, Calculus, Vector Analysis (both elective).

The course for the year 1887–88 is substantially the same as that of 1885. We quote from the catalogue the following account of it:

"In geometry the exercises consist in recitations from the text-book, the original demonstration of theorems, and applications of the principles to the solution of numerical problems.

"After the student has gained facility in the use of trigonometrical tables, the principles of plane trigonometry are applied to the problems of mensuration, surveying, and navigation, and those of spherical trigonometry to the elementary problems relating to the celestial sphere.

"In algebra the elementary principles of the theory of equations are illustrated graphically, and the student is exercised in the numerical solution of equations of the higher degrees and the graphical representation of the relations of quantities.

"In analytical geometry the student is carried through the elementary properties of the lines and surfaces of the second degree, and is introduced to the theory of map projection.

"These are studies of the Freshman and Sophomore years, and, together with the elements of astronomy which are pursued in Junior year, are regarded as essential parts of a liberal education.

"In the Junior and Senior years opportunity is given in the elective courses to obtain a wider knowledge of analytical geometry and trigonometry with their applications to geodesy and astronomy. A longer and shorter course are provided in Junior year in differential and integral calculus. The shorter course is designed for students who desire to become acquainted with the methods of the calculus, but whose principal studies are not of a mathematical character. The longer course is designed for such as expect to make a serious study of any department of pure or applied mathematics.

"In Senior year advanced subjects in the calculus and the elements of analytical mechanics form one line of study.

"An elementary and an advanced course are provided in what is called vector analysis. The object of these courses is to introduce the student to the methods of multiple algebra in geometry, mechanics, and physics. The matter taught is not entirely unlike that usually given in courses in quaternions, but the method followed is in some respects nearer to Grassmann's than to Hamilton's. The elementary course is confined to the simplest algebraic relations of vectors. The advanced course includes differentiation with respect to position in space, and the theory of linear vector functions.

"Students who show special aptitude are exercised in the working up of subjects which require the use of the library and more prolonged investigation than the daily exercises of the class-room. Such work begins in Freshman year. There is a considerable collection of models, which are used to assist the imagination in the various branches of study."

In November, 1877, a Mathematical Club was formed at Yale. Pro-

fessor Gibbs has been the leading spirit in it. He has, in recent years, presented papers showing the application of vector analysis to the computation of elliptic orbits. The work of the club has, however, not been confined to pure mathematics. Professor Newton has presented subjects on meteors and astronomy, and Professor Hastings has given results of experiments made by him on light.

COLLEGE OF NEW JERSEY.

In 1830 Albert B. Dod became professor of mathematics. He seems to have been a favorite teacher. His pupils cherish fondly the recollection of "his brilliant genius and the interest which he infused into the study of higher mathematics, as well as the magnetic charm of his manner, as by the wonderful acuteness and perspicuity with which he mastered and explained the most abstruse problems." The same qualities shone attractively in his lectures on architecture.* He discharged the duties of his office with signal ability till his death, in 1845. The family to which he belonged had for several generations been remarkable both for mathematical taste and talent. His father constructed the engine of the *Savannah*, the first steam-boat that crossed the Atlantic.

The scientific and mathematical departments of Princeton were first made prominent by the labors of Professors Henry and Alexander. Stephen Alexander was graduated at Union College, New York, in 1824, at the age of eighteen, with high honors. He then engaged in teaching. In 1830 and 1831 he was in Albany making numerous astronomical observations and communicating them to the Albany Institute. He and Joseph Henry were relatives. "Professor Henry was a son of the elder Alexander's sister, and in 1830 he married his cousin, Miss Alexander, thus establishing a double relationship, which unquestionably shaped the whole life and fortune of his younger and favorite cousin and brother-in-law.† In 1832 Professor Henry was elected to the chair of natural philosophy at the College of New Jersey. Alexander went with Henry and his family to Princeton. He there entered the Theological Seminary as a student, but in 1833 he was appointed tutor in the college. "In 1834 he was made adjunct professor of mathematics, and in 1840 he was promoted to the full professorship of astronomy, which he retained until 1876. During the long intervening period the style and duties of his professorship were several times more or less modified. For several years after the death of Professor Dod he was professor of mathematics and astronomy. When Professor Henry went to Washington he gave up the mathematics and became professor of natural philosophy and astronomy, but he always held fast to astronomy."

In 1847 John Thomas Duffield became connected with the mathemati-

* The Princeton Book, 1879.

† Biographical Memoirs of the National Academy of Sciences, Vol. II, p. 886, "Biographical Memoir of Stephen Alexander," by C. A. Young. Our remarks on Professor Alexander are drawn chiefly from this sketch.

cal department. He graduated at Princeton College in 1841, afterward studied theology, and then was appointed tutor in Greek. From 1847 to 1850 he served as adjunct professor of mathematics. During two years he had charge of a Presbyterian church in connection with his duties in the college. He published, also, a volume of sermons. He has been professor of mathematics since 1850. For many years the mathematical teaching at Princeton was in the hands of Professor Duffield and Professor Alexander. The former possessed great power in teaching young students, while the latter led their way into the more advanced mathematics and astronomy.

In 1850 the requirements for admission were arithmetic and the elements of algebra through simple equations. The *Freshmen* studied Hackley's Algebra and Playfair's Euclid; the *Sophomores* finished Euclid and then took up plane and spherical trigonometry, navigation, etc.; the *Juniors* studied analytical geometry (Young's), Alexander's Differential and Integral Calculus, and mechanics.

Princeton is one of the very few colleges in this country which have retained Euclid as a text-book in geometry to the present day. "Euclid is used as a text-book in geometry because of its historical associations and its decided superiority for the purpose of mental discipline to any modern text-book."*

Rev. Dr. R. G. Hinsdale, who was a student at Princeton from 1852 to 1856, gives the following reminiscences of the mathematical teaching there: "The requirements for admission were geometry—four books of Euclid, algebra through quadratics. The text-book in algebra during the Freshman year was Hackley's. The fact was, that but few who entered were fully prepared, and therefore we had a rapid review of the subjects *ab initio*, finishing Hackley the first year. In the Sophomore year we finished Euclid's geometry, also surveying and navigation (elementary). Both subjects were taught in a special way by Prof. John T. Duffield, whose syllabus taken down from dictation was a marvel of clearness. The notes of that syllabus I have with me. It has never been printed. The Junior class studied Young's Analytical Geometry and Conic Sections. The first half of the year we were taught by Professor Duffield, the last half by Professor Alexander, who had the chair of physics and astronomy.

"In the Senior class mathematics was taught by Professor Alexander, a gentleman of marked ability in the higher branches of his departments. He used no text-book in either department. Both subjects were taught orally. An elaborate compendium of mathematical physics was dictated to the class by the professor, accompanied by explanations of formulæ and experimental illustrations. The same way was adopted by the professor in teaching the mathematics of astronomy. His syllabus in *that* department, however, was 'printed, not published,' for the use of the class.

* Catalogue of the College of New Jersey, 1888–89, p. 42.

"Professor Alexander had a distinguished reputation among his confrères. Professor Peirce, of Cambridge, spoke of him repeatedly in public lectures as the 'Kepler of the nineteenth century,' always in connection with his theory as to the asteroids, accompanied by mathematical demonstrations that they once formed one wafer-shaped planet which, 'somewhere, somehow,' was shattered into fragments."

Rev. Horace G. Hinsdale says:* "He [Alexander] pushed his researches into the depths of mathematical and astronomical science, availing himself of his acquaintance with the principal languages of Europe. He printed for the use of his students treatises on ratio and proportion, differential calculus, and astronomy. He was unselfish in his devotion to the interests of the college and the advancement of learning. He aroused the admiration of his pupils by the evident extent of his knowledge and his ardor in imparting it, although it must be said that he often became so profoundly interested in setting forth the philosophy of mathematics as to forget that their acquaintance with the subject was, of necessity, far less than his own, and so to outrun their ability to follow and comprehend him. The closing lectures in his course in astronomy, in which he discussed the nebular hypothesis of Laplace, were characterized by a lofty and poetic eloquence, and drew to his class-room many others than the students to whom they were addressed. Even ladies from the village and elsewhere—so far did the traditional conservatism of Princeton give way before a wholesome pressure—invaded Philosophical Hall."

Professor Young says: "He was familiar not only with the ordinary range of mathematical reading, but with many works of higher order. He had large portions of the Mécanique Céleste almost at his finger's ends, and was well acquainted with the works of Newton, Euler, and Lagrange."

As was the case with all college professors in former years, and is still true with most of them, Professor Alexander's time and strength were so consumed by the routine duties of the office, that little remained for anything else. Still he accomplished a great deal. He published articles in various scientific journals, and presented a large number of papers, orally, before scientific societies; and the only record of these communications which we now have is a mere notice or a brief abstract of a paper read on such and such a date.

In 1848 he read before the American Academy for the Advancement of Science a paper on the Fundamental Principles of Mathematics. Prof. C. A. Young says of it: "It is an interesting, suggestive, and eloquent essay. The subject gives the author an opportunity to indulge his inherited Scotch love for metaphysics and hair-splitting distinctions, and he finds in it also opportunity for imagination and poetry to an extent which makes the paper almost unique among mathematical disquisitions."

*Quoted by Prof. C. A. Young in his memoir.

Professor Alexander was an astronomer, but his special forte was not that of the observer. In fact, he had no adequate instruments or observatory. Long did he labor to secure a good observatory for the college, and, at last, in 1882, a great telescope was pointed toward the stars. "There was something pathetic in his exclamations of satisfaction and delight, for the great instrument, so long dreamed of, had only come too late for him to use it."

In 1876 Alexander was made professor emeritus, and Charles Greene Rockwood became connected with the mathematical department. Professor Rockwood was graduated at Yale in 1864, and, before going to Princeton, was professor of mathematics in Bowdoin and Rutgers successively. He has acquired reputation by his studies of earthquakes, and has contributed articles on vulcanology and seismology to the Reports of the Smithsonian Institution, 1884–86.

From 1878 to 1883 Dr. G. B. Halsted was a teacher in mathematics; until 1881 as tutor, and from that time on as instructor in post-graduate mathematics.

The present mathematical corps consists of Professors John Thomas Duffield, C. G. Rockwood, H. B. Fine, and Tutor H. D. Thompson. C. A. Young is the successor to Alexander as professor of astronomy.

The conservatism of Princeton College is noticeable in some features of the mathematical instruction. Euclid has been retained as a text-book to the present day. Todhunter's edition has been used now for many years. Until recently Loomis's text-books were used largely, though not exclusively. In the academical under-graduate department the following mathematics were taught in 1881: *Freshman* year, Ray's University Algebra, Todhunter's Euclid, and Mensuration; *Sophomore* year, Loomis's Plane Trigonometry, Navigation, Surveying, Spherical Trigonometry, and Analytical Geometry; *Junior* and *Senior* years, Analytical Geometry of two and three dimensions, and Calculus. Under Professor Duffield oral instruction is made prominent. It might be more correct to say that mathematics is taught by him "mainly by lectures—the text-books being used by way of reference, and as furnishing examples for practice." "The students are required to take notes of the lectures and submit their note-books for examination at the end of each term." Until quite recently electives were introduced very sparingly. At present all studies are prescribed during the first two years; mathematics is elective during the last two years.

Modern higher mathematics was first introduced in Princeton College by Dr. G. B. Halsted. His examination papers on quaternions, determinants, and modern higher algebra, are the first ones that have ever been set at Princeton. One feature of the mathematical instruction at this institution that has been in vogue during the last ten years (perhaps longer) is, we think, to be recommended for more general adoption. Considerable attention is given to the study of the history of mathematics. The writer has before him examination papers, writ-

ten in answer to questions set by Halsted in 1881.* From the answers we infer that questions like these have been asked: Who wrote the first algebra that has come down to us? What was its nature? What part did the Hindoos play in the development of algebra? Its growth during the Renaissance? The laws underlying ordinary algebra? etc.

The present mathematical course, according to the catalogue of 1888–89, is as follows:

For admission to the academical department of the college, the mathematical requirements are: "Arithmetic, including the metric system; algebra, through quadratic equations involving two unknown quantities—including radicals, and fractional and negative exponents; geometry, the first and second books of Euclid, or an equivalent—that is, the propositions in other text-books relating to the straight line and rectilinear figures, not involving ratio and proportion."

Studies in the academical department: "In the Freshman year there are two exercises a week during the first and second terms, in algebra, and two exercises a week during the third term, in plane trigonometry, under Professor Fine; in geometry there are two exercises a week throughout the year, under Mr. Thompson. The text-book in algebra is Wells's University Algebra, to be supplemented by a course on the theory of equations, by the professor. Loomis's Trigonometry is the text-book in trigonometry. Euclid is used as the text-book in geometry because of its historical associations and its decided superiority for the purpose of mental discipline to any modern text-book. The first six and the eleventh books of Euclid are supplemented by a course in solid and spherical geometry. Since a thorough knowledge of geometry and familiarity with its more important propositions can be obtained only by extended practice in the demonstration of theorems and problems not contained in the text-book, this exercise occupies a prominent place in the course of instruction.

"The Sophomore class has three exercises a week throughout the year in mathematics, under Professor Duffield. For the first term the studies are analytical trigonometry, mensuration, and navigation; for the second and third terms, surveying, spherical trigonometry, analytical geometry, and the elements of the differential calculus.

"In the Junior year mathematics is an elective study. The class has two exercises a week throughout the year, under Professor Duffield. For the first and second terms the studies are analytical geometry and the differential calculus; for the third term, the integral calculus. Loomis's Trigonometry is the text-book during the first and second terms of the Sophomore year, Bowser's Analytical Geometry and Calculus during the third term Sophomore and Junior year—supplemented largely by oral instruction, and numerous exercises in addition to the examples for practice of the text-books.

* One of these was written by H. B. Fine, now assistant professor of mathematics at Princeton; another by A. L. Kimball, now associate professor of physics at the *Johns Hopkins University*.

"The Senior class in mathematics (elective) has two exercises a week throughout the year, under Professor Fine. The course for the current year is analytical geometry of three dimensions, differential and integral calculus. Williamson's text-books on the calculus are used, supplemented by lectures on determinants, differentiation and integration of functions of the complex variable, definite integrals."

In 1873 was founded, as a branch of Princeton College, a scientific school called the "John C. Green School of Science." Its courses lead to the degree of bachelor of science. Two years later a course in civil engineering was organized in this school. The mathematics in the scientific school is taught by Professor Rockwood. The course is framed so as to supply the necessary foundation in knowledge and training for the later studies of physics and mechanics, and especially finds its natural continuation in the applied mathematics of the course in civil engineering. Constant blackboard practice is a prominent feature of the instruction. Euclid is supplanted by Chauvenet's Geometry. Other text-books used are Wells's Algebra, Bowser's Analytical Geometry and Calculus. The calculus is begun at the end of the Sophomore year and then finished in the Junior. With the geometry, which is illustrated by models, is combined a thorough course in mensuration and an introduction to the elements of modern geometry. Thus, a synthetic course in conic sections is made to precede analytical geometry—an idea highly to be recommended. Calculus is required of all students in the scientific department. More advanced studies in pure mathematics are elective.

Descriptive Geometry is taught by Professor Willson from Warren's treatise.

In addition to the college courses, there are at Princeton University courses leading to the degrees of master and doctor.

Post-graduate mathematics have been taught since 1881.

"The University courses this year (1888–89) are in differential equations, in the theory of functions, and in higher algebraic curves and surfaces. They are based on the treatises of Forsyth and Boole, Hermite and Clebsch and Gordan, and Salmon and Clebsch, respectively. Professor Fine conducts these courses.

DARTMOUTH COLLEGE.

In 1833 Ira Young succeeded Ebenezer Adams as professor of mathematics and natural philosophy. His father was a carpenter, which trade he followed till he attained his majority. He early manifested much mechanical ingenuity. At twenty-one he began a course preparatory to entering college, and graduated at Dartmouth in 1828. He served in the college, first as tutor, then as professor, until his death in 1858. He is said to have been an admirable teacher.

From 1838 to 1851 Stephen Chase was a professor of mathematics at Dartmouth. He was a graduate of this college. While he was pro-

fessor he published an algebra. An old alumnus speaks of him as a teacher, "the light of whose genius, as it gleams through one of our text-books, yet lingers in our halls."

The catalogue for 1834 shows that, since 1828, a remodeling of the college course had taken place. There were now four departments, viz, the classical, mathematical and physical, rhetorical, and the department of intellectual and moral philosophy. The *Freshmen* in the mathematical and physical department studied Playfair's Euclid, reviewed Adams's Arithmetic, and commenced Day's Algebra during the first term; continued Day's Algebra in the second term; and completed Euclid in the third.

The *Sophomores* continued Day's Algebra, devoting their attention to applications to geometry and logarithms. They then took up plane trigonometry and its applications. During the second term Bridge's Conic Sections and Curvature, and Playfair's Spherical Geometry and Trigonometry occupied their attention. They began also Bezout's Calculus, which was finished during the third term. The *Juniors* pursued Olmsted's Natural Philosophy, Day's Mathematics (heights and distances, and navigation), Olmsted's Hydrostatics, and Astronomy. The Seniors had no mathematics, according to catalogue.

The next year (1839) indicates several changes. Legendre's Geometry and Bourdon's Algebra displaced old Euclid and Day's Algebra. Davies' Analytical Geometry and Calculus were also used. The influence of the Military Academy at West Point was now beginning to be felt at Dartmouth.

Regarding the mathematical teaching at this time, John M. Ordway, professor of applied chemistry and biology at the Tulane University of Louisiana, writes us as follows: "When I entered Dartmouth College in 1840, the Freshmen were instructed in algebra and geometry by a tutor. We used Davies' Bourdon's Algebra and Davies' Legendre's Geometry. In the Sophomore year we studied Davies' Surveying, and Plane and Spherical Trigonometry, Davies' Analytical Geometry and Davies' Calculus. The instruction was given by Professor Stephen Chase, an excellent scholar, but a somewhat peculiar man. He showed very little mercy to the duller students, and hence was not very popular. The analytical geometry and calculus had not been introduced many years, and it was a sort of traditional idea of the classes that preceded ours, that these subjects were very hard. We, however, did not perpetuate this tradition, for our class as a whole did not find these higher mathematics so very difficult. We had some field exercises in surveying and leveling. The professor went out first with half a dozen chosen students of the class, and they afterward went out with their respective sections of the class. Before our time there had been some solemn burnings of the mathematical text-books at the end of the year, but we had no such nonsense while I was in college.

"Professor Chase also gave the instruction in physics, which was

quite mathematical. He and Professor Young, the father of the present Professor Charles Young, of Princeton, had planned and partly written a work on physics, in which the demonstrations were to be made by the calculus and analytical geometry; but meanwhile Professor Olmsted published his Natural Philosophy, and as a matter of courtesy they dropped their own work and introduced the poorer one of Olmsted.* Olmsted used the common geometry and algebra, and his book was rather old-fashioned and contained some absurd errors. There was one question in the book, 'If the pebble that David threw weighed 2 ounces and Goliath weighed 800 pounds, with what velocity must the stone have moved to prostrate the giant?' The answer given was (about) 2,800 feet per second, or greater than that of a cannon ball. The professor called me up on this question, in the recitation, and asked me if I saw any absurdity in the matter. I told him yes, the answer should have been 2,800, and not 2,800 feet per second. Then the professor went on to explain that Goliath must have had a skull that would be penetrated by a stone moving with much less velocity. He had entirely overlooked the mathematical absurdity of getting a concrete answer out of mere abstract numbers. I went to him after recitation to explain my idea more fully, and told him that had Mr. Olmsted been a Frenchman he would have made the answer 2,800 *meters* per second, and that would have been just as correct, or 2,800 *miles* would have done just as well. This he acknowledged, but seemed never to have thought of it before, the physiological absurdity having shut out from his perception the mathematical error.

"While Professor Chase gave the mathematical teaching of physics, Professor Young lectured on the subject with the help of a very good set of apparatus.

"In the Junior year Professor Young taught astronomy, using Olmsted's Astronomy for a text-book. This work was better than the physics, but it rejected the calculus, which would have made many of the demonstrations much plainer. Professor Young was an excellent teacher and was very popular. He could be severe enough, but it was in a quiet, dry way that was not offensive. He would call up a fellow who had not studied the lesson well and put several questions, receiving the wrong answers without any sign of surprise or demur, and finally say, 'The reverse is true,' and call up another man.

"We had some astronomical instruments, but with the exception of the telescope very few of the students ever used any of them. Our examinations in those days were all oral. They were held in the presence of a committee of old graduates summoned to Hanover for the purpose, their expenses being paid by the college. These old fellows

* Olmsted's Introduction to Natural Philosophy was published in 1831; Young was elected professor in 1833 and Chase in 1838. While Young and, we believe, also Chase, served as tutors before they were appointed professors, it is, nevertheless, not likely that reference can be had, in the above, to the *first* edition of Olmsted's work on natural philosophy.

were rather rusty sometimes and gave the boys some amusement by their occasional old-time questions. The examinations were really a farce, though the results were counted in with the rest of the marks. It was rather funny to see how some fellows who had been rated very low all the year would be made out by the examiners to be among the very best."

In 1849 Chase's Algebra appeared, and began to be used at Dartmouth. Three years later Loomis's series was introduced, excepting his Algebra.

In 1851 Chase was succeeded by John Smith Woodman, of the class of 1842. After graduation he taught school in Charleston, S. C., afterward made a tour through Europe on foot, then studied and practiced law, and finally was elected professor at his *alma mater*.

In 1854 James Willis Patterson, of the class of 1848, became professor of mathematics. He had previously been tutor two years. From 1859 till 1865 he was professor of astronomy and meteorology. He afterward entered politics, was elected to the Legislature and finally to the United States Senate.

About the middle of the present century attempts were made to organize a system of education based chiefly upon the pure and applied sciences, modern languages, and mathematics. Of this class were the scientific schools connected with colleges, such as the Lawrence Scientific School at Harvard, the Sheffield Scientific School at Yale, the School of Mines at Columbia, and the Chandler Scientific School at Dartmouth. These schools have done efficient work and supplied a long-felt want.

The Chandler Scientific School was established in 1851. The instruction was designed to be "in the practical and useful arts of life, comprised chiefly in the branches of mechanics and civil engineering." At first J. W. Patterson is given in the catalogue as Chandler professor of mathematics, but Professor Woodman was the one who labored longest in this school. He taught in it from its establishment, became professor of civil engineering in 1856, and was practically at the head of it. He retained those posts until his death in 1871.

The mathematical course in this school was low at the beginning. Loomis's books were used, also Puissant's Mathematics. Descriptive geometry, shades, and shadows were also introduced.

In the catalogue of 1865, Robinson's series, from the Algebra to the Differential and Integral Calculus, is given. In 1866 Church's Analytical Geometry and Calculus were studied in the Chandler Scientific School. Two years later the college dropped Robinson's series and returned to Loomis's.

Since 1870 the text-books used have been, in algebra, Olney, Quimby; in geometry and trigonometry, Olney; in analytical geometry, Loomis, Church, Olney; in calculus, Church, Olney; in analytical mechanics, Peck, Wood; in descriptive geometry, Church; in quaternions, Hardy.

The terms of admission to the college were, in 1828, arithmetic, algebra through simple equations; in 1841, the same; in 1864, the same, with the addition of two books of (Loomis's) geometry; in 1886 and for some years previous, all of plane geometry was required; in 1888, arithmetic, including the metric system, algebra to quadratics, and plane geometry.

The college offers now two courses, one leading to the degree of bachelor of arts, the other (the Latin-scientific course) to the degree of bachelor of letters. The course of study for the year 1888–89 is as follows:

In the Prescribed Courses, I *is in each case an advanced division for students judged to be qualified to pursue a more extended course.*

PRESCRIBED COURSES.

FRESHMAN YEAR.

1. I. Algebra, including Theory of Equations (Quimby). *Sixty-five hours.*
   II. Algebra. *Sixty-five hours.*
2. I. Solid, with advanced, Geometry (Olney). *Forty-five hours.*
   II. Solid Geometry. *Forty-five hours.*
3. I. Plane trigonometry (Olney), including applications to Surveying; Spherical Trigonometry. *Sixty-two exercises (including ten exercises of field work of three hours each).*
   II. Same as 3, I, omitting Spherical Trigonometry.

SOPHOMORE YEAR.

4. I. Analytic Geometry (Olney). *Forty hours.*
   II. Spherical Trigonometry and Conic Sections. *Forty hours.*
5. Surveying with field work and plotting. *Eighty-seven hours.*
   Course 5 *is open only to students of the Latin-Scientific Course.*
6. Descriptive Geometry; Drawing. *Sixty hours.*
   Course 6 *is open only to students of the Latin-Scientific Course.*

ELECTIVE COURSES.

7. a. Differential Calculus. } Applications to Analytic Geometry. Lectures.
   b. Integral Calculus. } *Ninety-four hours.*
   Course 7, a *and* b, *is elective with French* 2 *followed by Mathematics* 8.
8. Elementary Mechanics (Wood). *Fifty hours.*
   Course 8 *preceded by French* 2 *is elective with Mathematics* 7, a *and* b.

JUNIOR YEAR.

9. Analytic Mechanics; Lectures. *Sixty hours.*
   Course 9 *is open only to students who have completed Course* 7, *and is elective with Latin and Greek.*
10. Descriptive Geometry; Shades, Shadows, and Perspective (Church). *Forty-four hours.*
    Course 10 *is elective with Latin, Greek, German, Physics.*

The minimum amount of mathematics on which a degree can be obtained, is a course ending with spherical trigonometry and conic sections. Analytic geometry is not necessary.

The course in pure mathematics in the Chandler Scientific School is much the same as the above. The catalogue mentions in that department Olney as the text-book in calculus and Peck as that in analytical mechanics.

In 1860, John E. Varney was appointed professor of mathematics, and served for three years. During the next six years John E. Sinclair filled this position. In 1872 F. A. Sherman, the present professor of mathematics in the Chandler School of Science, was elected. From 1872 to 1878 C. F. Emerson was connected with the mathematical department. Since then he has occupied the chair of natural philosophy and has devoted his energies chiefly to the development of the physical laboratory. P. H. Pettee has been professor of mathematics since 1877, and is now teaching mathematics and engineering in the New Hampshire Agricultural Experiment Station, which is a branch of Dartmouth College. At present T. W. D. Worthen is associate professor of mathematics in the college.

Since 1878 Arthur S. Hardy has been the head of the mathematical department at Dartmouth. He is professor of mathematics and of civil engineering. Previous to the above date he held the professorship of civil engineering in the Chandler Scientific School. Professor Hardy was graduated at West Point in 1869. For three years he was professor of civil engineering at Iowa College. He then spent one year in study at the École impériale des ponts et chaussées in Paris, and on his return went to Dartmouth. In 1881 appeared his Elements of Quaternions, the first American book on this subject. It is elementary and well adapted for use of those students in our colleges who may desire to know something of the wonderful researches of Sir William Rowan Hamilton. A neat little publication of much interest is Professor Hardy's translation from the French of Argand's Imaginary Quantities. He published also New Methods of Topographical Surveying, 1884. Professor Hardy possesses two qualifications that are rarely combined; he is a successful mathematician and also a successful novelist.

BOWDOIN COLLEGE.*

In 1825 William Smyth became adjunct professor of mathematics, and in 1828 was given the full chair, which he held until his death in 1868. He was an alumnus of the college. After his graduation, in 1822, he studied theology at Andover, and then became tutor at his *alma mater*. He was led to abandon Greek and take the department of mathematics as an instructor, from his success in popularizing algebra by means of the blackboard.

The introduction of the blackboard in our colleges must have caused important changes in the methods of teaching mathematics, especially geometry. Unfortunately no record of these changes has been preserved except at one or two institutions. We are happily able to quote the following account of its introduction at Bowdoin, taken from the history of the college, written by A. S. Packard. He says (p. 91) that the blackboard was introduced by "Proctor (afterward Professor)

* We are indebted for the information herein contained chiefly to a communication from Prof. George T. Little, of Bowdoin College.

Smyth in 1824. That novelty, let me here say, made a sensation. When he had tested the experiment in the Sophomore algebra, and with great success, a considerable portion of the Juniors requested the privilege of reviewing the algebra under the new method at an extra hour—a wonder in college experience; and that blackboard experiment, I am sure, led to his appointment as assistant professor of mathematics a year after. Of this also I am sure, that he had then first detected a mathematical element in his mental equipment. His forte had been Greek."

Professor Packard gives also an interesting account of the modes of teaching immediately before the blackboard came to be used. "The blackboard caused important changes in the manner of teaching generally, but especially in the mathematical branches. In arithmetic, a Freshman study, and algebra, to which we were introduced at the opening of the Sophomore year, each student had his slate, and when he finished his work he took the vacant chair next the teacher's and underwent examination of the process or principle involved. In geometry we kept a MS., in which we drew the figures and demonstrated from that. I have been shown a very neat MS. kept at Harvard by the late Dr. Lincoln, the father, and bearing the date 1800. * * * It may surprise my hearers that I professed to teach the algebra of the Sophomore class in Webber's Mathematics—the first tutor, I believe, to whom the duty was entrusted. That was the class of 1824. Franklin Pierce, of the class, in his earlier years of college life, more fond of fun than of surds and equations, took his seat by my side for a quiz with his slate and solution of a problem. When asked how he obtained a certain process; he replied very frankly, 'I got it from Stowe's slate.' * * * With blackboard such transfers are less easy. * * * It will cause more surprise that conic sections in Webber, a Junior branch, fell under my charge. The manner of reciting was simply to explain the demonstration in the text-book."

In 1834 the requirements for admission were increased so as to include "six sections of Smyth's Algebra." These six sections include nearly the entire Algebra, logarithms and the binomial theorem being excluded. In 1867 the requirements were raised so as to read arithmetic, the first eight sections of Smyth's New Elementary Algebra (to equations of the second degree), and the first and third books of Davies' Legendre. The requisites remained practically the same from 1867 to 1887, though the text-books recommended were several times changed. Since that time all of plane geometry has been required.

The calculus first appears as a study in the annual catalogue of 1830, the notation of Leibnitz being then used. Fluxions were probably never taught at Bowdoin.

Professor Smyth became an exceedingly able teacher and gained celebrity as a successful writer of mathematical text-books. His publications were, a work on Plane Trigonometry, followed by his Algebra, Analytical Geometry (1855), and Calculus (1856). All of these passed

through repeated editions and enjoyed an extensive sale. As they came from the press they took the place of the Cambridge Mathematics at Bowdoin. In the preparation of his Algebra he followed Bourdon and Lacroix as models, and it contains many of the excellences and some of the defects of these works. A remarkable feature is the very late introduction and explanation of negative quantities. They appear on page 89, after the solution of simultaneous linear equation. In his calculus he uses infinitesimals. "As a logical basis of the Calculus," says he (p. 229), "the method of Newton, and especially that of Lagrange, has some advantage. In other respects the superiority is immeasurably on the side of the method of Leibnitz." At the end of the book he very briefly explains the methods of Newton and Lagrange. A few pages are also given to the "Method of Variations" and "Applications to Astronomy."

The following account of Smyth and his works is taken from an obituary address by his colleague, Professor Packard: "As the first fruits, he issued a small work on Plane Trigonometry, availing himself of the ingenuity of the late Mr. L. T. Jackson, of this town, in preparing blocks on a novel plan for striking off the diagrams. The first edition of his Algebra from the press of Mr. Griffin, of this town, appeared in 1830, which first adapted the best French methods to the American mind, received warm commendation from Dr. Bowditch, and was adopted as a text-book at Harvard and other institutions. It passed through several editions and then gave place to two separate works, the Elementary Algebra and the Treatise on Algebra. Then followed an enlarged edition of the Trigonometry and its application to Surveying and Navigation, and treatises on Analytic Geometry and on the Calculus, the last being so clearly and satisfactorily developed and with so much originality as to receive emphatic approval in high quarters, particularly from the late Professor Bache."

"In explanation he was precise, simple, and clear. He had great power of inspiring interest; his own enthusiasm, which often kindled, especially in certain branches of his department, at the blackboard, being communicated to his class. Later classes will carry through life his setting forth of what he termed the 'poetry of mathematics,' as exemplified in the Calculus."

Of the graduates of Bowdoin during Smyth's time who distinguished themselves in the mathematical line, we mention John H. C. Coffin (class of 1834), who, soon after graduation, was appointed professor of mathematics in the U. S. Navy. He was for many years in the Naval Observatory and, in 1866, took charge of the American Ephemeris and Nautical Almanac.

Professor Smyth's successor, in 1865, was Charles Greene Rockwood, who had graduated at Yale in 1864, and in 1866 received the degree of Ph. D. When he left Bowdoin to accept a position at Rutgers College, in 1873, Charles Henry Smith took his place. In 1887, Professor Smith

was succeeded by Prof. William Alboin Moody, the present incumbent of the chair of mathematics. Professors Rockwood and Smith left the college with the reputation of able and skillful teachers. "The latter was, in my judgment, remarkably successful," says Professor Little, "in securing good and faithful work from all." The writer has before him a report on geometry by Professor Smith, presented to the Maine Pedagogical Society in 1884, and containing some good recommendations on the study of its elements. He strongly recommends a course in empirical geometry of the sort marked out by G. A. Hill's Geometry for Beginners, Mault's Natural Geometry, and Spencer's Inventional Geometry, to precede the course in demonstrative geometry.

Mathematics have never been taught at Bowdoin by lectures, though the instruction has been frequently supplemented by lectures. Since 1880 all mathematics have been elective after the Sophomore year; since 1886, all after the Freshman year. An elective in calculus, not then a required study, was offered from 1870 to 1880. In the year 1882–83 the *Freshmen* studied Loomis's Algebra, and Loomis's Geometry and Conic Sections, in two parallel courses during the first two terms; the third term of the year being given to Plane Trigonometry (Olney). The *Sophomores* had Olney's Spherical Trigonometry during the first term; during the second and third term they had the choice between analytical geometry, and Latin and Greek. Calculus was elective for *Juniors*. In the *Senior* year no mathematics were offered. The text-book in astronomy was Newcomb and Holden. A feature in this mathematical course to be recommended is that analytical geometry is preceded by a short course in conic sections (treated synthetically). The course for the year 1888–89 differs from the preceding in this, that plane geometry is required for admission; that Wentworth's Algebra has taken the place of Loomis's; that differential and integral calculus are studies in the second and third terms of the Sophomore year; that an advanced course in calculus (Williamson's) is offered during the first two terms of the Junior year, and quaternions during the third term.

GEORGETOWN COLLEGE.*

From 1831 to 1879 Father James Curley was the head of the mathematical and astronomical department at Georgetown College. He was born in Ireland, October 25, 1796. He entered the Society of Jesus September 29, 1827, and came here in 1830. In 1843 Father Curley built the college observatory. Here he calculated, from his observations, the longitude of Washington. The astronomers at the U. S. Naval Observatory had found a longitude differing a little from Father Curley's result. When, however, the laying of the Atlantic cable

* For what material we possess on the teaching at this college we are indebted to the kindness of Prof. J. F. Dawson, S. J., professor of physics and mechanics at Georgetown College.

brought Washington into telegraphic communication with Greenwich, it was found that Father Curley's calculation was the correct one. Since 1879 Father Curley has not been able to teach; he is still living at Georgetown, and is in the full possession of all his faculties.

Since 1839 Father Curley has generally had two assistants, or associates, in mathematics.*

Rev. James Clark was born October 21, 1809. He entered West Point at the age of sixteen, and graduated in the class of 1829. He served in the Army several years. In 1844 he entered the Society of Jesus, and came to Georgetown in 1845. In 1849 he went to Worcester College, Massachusetts, then recently established, but remained there only one year, returning to Georgetown in 1850. From 1862 to 1867 he was president of Worcester College. He returned to Georgetown in 1867, but was appointed president of Gonzaga College, Washington, in 1869. This office he held until 1875, when he again took his old position in Georgetown. In 1879 Father Clark became unable to teach, and on September 9, 1885, he died at Georgetown. For some years he taught calculus from his own manuscript, and intended to publish a text-book but for some reason did not do so.

About the year 1848, political troubles in Europe induced a considerable emigration to America of some of the most able members of the Society of Jesus, and the faculty of Georgetown College was increased by a considerable accession of learning and talent. We mention as the most conspicuous, Fathers Sestini and Secchi.

Rev. Benedict Sestini was born in Italy, March 20, 1816. He entered the society in 1836. In 1847 he was astronomer of the Roman Observatory. In 1848 he was compelled to leave Italy by the revolutionists, and came to Georgetown. He taught here until 1857; then he taught

* During 1839–'42, 1843–'45, 1847–'48, and 1865–'67, Rev. James Ward, S. J., gave instruction in mathematics; 1841–'46, Rev. Thomas Jenkins, S. J; 1840–'41 and 1842–'43, Rev. Augustine Kennedy, S. J.; 1844–'45, Rev. George Fenwick, S. J.; 1846–'47 and 1869–'71, Rev. Joseph O'Callaghan, S. J.; 1848–'49, Rev. Angelo Secchi, S. J.; 1849–'52, Rev. Edward McNerhany, S. J.; 1852–'54, Rev. Anthony Vanden Heuvel, S. J.; 1854–'60, Rev. John Prendergast, S. J.; 1860–'61, 1862–'63, 1870–'71, 1879–'8[illegible], Rev. C. Bahan; 1861–'63 and 1871–'74, Rev. G. Strong, S. J.; 1845–'49, 1850–'62, and 1875–'79, Rev. James Clark, S. J.; 1848–'57, and 1863–'69, Rev. B. Sestini; 1863–'64, Rev. Aloysius Varsi, S. J.; 1864–'65, Rev. James Major, S. J.; 1867–'69, Rev. Antonio Cichi, S. J.; 1869–'70, Rev. Patrick Forhan, S. J.; 1871–'72, Rev. Patrick Gallagher, S. J.; 1872–'73, Rev. Jerome Daugherty, S. J.; 1873–'74, Rev. Edmund Young, S. J.; 1874–'78, Rev. J. Ryan, S. J.; 1874–'78, Rev. M. O'Kane, S. J.; 1878–'83, Rev. J. R. Richards, S. J.; 1879–'83, Rev. Henry T. Tarr, S. J.; 1881–'86, Mr. Thomas McLoughlin, S. J.; 1883–'84, Rev. Timothy Brosnaham, S. J.; 1883–'84, Rev. John O'Rourke, S. J.; 1884–'85, Rev. Edward Devitt, S J.; 1884–'85, Rev. Thomas Stack, S. J.; 1885–'88, Rev. Samuel H. Frisby, S. J.; 1885–'87, Mr. Joseph Gorman, S. J.; 1887–'88, Mr. David Hearn, S. J.; 1888–'—, Rev. John Hagen, S. J., Rev. John Lehy, S. J., Mr. James Dawson, S. J., Mr. J. Gorman, S. J.

The frequent changes in the corps of instructors are due to the custom of the Society of Jesus. "In the society a teacher is liable any year to be sent to another college, and is rarely left more than four or five years in one place."

three years in Gonzaga College, two years in Worcester College, and one year in Boston College. In 1863 he returned to Georgetown, where he taught until 1869; he was then removed to Woodstock College. In 1886, advanced in years and broken down in health, he was sent to Frederick, to the novitiate of the society, where he still remains awaiting his end.* His books were used several years at Woodstock College (the scholasticate of the society). At one time they were in rather extensive use, but at present they have gone out of use almost completely.

Rev. Benedict Sestini published the following mathematical works: A Treatise on Analytical Geometry, Washington, 1852; A Treatise on Algebra, Baltimore, 1885; Elementary Algebra, second edition, 1855(?); Elementary Geometry and Trigonometry, 1856; Manual of Geometrical and Infinitesimal Analysis, Baltimore, 1871. The method of treatment of the various subjects in these works is not entirely conformable to that generally in vogue in this country at the time of their publication. The last named work is a thin volume of 130 pages, making no pretension of being a complete work on the subject. It was intended primarily for students in the author's own classes at Woodstock College, in Maryland, and as an introduction to the study of physical science.

With Father Sestini came Rev. Angelo Secchi, the astronomer. He was born in 1818, and entered the society in 1833. He was compelled to leave Italy in 1848, on account of the revolution. He remained at Georgetown very little more than a year. In 1850 he returned to Italy, and was placed in charge of the Roman Observatory, where he labored until his death, February 26, 1878.

At present the mathematical course consists of geometry, plane and spherical trigonometry, analytical geometry, differential and integral calculus, mechanics, and astronomy. Algebra is taught in the preparatory department. This course has remained practically the same since 1829, except that the time given to mechanics has been increased.

Elective studies have never been offered at the college, nor has the practice of lecturing ever been in vogue. Since 1829 more time has been given to mathematics than formerly. About the year 1820 the Society of Jesus adopted a new "*ratio studiorum*," or plan of studies, giving to mathematics more attention than had hitherto been accorded to them. This brought about the change at Georgetown in 1829. The methods of the Society of Jesus have been strictly adhered to. "The professor first explains the lesson, pointing out the important parts, the proofs, the connection with other parts of the subject, etc., and giving other proofs if those in the book do not suit him. On the following day he calls on one of the class for a repetition; after the repetition

* Professor Dawson has endeavored to find out something about the early life and education of Father Sestini, but with no success. Father Sestini himself can not give any information on the subject; his health has failed very much, and his memory can not be relied upon.

the members of the class bring forward their difficulties, suggestions, etc., after which the following lesson is explained. Problems are frequently given to test the knowledge and inventive powers of the students."

Father Sestini's text-books were used several years. They were replaced by those of Davies. In 1860 Gummere's surveying was introduced, and the Algebra, Geometry, and Trigonometry of Davies; Sestini's Analytical Geometry and Calculus were retained. In 1870 Sestini's Analytical Geometry and Calculus were replaced by Davies' Analytical Geometry and Church's Calculus. In 1872 and 1873 Loomis's Analytical Geometry was used. In 1874 Olney's Algebra, Trigonometry, and Calculus were introduced; Davies' Geometry and Gummere's Surveying were retained. In 1878 the Algebra and Geometry of Loomis were used, and in 1879 his Trigonometry, Analytical Geometry, and Calculus. Two years ago Wentworth's series was introduced, with Taylor's Calculus. Peck's Mechanics was used until 1881, when it was replaced by Dana's. In calculus the notation of Leibnitz has been employed "as far back as we have any records."

At the college observatory no work has been done for some years; but in January, 1889, Rev. John Hagen, S. J., was placed in charge of the observatory and will make regular observations. Father Hagen was formerly at Prairie du Chien, Wis. He is a mathematician of considerable ability and has contributed articles to the American Journal of Mathematics.

CORNELL UNIVERSITY.*

When Dr. Andrew D. White entered upon the organization of Cornell University and the selection of a faculty, the first professor appointed was Evan William Evans. He occupied the chair of mathematics at Cornell from the time of its opening, in 1868, till 1872, when he resigned on account of failing health. Professor Evans was a native of Wales, came to this country with his parents when a child, was graduated at Yale in 1851, and studied theology for a year. He then became principal of the Delaware Institute, Franklin, N. Y., was tutor at Yale from 1855 to 1857, and, later, professor of natural philosophy and astronomy in Marietta College, Ohio, where he remained until 1864. Before entering upon his work at Cornell University he was occupied for three years as mining engineer, and spent one year in European travel. He died not long after resigning his position at Cornell.

In the same year that Professor Evans was selected to the mathematical chair, Ziba Hazard Potter, a graduate of Hobart College, was appointed assistant professor of mathematics. This position he held for fourteen years.

* The writer is indebted to the kindness of Professor Oliver for sending annual reports and giving information on the mathematical courses of study at Cornell University.

In 1869 William E. Arnold, major U. S. Volunteers, entered upon the duties of assistant professor of mathematics and military tactics, and served seven years in that capacity.

Appointed as assistant professor at the same time as Professor Arnold, was Henry T. Eddy. He is a native of Massachusetts, was graduated at Yale in 1867, and then studied engineering at the Sheffield Scientific School. In 1868 he became instructor of mathematics and Latin at the University of Tennessee, at Knoxville. At Cornell he received the degrees of C. E. and Ph. D. for advanced studies in pure and applied mathematics. In 1872 he went to Princeton, where, for one year, he was associate professor of mathematics. Since 1874 he has held the chair of mathematics at the University of Cincinnati. The year 1879–80 was spent by him in study abroad.

Professor Eddy has won distinction as an original investigator. His Researches in Graphical Statics (New York, 1878) and his *Neue Constructionen in der graphischen Statik* (Leipzig, 1880) are contributions of much value, and, we believe, the first original work on this subject by an American writer. Professor Eddy is contributing largely to scientific and technical journals. In 1874 appeared his Analytical Geometry. At the meeting of the American Association for the Advancement of Science, in Philadelphia in 1884, Eddy was Vice-President of Section A, and delivered an address on "College Mathematics." Having been connected as student or teacher with several higher institutions of learning, both classical and scientific, he was able to speak from his own observation and experience of the defects of the mathematical instruction in the United States. His address contains many valuable suggestions.

In 1870 Lucien Augustus Wait, who had just graduated at Harvard, was appointed assistant professor. He held this position for about ten years, when he was made associate professor, which position he still holds. Some time ago he spent one year in Europe on leave of absence. Professor Wait is an energetic and excellent teacher of mathematics.

For three years succeeding 1873 William E. Byerly, a graduate of Harvard, and now professor there, was assistant professor at Cornell University. Professor Byerly is a fine teacher, and by his publications has made his name widely known among American students of the more advanced mathematics.

Since 1877 George William Jones has been assistant professor. He is a graduate of Yale, 1859. He "is thoroughly logical, and the best drill-master" in the mathematical faculty at Cornell University.

As has been seen from the above, two of the former assistant professors at Cornell have since won distinction elsewhere. The same is true of some of the instructors in mathematics. Before us lie the names of the following former instructors in mathematics at Cornell: George Tayloe Winston (one year, 1873, now at the University of North Carolina), Edmund De Breton Gardiner (one year, 1876), Charles Ambrose Van Velzer (one year, 1876, now professor of mathematics at University

of Wisconsin), Madison M. Garver, and Morris R. Conable (each for part of one year, about 1876).

At present there are four instructors, viz: James McMahon (since 1884), Arthur Stafford Hathaway (since 1885), Duane Studley (since 1887), George Egbert Fisher (since 1887).

Mr. McMahon is a graduate of the University of Dublin, Ireland, 1881. He has a fine mathematical mind, and has obtained gold medals for his proficiency in mathematics and mathematical physics, and also an appointment to a scholarship at his *alma mater*. He has not published much, but has assisted in the preparation of text-books on mathematics issued by the Cornell professors.

Mr. Hathaway graduated at Cornell in 1879, and then pursued graduate studies at the Johns Hopkins University, under Sylvester and his associates till 1884. While in Baltimore he frequently contributed papers to the mathematical society at the university, which were subsequently published in the Johns Hopkins University Circulars. He has made the theory of numbers his specialty, and has contributed several original articles on the subject to the American Journal of Mathematics. He gives a new theory of determinately-combining ideals. Mr. Hathaway is not only an able mathematician, but also an expert stenographer. When Sir William Thomson, of the University of Glasgow, delivered a course of lectures on Molecular Dynamics at the Johns Hopkins University, in October, 1884, Mr. Hathaway exercised his "power to seize on every passing sound." These stenographic notes of Thomson's lectures were printed by the papyrograph process and published. At Cornell, Hathaway has assisted in the preparation of text-books, and is now, with Professor Jones, preparing a Projective Geometry.

It will be noticed that Harvard University has contributed the largest share of mathematical talent to the faculty of Cornell. Not only are Byerly and Wait graduates of Harvard, but also Oliver, the present occupant of the mathematical chair at Cornell. These three sat at the feet of that Gamaliel, Benjamin Peirce, and caught the inspiring words of their great master.

James Edward Oliver was born in Maine, in 1829, and was graduated at Harvard in 1849. He had then already displayed extraordinary mathematical power, and was at once appointed assistant in the office of the American Nautical Almanac, at that time in Cambridge. In the Harvard catalogues of 1854 and 1855 we find J. E. Oliver and T. H. Safford enrolled as mathematical students in the Lawrence Scientific School, and taking advanced courses of mathematics, such as were offered at that time by no other institution in the land. In 1871 Oliver became assistant professor of mathematics at Cornell, and two years later was given full possession of the chair.

Professor Oliver is an extraordinary man, and it is interesting to listen to what his former pupils have to say of him. Says Prof. C. A. Van

Velzer: "He is indeed a wonderful man. If Professor Oliver had some of Sylvester's desire for reputation, he would have been heard from long ago, and would have been known all over the world." Says Mr. A. S. Hathaway: "Professor Oliver is a rare genius, powerful, able, but without the slightest ambition to publish his results. He works in mathematics for the love of it. I have seen work of his done one or two years ago. Practically the same work appeared in the American Journal of Mathematics, written by prominent authors, that I had urged him to publish, and which he had promised to do, but which, with his characteristic dilatoriness and diffidence in this respect, he failed to do until it was too late. I consider him fully equal in point of natural ability to Professor Sylvester, and he is better able than Professor Sylvester, I think, to acquire a knowledge of what others have done. He lacks, however, the energy and ambition of Professor Sylvester, and does not concentrate his powers on any one subject. His work is im-methodical, and leads in whatever direction his mind is bent at the moment. The result is that he is a far more amiable and congenial person to meet than Professor Sylvester. He never obtrudes self upon you, and wherever you may lead he will follow. Indeed, his simplicity of character and interest in everything that interests anybody else is one of his greatest charms. There are few subjects in which he does not know more than most people—you find it out when you are talking with him—but he does not seem to know it, at least he never obtrudes it."

Professor Wait is described as a "live energetic business manager, who was appointed to the position of associate professor to supplement Professor Oliver's shortcomings, and to take care of the practical management of the department. A better man could not have been chosen to associate with Professor Oliver. The latter finds in Professor Wait a ready promoter of his ideas and plans, and one who is capable of carrying them out in the smallest detail, and of taking charge of the department without troubling the chief."

Professor Jones is a good drill-master. The bulk of the work on mathematical text-books is done by him. His style has been adopted throughout. Professsor Oliver's style is more classical and polished, but that of Professor Jones is more suitable for elementary text-books. In consequence, everything written by any one else, has been re-shaped more or less by him.

The mathematical faculty of Cornell have published several text-books, going by the name of "Oliver, Wait, and Jones's Mathematics." The works in question are, a Treatise on Trigonometry, a Treatise on Algebra, and Logarithmic Tables. In preparation are also a Drill-Book in Algebra, which will be specially adapted to the work of the preparatory schools, and a Treatise on Projective Geometry.

The Treatise on Trigonometry has been used successfully at Cornell for eight years, and their Treatise on Algebra for two years. "For the

regular classes (in algebra) the more difficult parts have been cut out; but every year nearly all that was omitted by them has been taken up by volunteer classes (all Freshmen) with great satisfaction and profit." After eight years of use the Trigonometry has been wholly rewritten.

The Treatise on Algebra is not a book intended for beginners, but primarily for students entering the Freshman class at Cornell, and who have had extensive drill in elementary algebra. Most of our American colleges would find the book too difficult for use, on account of deficient preparation on the part of students entering.

If we compare Oliver, Wait, and Jones's Algebra with algebras used in our colleges ten or fifteen years ago, we discover most radical differences and evidences of a speedy awakening of mathematical life among us. A great shaking has taken place among the "dry-bones" of American mathematical text-books, and no men "shake" more vigorously than the professors at Cornell. Among the improvements we would mention a clearer statement of first principles and of the philosophy of the subject, the introduction of new symbols, a more extended treatment and graphic representation of imaginaries, and a more rigid treatment of infinite series. With some corrections and alterations in a subsequent edition, we have little doubt that the book will become the peer of any algebra in the English language.

At Cornell great efforts are made to teach the logic of mathematics, but it is hard to attain the desired standard on account of the way that preparatory schools train their pupils. The preliminary training in algebra generally gives students the idea that algebra is merely a mass of rules, and that students have simply to learn the art of applying them. In consequence of this, there is a constant rebellion among the average Freshmen to the logical study of algebra. Formulæ and substitutions are his stand-by.

The attendance of students has been very large at Cornell. Compared with some other departments of the university, the teaching force in mathematics has been rather small. In consequence of this, the time and energy of the professors have been taxed unusually by work in the class-room. In the appendix to the Annual Report of the President of Cornell University for 1886–87, Professor Oliver speaks of this subject, and also of the general work of the mathematical department. He says:

"We are not unmindful of the fact that by publishing more, we could help to strengthen the university, and that we ought to do so if it were possible. Indeed, every one of us five is now preparing work for publication or expects to be doing so this summer, but such work progresses very slowly because the more immediate duties of each day leave us so little of that freshness without which good theoretical work can not be done.

"A reprint of our algebra, increased to 412 pages, has, however, appeared this year, and has attracted favorable notice from the press and

from distinguished mathematicians. All five of us have in some way contributed to the work, but much more of it has been done by Professor Jones than by any one else. The chapters with which we propose to complete the book deal mainly with special applications, or with topics peculiar to modern analysis. Meanwhile we have successfully used the volume in all the Freshman sections this year. * * *

"The greatest hindrance to the success of the department, especially in the higher kinds of work, lies, as we think, in the excessive amount of teaching required of each teacher; commonly from seventeen to twenty or more hours per week. The department teaches more men, if we take account of the number of hours' instruction given to each, than does any other department in the university. Could each teacher's necessary work be diminished in quantity, we are confident that the difference would be more than made up in quality and increased attractiveness."

From the Report for 1887–88, p. 75, we clip the following:

"Of course one important means toward this end [of securing the attendance of graduate students] is the publication of treatises for teaching, and of original work. A little in both lines has been done during the past year, though less than would have been but for the pressure of other university work, and less than we hope to accomplish next year. Professor Oliver has sent two or three short articles to the Analyst,* and has read, at the National Academy's meeting in Washington, a preliminary paper on the Sun's Rotation, which will appear in the Astronomical Journal. Professor Jones and Mr. Hathaway have lithographed a little Treatise on Projective Geometry. Mr. McMahon has sent to the Analyst a note on circular points at infinity, and has also sent to the Educational Times, London, solutions (with extensions) of various problems. Other work by members of the department is likely to appear during the summer, including a new edition of the Treatise on Trigonometry."

As to the terms for admission to the university, in mathematics, the requirements in 1869 were arithmetic and algebra to quadratic equations; but plane geometry also was required for admission for the course in arts. "I judge from an old 'announcement,'" says Professor Oliver, "that in 1868, when the university opened, some students were admitted with only arithmetic." In recent years the requirements have been arithmetic, algebra through quadratics, radicals, theory of exponents, and plain geometry. In the engineering and architectural courses solid geometry has been added.

In and after 1889, candidates will have two examinations, the "primary" and the "advanced." The "primary" examination will cover the following subjects in mathematics:

In *Arithmetic*, including the metric system of weights and measures; as much as is contained in the larger text-books.

* The name of the mathematical journal in question is not *Analyst*, but *Annals of Mathematics*.

In *Plane Geometry;* as much as is contained in the first five books of Chauvenet's Treatise on Elementary Geometry, or in the first five books of Wentworth's Elements of Plane and Solid Geometry, or in the first six books of Newcomb's Elements of Geometry, or in the first six books of Hamblin Smith's Elements of Geometry.

In *Algebra*, through quadratic equations, and including radicals and the theory of exponents; as much as is contained in the corresponding parts of the larger treatises of Newcomb, Olney, Ray, Robinson, Todhunter, Wells, or Wentworth, or in those parts of Oliver, Wait, and Jones's Treatise on Algebra that are indicated below, with the corresponding examples at the ends of the several chapters: Chapters I, II, III; Chapter IV, except theorems 4, 5, 6; Chapter V, except §§ 3, 5, and notes 3, 4, of problem 2; Chapter VII, § 11; Chapter VIII, §§ 1, 2, the first three pages of § 8 and § 9; Chapter XI, except § 9, problem 9 of § 12, and §§ 13, 17, 18.

For admission to the course leading to the degree of bachelor of arts, no further knowledge of mathematics will be necessary, in any case.

For admission to the courses leading to the degrees of bachelor of philosophy, bachelor of science, bachelor of letters; to the course in agriculture; and (in and after 1890) for all optional students, there will be required, in addition to the "primary" examination, an "advanced" examination in two advanced subjects, "one of which must be French or German or mathematics." If the applicant chooses mathematics, he will be examined on all the Freshman mathematics, namely, solid geometry and elementary conic sections, as much as is contained in Newcomb's Elements of Geometry; advanced algebra, as much as is contained in those parts of Oliver, Wait, and Jones's Treatise on Algebra that are read at the university (a list is sent on application to the Registrar); and trigonometry, plane and spherical, as much as is contained in the unstarred portions of Oliver, Wait, and Jones's Treatise on Trigonometry.

It was the desire of Mr. Cornell and President White to establish a university giving broad and general training, in distinction to the narrow, old-fashioned college course with a single combination of studies. The idea was well expressed by Cornell when he said that he trusted the foundation had been laid to "an institution where any person can find instruction in any study." We shall proceed to give the course of study in mathematics, and let the reader judge for himself whether or not the idea of the founder has been carried out in the mathematical department.

We begin with studies which have been *required* for graduation. The mathematical course has always included, for all candidates for baccalaureate degrees except (at one time) a few natural history and analytic chemistry students, one term each of solid geometry, advanced algebra, and trigonometry (either plane, or plane and spherical). At one time students in history and political science had one term of theory of probabilities and statics instead of spherical trigonometry. There have also always been required in all engineering courses and in architecture, analytic geometry and calculus; and, sometimes, analytic geometry in certain other courses, as those in science and philosophy. At present the amount required is one term of analytic geometry and one term of

calculus, in the course of architecture, and one term of analytic geometry and two terms of calculus in the engineering course. At present, the students in mechanical and electric engineering take also an extra term in projective geometry and theory of equations in the Freshman year. These are the "required" mathematics in the different courses.

In addition to these, "elective" mathematics has always been offered by the university to upper classmen, and also, of late years, to Freshmen and Sophomores. The number of these elective courses has gradually increased, till now they are as follows (Register 1888–89):

ELECTIVE WORK.*

[Any course not desired at the beginning of the fall term by at least three students, properly prepared, may not be given.]

11. Problems in Geometry, Algebra and Trigonometry, supplementary to the prescribed work in those subjects, two hours a week. Professor JONES.

12. Advanced work in Algebra, including Determinants and the Theory of Equations, two hours a week. Professor WAIT.

13. Advanced work in Trigonometry, one hour a week. Professor WAIT.

[The equivalents of courses 8, 12, and 13 are necessary, and course 11 is useful, as a preparation for most of the courses that follow.]

14. Advanced work in Analytic Geometry of two and three Dimensions, viz:—

(*a*) First year, Lines and Surfaces of First and Second Orders. 3 hours. Professor JONES.

(*b*) Second year, General Theory of Algebraic Curves and Surfaces. 3 hours. Professor OLIVER.

15. Modern Synthetic Geometry, including Projective Geometry. 2 hours. Professor JONES.

16. Descriptive and Physical Astronomy. 3 hours. Mr. STUDLEY.

17. The Teaching of Mathematics. Seminary work. 1 hour. Professor OLIVER, and most of the teachers in the Department.

18. (*a*) Mathematical Essays and Theses: (*b*) Seminary for discussion of results of students' investigations. Professor OLIVER.

19. Advanced work in Differential and Integral Calculus. 3 hours. Mr. FISHER.

20. Quantics, with Applications to Geometry. Requires courses 8, 12, 14 (*a*), and preferably also 11, 13, 19. May be simultaneous with 14 (*b*). 3 hours. Mr. MCMAHON.

21. Differential Equations: to follow course 19. 3 hours. Mr. HATHAWAY.

22. Theory of Functions. Requires course 19, and preferably 21. (*a*) First year, 3 hours. (*b*) Second year, 2 hours. Professor OLIVER.

23. Celestial Mechanics. 3 hours. Professor OLIVER.

25. Finite Differences. 2 hours. Professor OLIVER.

27. Rational Dynamics. Professor WAIT.

28. Molecular Dynamics; *or*, 29, Theory of Numbers. 3 hours. Mr. HATHAWAY.

30. (*a*) Vector Analysis; *or*, (*b*) Hyper-Geometry; *or*, (*c*) Matrices and Multiple Algebra. 2 hours. Professor OLIVER.

31. Theory of Probabilities and of Distribution of Errors, including some sociologic applications. 2 hours. Professor OLIVER, or Professor JONES.

41. Mathematical Optics, including Wave Theory and Geometric Optics. 2 hours. Professor OLIVER.

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* Numbers 1 to 10, inclusive, refer in the catalogue to *required* studies in mathematics.

43. Mathematical Theory of Sound. 3 hours. Mr. McMAHON.

44. Mathematical Theory of Electricity and Magnetism. Professors OLIVER and WAIT.

In most of the above branches of pure mathematics, an additional year's instruction, 1 or 2 hours per week, may be given if desired.

For several years (from 1874 to 1887, we believe) there has been also a "course in mathematics," with a fixed curriculum, leading to the degree of "bachelor of science in mathematics," but it was dropped when the numerous prescribed curricula and resulting degrees were consolidated into a few "general courses," of which the work is mainly prescribed in the first two years and mainly elective in the last two, and a few "technical courses," whose work is mainly prescribed throughout. That old "course in mathematics" comprised some language and culture studies, botany, geology, logic, English literature, descriptive geometry, analytical mechanics, lectures and laboratory work in physics, while, perhaps, two-fifths of all the student's time was given to pure mathematics, including analytical geometry, calculus, differential equations, finite differences, quaternions, imaginaries, mathematical essays, seminary-work, etc. The object of this course was to give the best equipment to students intending to become teachers of mathematics, professors, and investigators. The students in this course were few, but earnest, and some of them have since been making their mark as teachers and investigators.

As to the mathematical text-books which have been used at different times, we make the following statement:

In Elementary Geometry, Loomis till about 1873; since then, Chauvenet.

In Elementary Geometric Conics, Loomis, then Peck, though the present professors "don't much like either."

In Modern Synthetic Geometry, Professor Evans used no book, but gave lectures. The same has sometimes been done since. At other times, Cremona's *Géométrie Projective*, or the recent English translation, was used. But now a little lithographed treatise on Projective Geometry, written for the purpose by Jones and Hathaway, is being used. Professor Oliver has taught, also, Casey's Sequel to Euclid, and, once, Steiner's Conics.

In Algebra, first Loomis, then Davies' Bourdon, Olney, Wells, Newcomb, and now Oliver, Wait, and Jones's, Todhunter's, Burnside and Panton's Theory of Equations.

In Determinants, Muir, Dostor, Hanus, and lectures.

In Quantics, Salmon's Higher Algebra.

In Trigonometry, first Loomis's (including a little of mensuration, surveying, and navigation), then Greenleaf's, Chauvenet's, Wheeler's, and now Oliver, Wait, and Jones's, Todhunter's.

In Analytic Geometry, first Loomis (for two dimensions) and Davies (for three dimensions), also Church, then Peck, Todhunter, Aldis, and now Smith (English work) with the three dimensions by lecture. With

more advanced students have been used also Salmon's Conic Sections, Higher Plane Curves, and Analytic Geometry of Three Dimensions.

In Differential and Integral Calculus, first Loomis and Church, then Peck, Todhunter, Williamson, Taylor, Meunier-Joannet, Homersham Cox, Woolhouse, Smyth, Byerly; and now Taylor for the few students in the one-term course, the abridged Rice & Johnson's Differential and Rice's Integral Calculus (one term each) for the two-term course for engineers, and Williamson and Todhunter for advanced work, with Bertrand for occasional reference and special work.

In Imaginaries, Argand was used, but now preference is given to Chapter X of Oliver, Wait, and Jones's Algebra.

In Equipollences, Belavitis was once used.

In Quaternions, Kelland, Tait, Hardy, Hamilton's Lectures, Hamilton's Elements.

In Theory of Functions there have been used Laurent's *Fonctions Elliptiques*, Hermite's *Cours d' Analyse;* and now Briot and Bouquet's *Théorie des Fonctions Elliptiques* and Halphen's *Traité des Fonctions Elliptiques.*

In Theory of Numbers, Dedekind's edition of Lejeune Dirichlet's *Zahlentheorie* has been used recently.

In Least Squares, Merriman.

In Differential Equations, Boole, Forsyth.

In Finite Differences, Boole.

In Descriptive Astronomy, Loomis, Newcomb, and Holden, with Young's "The Sun" and Chauvenet (for eclipses) for collateral reading.

In Mechanics, Duhamel's *Mécanique Analytique*, and now Minchin's Analytical Statics and Williamson's Analytical Dynamics.

Quaternions have not been taught now for several years, because the professors are convinced that the benefit of that study is with most students better gotten with a mixed course in matrices, vector addition and subtraction, imaginaries, and theory of functions.

Among the fundamental ideas of President White, in organizing the university, was a close union of liberal and practical education. There have, therefore, from the beginning, existed separate departments of civil engineering, of mechanic arts, and of physics, each with a separate professor at its head. Astronomy is taught partly in the department of civil engineering and partly in that of mathematics.

Pupils in mathematics are always encouraged to do original work, but it is only by older and maturer students that researches are made which are of sufficient value to merit publication. The writer has before him two printed theses, written to secure the degree of doctor of philosophy at Cornell University. One is by C. E. Linthicum, "On the Rectification of Certain Curves, and on Certain Series Involved" (Baltimore, 1888); the other is by Rollin A. Harris, on "The Theory of Images in the Representation of Functions" (Annals of Mathematics, June, 1888). Both of these are very creditable to the writers and to the university, and the latter appears to us to fill a gap.

There are always some under-graduate students who do good work in the more advanced mathematical electives, but at present it is by resident graduates of Cornell and other colleges that the best advanced work is expected to be done. Great efforts have been and are being made to secure the attendance of graduate students in advanced courses in mathematics. During the year 1885–86 eleven graduate students were engaged in the study of the higher mathematics. The number for the year following is not known to the writer, but the president's report indicates that the attendance on advanced courses in mathematics was increasing, and that about one-fifth of the graduate students were taking their chief work in mathematics. In the last report Professor Oliver says:

"During 1887–88 eleven graduate students have taken more or less of their work with us. Allowing for such as were partly in other departments or remained but part of the year, we find that the mathematical department has had about one-seventh of all the graduate work in the university. This would seem to be our full share of this desirable kind of teaching, when it is considered that the higher mathematics is difficult, abstract, and hard to popularize; that of course we can not attract students to it by laboratories and large collections (except of books), nor by the prospect of lucrative industrial applications; and that our department's whole teaching force, composed of only about one-eleventh of all the active resident professors and instructors in the university, and including only one-thirteenth of the resident professors, has to do about one-ninth of all the teaching in the university."

We are sure that many, perhaps all of our professors of mathematics will see in the following remarks by Professor Oliver the reflection of their own experience as teachers:

"We have always had to contend with one other serious difficulty. There is a wide-spread notion that mathematics is mainly important for the preliminary training of certain crude powers, and as auxiliary to certain bread-winning professions, and that only literary studies can afford that fine culture which the best minds seek for its own sake. Time, no doubt, will rectify this misapprehension; but meanwhile it hinders our success."

The methods of teaching mathematics at Cornell are various. The professors sometimes lecture, especially when there is no suitable text-book at hand. This method, when a rather full syllabus is given out beforehand, and plenty of problems are assigned to the students for solution, has sometimes proved very successful. The lecturer perhaps calls upon the class for suggestions as he proceeds with his topic, and then assigns to them for home study some problem very much alike in principle to the one they have just been discussing together.

But oftener it is preferred to base the teaching upon a book that the students can study for themselves, supplementing it by lectures and explanations, and holding the class to recitations and examinations upon it. In all the work, and especially in that for advanced classes, the pupils

are treated by the professor as fellow-students, and he avoids assuming toward them the air of master and dictator. Independent thought is constantly encouraged, even when this leads the students to criticise the things they are being taught. Mere memory-work and rote-learning—still in vogue in many of our schools—is discouraged in every way possible.

Some of the mathematical teachers at Cornell have been accustomed to test their pupil's mastery of the subject by written examinations, given in the midst of the term's work without warning, or on weekly reviews. There is also a written examination at the close of each term; but students who have done their term's work with a certain degree of excellence beyond what would be strictly requisite to "pass them up" in the subject are often exempt from this examination.

Since 1874 mathematical clubs have existed at times. Different members of it would give in turn the results of their mathematical studies in lines a little outside of the regular work of the class-room, and the matter thus presented was then open to discussion by the whole company present. Professor Oliver has generally presided at these meetings and taken his turn at presenting topics and work for discussion. The attendance upon these clubs has generally been small, including only the professors, instructors, and a few advanced students. Sometimes the meetings would be kept up for a few months or a year with a good deal of spirit, and then with change of membership the interest would flag, and the club would be discontinued for a while. Much of the work presented was the work of immature students and not worth publishing. But these clubs have helped to keep up an interest in mathematics and to stimulate the spirit of originality.

For the past three or four years the club has been merged into a "seminary" for the discussion of aims and methods in teaching mathematics. Here the professor proposes such problems as these: "Why do we teach mathematics at all, and what practical rules does this suggest to us in order that our teaching may be most effective and useful toward the end proposed?" "What is the place of memory in mathematical teaching?" "What are the relative advantages of lecturing and book work, and how are they best combined?" "How can we best teach geometry?" "What is the nature of axioms in geometry, and how modified when we consider the possibility of non-Euclidian space?" The professor proposes some such problem, then calls for discussion and adds his own views. If possible he develops on the blackboard a syllabus or tabular view of the different heads under which the theory must fall. Then these are discussed in order, either at that or at subsequent meetings. In the latter case the discussions are often opened by essays from members of the seminary. This method of conducting the seminary is most fruitful of results, especially if we remember that the chief object of the graduate department in mathematics is to train teachers of this science. The coming teacher will acquire possession of better methods and higher ideals of mathematical teaching.

VIRGINIA MILITARY INSTITUTE.

The Virginia Military Institute at Lexington, Va., is a State institution, and was organized in 1839 as a military and scientific school. It is a foster child of the U. S. Military Academy at West Point. At its organization General Francis H. Smith was made its superintendent. This position he has now held for half a century. What the Virginia Military Institute has been and is, is due chiefly to his long and faithful service as superintendent.

General Francis Henney Smith is a native of Virginia. He graduated at West Point in 1833, and was assistant professor of mathematics there during the first two years after graduation. He then occupied the chair of mathematics for two years at Hampden-Sidney College. At the military institute he added to his duties as superintendent those of professor of mathematics and moral philosophy.

Smith has published a number of mathematical text-books. Some of his books have suffered from frequent typographical errors. In 1845 appeared his American Statistical Arithmetic, in the preparation of which he was aided by R. T. W. Duke, assistant professor of mathematics at the institute. The book was called "Statistical Arithmetic," because the examples were selected as far as practicable from the most prominent facts connected with the history, geography, and statistics of our country. This novel idea made that arithmetic the medium for communicating much important information and a better appreciation of the greatness and resources of our country.

Other arithmetics appeared by the same author, which enjoyed quite an extensive circulation. About 1848 was published also a series of algebras, as a part of the mathematical series of the Virginia Military Institute.

A valuable contribution to the list of college text-books was the translation, by Professor Smith, in 1840, of Biot's Analytical Geometry. The original French work of Biot was for many years the text-book for the U. S. Military Academy at West Point. When, about ten years previous, Professor Farrar prepared his Cambridge mathematics, he chose Bezout's work on the "application of algebra to geometry," in preference to the works of Lacroix and Biot, for the reason that these works were thought to be too advanced for our American colleges, which had up to that time paid no attention whatever to *analytical* geometry. Bezout's work can hardly be called an analytical geometry. The only works on this subject which were published in this country after the Cambridge mathematics and previous to Smith's Biot were the elementary treatise of J. R. Young (which followed Bourdon as a model) and the work from the pen of Professor Davies, of West Point. Smith's translation of Biot reached a second edition in 1846. Afterward the book was revised. An edition of it appeared in 1870.

In 1867 Smith published an edition of Legendre's Geometry. Edi-

tions of this work had appeared in this country by Farrar and Davies. Smith's translation was from a later French edition, which contained additions and modifications by M. A. Blanchet, an élève of the École Polytechnique.

In 1868 appeared from the pen of General Smith a Descriptive Geometry. The study of this subject had been introduced in the institute at a time when it was hardly known by name in other schools and colleges of Virginia.

The organization of the Virginia Military Institute and the methods of teaching have been much the same as at the U. S. Military Academy. Indeed, the institute is frequently called the "Southern West Point." The division of classes into sections and the rigid and extended application of the "marking system" have been adopted from West Point. The marking system seems to have originated in France, and to have been introduced into this country by West Point.

The relative weight given to the different subjects of instruction forming the general merit-roll of each class is, according to the Official Register of 1887–88, as follows:

1. Mathematics (grade)	3	12. Surveying	1
2. Civil engineering	3	13. Moral and political philosophy	1
3. Military engineering	1	14. Ordnance and gunnery	1
4. Chemistry	2	15. Drawing	1
5. Mechanics	2	16. Geography	1
6. French	1	17. Infantry tactics	0.5
7. German	1.5	18. Geology	0.5
8. English	1	19. Descriptive geometry	1
9. Physics	1.5	20. Logic	0.5
10. Mineralogy	1	21. Rhetoric	0.5
11. Astronomy	1	22. Latin	1.5

The success of the educational work of the school turns largely upon the method of dividing classes into *sections*, whereby the students are accurately *graded* according to scholarship, and each secures a proportionately large share of the personal attention of the instructor. Each section is "under the command of a 'section-marcher,' taken from the first cadet on the section-roll. The sections are formed on parade, at the appointed hours; the roll is called by the section-marcher, absentees are reported to the officer of the day, whose duty it is to order all not properly excused to the class duty. The section-marcher then marches his section to the class-room, reports the absentees to the professor, and then transfers to him the responsibility which he had thus far borne. The professor examines the section on the appointed lesson, is responsible for the efficiency of his instruction, and once a week makes an official report to the superintendent of the progress of his section. These reports are duly recorded, and constitute an important element in the standing of each cadet at his semi-annual or general examinations." *

* The Inner Life of the Virginia Military Institute Cadet, by Francis H. Smith, LL. D., 1878, p. 22.

As at West Point, so at this institution, a candidate for admission has been required to know no other mathematical study than arithmetic. "The four ground rules of arithmetic, vulgar and decimal fractions, and the rule of three" admitted a candidate, as far as mathematics is concerned.

The course of study has been the same as at West Point, but the books used have not always been the same. The books used at the beginning were as follows: Bourdon's Algebra, Legendre's Geometry, Boucharlat's Analytical Geometry (in French), Boucharlat's Differential and Integral Calculus (in French), Davies' Descriptive Geometry. These were, later, displaced by other books, chiefly Smith's own works, viz, Smith's Algebra, Smith's Descriptive Geometry (after De Fourcy), Smith's Legendre's Geometry, Smith's Biot's Analytical Geometry, Courtenay's Differential and Integral Calculus, Buckingham's Calculus. As at West Point, so here, there have been no elective studies.

During the first twenty years of its existence the Virginia Military Institute was flourishing. It "had just placed itself before the public as a general school of applied science for the development of agricultural, mineral, commercial, manufacturing, and internal improvement interests of the State and country when the army of General Hunter destroyed its stately buildings and consigned to the flames its library of ten thousand volumes, the philosophical apparatus used for ten years by 'Stonewall' Jackson, and all its chemicals. The cadets were then transferred to Richmond, and the institution was continued in vigorous operation until the evacuation of Richmond on the 3d of April, 1865."*

The War left sad traces on the institution, besides the destruction of its buildings, library, and apparatus. Three of its professors had been slain in battle: Stonewall Jackson, who had been professor of natural and experimental philosophy since 1850; Maj. Gen. R. E. Rodes, a graduate of the institute, and, in 1860, appointed professor of civil and military engineering; Col. S. Crutchfield, also a graduate of the institute, and, since 1858, professor of mathematics. Among the slain were also two assistant professors and two hundred of its alumni.

Notwithstanding the impoverishment of the people immediately after the War, it was decided in 1865 to re-open the institution. Without one dollar at command to offer by way of salary to the professors, the board of visitors called back all who survived, and filled the vacancies of those who had died. Work was begun with earnestness. On the 18th of October, 1865, the day designated for the resumption of academic duties, sixteen cadets responded. At the end of the academic year the number of cadets was 55. Such vitality under such discouragements prompted the legislature to restore the annuity the next winter. It was not very long before the Virginia Military Institute was restored to all its former lustre. In 1870 the buildings of the institute were restored and equipped with laboratories and instruments.

* Official Register, 1887-88.

The Official Register for 1887–88 gives 119 cadets in the academic school. The studies in mathematics for that year are as follows: *Fourth class—First year:* Smith's Algebra, Davies' Legendre's Geometry and Trigonometry (revised by Van Amringe), Exercises. (Recitations from 8 to 11 daily.) *Third class—Second year:* Smith's Biot's Analytical Geometry, Buckingham's Differential and Integral Calculus. (Recitations from 9 to 11 daily.) *Second class—Third year:* Mahan-Wheeler, Davies' Surveying (Van Amringe), Gillespie's Surveying, field work. (Recitation from 10 to 11.) *First class—Fourth year;* Rankine's Applied Mechanics and Rankine's Civil Engineering, lectures, and field practice.

UNIVERSITY OF VIRGINIA.

President Jefferson devoted the golden evening of his life to the founding and building up of the University of Virginia as a nursery for the youth of his much-loved State. This greatest university of the South has from its beginning had features peculiar to itself. The entire abandonment of the class system, and the course arrangement of its studies, are its most prominent distinguishing features. From the very beginning the method of instruction has been by lectures and examinations. "Text-books are by no means discarded, but the professor is expected to enlarge, explain, and supplement the text. Every lecture is preceded by an oral examination of the class on the preceding lecture and the corresponding text. This method stimulates the professor to greater efforts, and excites and maintains the interest and attention of the students a hundred fold."*

The university was opened for students in March, 1825. It then had eight distinct schools, but at the present time it has nineteen, "each affording an independent course under a professor, who alone is responsible for the system and methods pursued." One of the eight original schools was that of mathematics, pure and applied. The first professor of mathematics (from 1825 to 1827) was Thomas Hewett Key, of England. He was a graduate of Trinity College, Cambridge. Besides his ability as a mathematician, he possessed great classical and general attainments. He resigned his position in order to accept the professorship of Latin in the London University.

His successor was Charles Bonnycastle, of England, who, upon Mr. Key's resignation, was transferred from the chair of natural philosophy to that of mathematics, which he continued to fill until his death, in 1840. He was the son of John Bonnycastle, who was widely known in England and America for his mathematical text-books, and was educated at the Royal Military Academy at Woolwich, where his father was professor. His father's books exhibit those faults which were common to English works on mathematics in his day. It is fair to presume,

* Dr. Gessner Harrison, in Duyckinck's Cyclopædia of American Literature; Article, "University of Virginia."

however, that Charles belonged to that coterie of English mathematicians of which Herschel, Peacock, Whewell, and others were members, and which introduced the Leibnitzian notation and also the ratio definition of the trigonometric functions into Cambridge. At the University of Virginia he enjoyed the reputation of a man of great ability in mathematics and of broad general knowledge. His lighter writings indicate that he could have shone also in the fields of literature. We are happy in being able to quote the following, from Dr. James L. Cabell, professor of physiology and surgery at the University of Virginia:*

"Though apparently an earnest and enthusiastic student of the higher mathematics, it was the constant habit of Professor Bonnycastle to make extensive and varied excursions into other fields of study, such as history, metaphysical philosophy, and general literature. I remember to have seen in his private library after his death several volumes of works on moral philosophy with copious marginal notes written by him. I recall in this connection the fact that he used to speak with emphasis and some indignation on the absurd charge that the study of mathematics tends to render its votaries insensible to the force of probable evidence, and that when strict mathematical investigation cannot be had, persons whose mental discipline has been secured by such training become either obstinately skeptical or wildly credulous. He insisted that all one-sided training had a natural tendency to narrow the intellect and that this applied to all other branches of learning and all professional pursuits as well as to mathematics. The obvious remedy lies in a liberal and broad culture. It was doubtless with a view to enforce his precepts by occasional examples that he was in the habit of delivering at the opening of each session of the university a popular lecture, the topics of which, having apparently a very remote connection with mathematical studies, were actually suggested by some recent publications in the department of general literature. These addresses were greatly admired by the crowds of young men who attended them, including, in addition to his own class, representatives from all the other departments of the university. He was also a contributor to a literary magazine published by the faculty in 1828–29. Some of his articles were stories of more than ordinary merit in this class of literary productions, and would probably have made his fortune if such magazines as Harper's, Scribner's, etc., had existed at that day with a competent development of public taste.

"The only distinct impression which I can now recall as to Professor Bonnycastle's method of teaching has reference to his attempts to indoctrinate his pupils at every stage of their studies with the philosophy and essential principles of the subject under consideration. At that time most, if not all, the usual text-books and all the school teachers gave only rules which the student was to apply. So far as the students knew, these rules might be wholly arbitrary. Professor Bonnycastle

* Letter to the writer, January 4, 1889.

insisted on the necessity of placing the student in a position to recognize the true significance of every principle laid down. This was done by oral lectures characterized by remarkable lucidity of statement and by a marvellous fertility of striking illustrations. These lectures were fully appreciated by the better sort of students in the advanced classes, but were thought by most of us to be thrown away upon the younger and less ambitious members of the lower classes. The general verdict of all classes of hearers ascribed to Mr. Bonnycastle genius and attainments of the highest and most varied character."*

The text-books used by Bonnycastle in pure mathematics, in connection with his lectures, were the Arithmetic, Algebra, and Differential Calculus of Lacroix, the first two in Farrar's translation. The theory of the integral calculus was taken from Young, the examples from Peacock's Collection. In geometry he used his own work on Inductive Geometry (1834).

In pure mathematics there were in his time three classes: the "First Junior," "Second Junior," and "Senior." "Of these the First Junior begins with arithmetic; but as the student is required to have some knowledge of this subject when he enters the university, the lectures of the professor are limited to the theory, showing the method of naming numbers, the different scales of notation, and the derivation of the rules of arithmetic from the primary notion of addition; the addition, namely, of sensible objects one by one. The ideas thus acquired are appealed to at every subsequent step, and much pains are taken to exhibit the gradual development from elementary truths of the extensive science of mathematical analysis." (Catalogue for 1836.) After a thorough course in arithmetic students were well prepared for algebra. In teaching the rules for adding and subtracting, etc., they were compared with the corresponding rules in arithmetic, and the agreement and diversity were noticed and explained. The elements of geometry were taught and illustrated by models. The book on Inductive Geometry was prepared especially for the use of his students. It includes geometry, trigonometry, and analytical geometry. In the definition of the trigonometric functions the ratio system is used. "The chief result which the author hoped to secure by the proposed innovation was such an arrangement of the subject as would enable him to

* In another part of his letter Dr. Cabell says: "I felt bound to tell you that owing to my complete want of mathematical knowledge, even to the extent of ignorance of the terminology of the science, I was utterly incompetent to form a critical judgment of Professor Bonnycastle's method of teaching. I can, however, recall with some vividness the impression made upon me at the time when he caused me and my fellow-students to understand the significance of processes which we had previously applied in a purely arbitrary method. It is probable that we exaggerated the merit of our new professor by contrasting him with the very imperfect and defective standards of the common schools of Virginia at that day. I believe, however, that these defects were common to the whole country when Professor Bonnycastle introduced a reform which in a few years may have become general."

dispense with the distinctions hitherto made between the different branches of geometry, and thus permit him to treat the problems embraced under the heads of synthetic geometry, analytic geometry, and the two trigonometries, as composing one uniform doctrine, the science of Quantity and Position."*

The general plan appears to be a good one, in the main. But its execution is not satisfactory. The work covers 631 crowded pages. The *form* in which the subject is presented is bad. Theorems and their demonstrations are in the same kind of type, and the eye finds nothing to assist and relieve it in passing over the crowded pages of prolix explanations. Nor is the reasoning always good.†

His Inductive Geometry is, we believe, the only mathematical work which he published while he was professor at the University of Virginia.

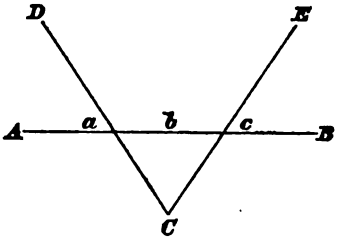
Both algebra and geometry were begun in the "First Junior" class (catalogue 1836), and then continued in the "Second Junior" class. Calculus was begun in this class and then completed in the "Senior" class.

The notation of Leibnitz was used at the University of Virginia from the very beginning.

In the Virginia Literary Museum, a weekly journal issued in 1829 by the professors of the university, we read of an examination of the

* Preface to Inductive Geometry.

† "Angles are so evidently portions of space surrounding their vertex, and this space so manifestly the same in all cases, that we are forced to regard it, directly or indirectly, as the standard to which all angles should be referred" (p. 112). The reasoning by which the sum of the three angles of a triangle is shown to be two right



angles, is as follows (p. 123): "The lines AB, CD, CE, * * * that enclose a small triangle at C, are separated by the openings a, b, c, that are nearly equal to the angles of the triangle; two of these openings, namely, a and c, are identical with angles of the triangle, and the third, b, which forms a space indefinitely extended, differs from the opening we call the angle C merely by the small space included in the triangle.

"This last, by bringing the triangle nearer to C, may be rendered as small as we please; and thus a triangle can always be assigned whose angles shall differ from a, b, c, and, consequently, the sum of whose angles shall differ from two right angles by less than any assignable quantity. Some difference between the results appears, it is true, always to remain; but if we examine more attentively the idea that we are able to form of infinite space, we shall find the difference in question merely apparent, and shall perceive the sum of the three angles to be rigidly equal to two right angles." This reasoning is bad. It involves, unnecessarily, the consideration of infinite spaces.

Senior class in mathematics, on Thursday, July 16, 1829: "The members of the class were examined in application of algebra to geometry and the theory of curves, as contained in the IV chapter of Lacroix's *Traité du Calcul Différentiel et du Calcul Intégral.* In the differential and integral calculus they were examined by examples taken from the questions on these subjects published by Peacock & Herschel. The class have studied the differential calculus chiefly from the treatise of Boucharlat, and the integral from Boucharlat, Lacroix, and the examples before mentioned. They have proceeded to the integration of partial differential equations of three or more variables, and the questions proposed were chosen to this extent." These extracts show that the course of mathematics taught by Professor Bonnycastle was remarkably far advanced, compared with the work done in the ordinary college or university in this country at that time.

Besides the three classes above given there was from the beginning a class in mixed mathematics (really a graduate class). Under Bonnycastle the text-books in this study were Venturali's Mechanics and the first book of Laplace's *Mécanique Céleste.* The principles were applied to various problems. A separate diploma has been given to students completing this course of mixed mathematics.

Professor Bonnycastle left a large number of mathematical MSS. in the keeping of Professor Henry, of the Smithsonian Institution, who a short time before his death sent them to be deposited in the library of the University of Virginia.

After the death of Bonnycastle, Pike Powers, now a minister at Richmond, held the chair until J. J. Sylvester was elected professor, in 1841. Mr. Powers was a young mathematician of fine gifts and attainments, and a pupil of Bonnycastle. Professor Sylvester was then already generally recognized as a man of brilliant genius and profound mathematical learning. He resigned in about half a year, and afterward accepted a professorship in the Royal Military Academy at Woolwich. We shall have to say more about him in connection with the Johns Hopkins University. Prof. Pike Powers was again appointed, temporarily, to teach the mathematics.

The next possessor of the mathematical chair was Edward H. Courtenay, from 1842 to 1853. He was the first regular occupant of this chair who was educated in this country. He was born in Baltimore, in 1803. After having been examined for admission to the U. S. Military Academy at West Point, in 1818, the examiner remarked: "A boy from Baltimore, of spare frame, light complexion, and light hair, would certainly take the first place in his class." Courtenay completed the four years' course in three years, and graduated at the head of his class in 1821. From that time till 1834 he was connected as teacher with the Military Academy, excepting the period from 1824 to 1828. After leaving West Point he was for two years professor of mathematics at the University of Pennsylvania, then he became division engineer for the

New York and Erie Railroad. He was employed by the United States Government as civil engineer in the construction of Fort Independence, Boston Harbor, from 1837 to 1841. Just before his appointment to the professorship at the University of Virginia he was chief engineer of dry-dock, Navy-Yard, Brooklyn, N. Y.*

Mr. Courtenay was a mathematician of noble gifts and a great teacher. "His mind was quick, clear, accurate, and discriminating in its apprehensions, rapid and certain in its reasoning processes, and far-reaching and profound in its general views. It was admirably adapted both to acquire and use knowledge."† He was modest and unassuming in his manner, even to diffidence. He would never utter a harsh word to pupils or disparage their efforts. "His pleasant smile and kind voice, when he would say, 'Is that answer *perfectly* correct?' gave hope to many minds struggling with the difficulties of science, and have left the impression of affectionate recollection on many hearts."‡

Regarding his work at the University of Virginia, Professor Venable (at one time a pupil of Courtenay) says that his course in pure mathematics was prepared and written out (or rather printed on white cloth in large letters) with great care—following Bonnycastle in the use of Young in the treatment of the differential and integral calculus. His course in this branch embraced differential equations and the calculus of variations. His MSS. on these two subjects for the Senior class fill nearly one hundred and fifty pages of his printed work. His notes on the calculus were published in 1857, after his death, and became a valued text-book in many institutions. "In its publication the plan, language, and even the punctuation have been followed with a fidelity due to the memory of a friend." The work was more extensive than any which had yet appeared in this country on the same subject. Courtenay added descriptive geometry to the regular course of pure mathematics. He prepared extensive notes for his class in mixed mathematics, which embraced a full course in the applications of the calculus to mechanics and to the planetary and lunar theories (perturbations).

In 1845 the course in the School of Mathematics was as follows: *Junior class*, theory of arithmetic, algebra, synthetic geometry; *Intermediate class*, plane and spherical trigonometry, land surveying, navigation, descriptive geometry and its application to spherical projection, shadows, perspective; *Senior class*, analytical geometry, calculus. The class in mixed mathematics studied selections from Poisson, Francœur, Pontecoulant, and others. This embraced the mathematical investigations of general laws of equilibrium and motion, both of solids and fluids. The text-books for that year were, Lacroix's Arithmetic, Davies' Bourdon, Legendre's Geometry, Davies' Surveying and Descriptive Geometry, Davies' Analytical Geometry, Young's Differential and Integral Calculus.

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* Courtenay's Calculus, p. iv. † *Ibid.*, p. v. ‡ *Ibid.*, p. vii.

After the death of Courtenay the chair of mathematics was filled by Albert Taylor Bledsoe. He was a native of Kentucky, and graduated at West Point in 1830. He was one year adjunct professor of mathematics and French at Kenyon College, Ohio; then one year professor of mathematics at Miami University, Ohio. Afterward he practiced law for eight years at Springfield, Ill. Before his coming to the University of Virginia he was professor of mathematics and astronomy at the University of Mississippi. He remained in his new position till 1863, then became assistant secretary of war in the Southern Confederacy. After the War he became principal of a female academy in Baltimore and editor of the Southern Review. He died in 1877 at Alexandria, Va.

Prof. Francis H. Smith, of the University of Virginia, who was associated with Bledsoe in the faculty of the institution, writes us about him as follows: "He succeeded here an eminent teacher, Prof. Edward H. Courtenay; and, while the two men were most unlike in every respect, Dr. Bledsoe's evident ability so impressed his class, that the prestige of the mathematical class suffered no loss in his hands. From his life-long addiction to metaphysical studies, he entered with great zeal upon the philosophy of mathematics, a subject which every infantile mathematician is bound to have an attack of, but which in its widest relations may very well tax the powers of the most mature and advanced geometer. In this field I think Dr. Bledsoe won a place by the side of Bishop Berkeley and Auguste Comte. His treatise on the Philosophy of Mathematics was put in print and had a considerable circulation. He established a new course of lectures here in connection with the usual mathematical curriculum, upon the History and Philosophy of Mathematics. That feature survives to this day. As a manipulator of mathematical formulæ and solver of mathematical problems, Dr. Bledsoe was not strikingly able. I have known many men of far less strength who were his superiors in mere algebraic dexterity. Yet, I was convinced from several incidents which came to my knowledge during his teaching here that had his life, after he left West Point, been devoted to the science, he would have left the pure mathematics simplified in statement and improved in form. His originality and force were obvious to me, to whom he freely communicated his difficulties and successes, during his entire residence here. I learned that while at the Military Academy these traits were strikingly exhibited by his solving a problem in the tangencies of circles which had up to that time baffled the geometrical skill of the academy, and which had been left unsolved by Archimedes himself. The solution given by Dr. Bledsoe was afterward published in the Southern Review, of which the doctor was editor and proprietor for a number of years before his death. He had in the latter years of his life completed a treatise on synthetical geometry, of the Euclidian type, and, I think, had found a publisher, but whether it ever got printed I am not aware. Dr. Bledsoe's greatest work was in the field of metaphysical theology, constitutional law, and review articles."

His Philosophy of Mathematics, published in 1867, exhibits brilliant controversial powers. It initiated a reactionary movement among us against the unphilosophical exposition of the calculus in the colleges of our land. The book is somewhat verbose in its style. The bulk of it consists of criticisms of various text-books. Comparatively little space is given to what the author considers to be the true explanation of the subject. It seems to us that the criticisms which he makes are generally good and well founded, but that he fails in proposing a sound substitute for the explanations which he rejects. The influence of the book has been beneficial in so far as it has caused many teachers to meditate upon the philosophy of the calculus.

He gave lectures also on the history of mathematics—a subject which received little or no attention in our colleges at that time. He prepared, but never published, a work on analytical geometry, in which, by the discussion of one equation which contained, wrapped up within itself, the whole folio of Apollonius on conic sections, he developed the properties of the circle, ellipse, hyperbola, and parabola.*

Bledsoe pursued, in the main, the course in pure mathematics laid down by his predecessor, except that Courtenay's Calculus was used in place of Young's. For the class in mixed mathematics he used (in 1854) Bartlett's Analytical Mechanics, Newton's Principia, and Pratt's Mechanical Philosophy. Pontecoulant's *Système du Monde* was also used by him for his class.

Professor Bledsoe was not very strict with students in their daily work, but on approach of examination day he knew how to prepare a tough set of questions.

By temporary appointment, Alexander L. Nelson taught mathematics during part of the session 1853–54; Robert T. Massie during part of the session 1861–62, and Francis H. Smith, of the School of Natural Philosophy, from 1863 to 1865.

During the War the university barely subsisted; but scarcely was peace restored ere the institution, amidst perplexing pecuniary embarrassments, prepared with resolute energy to enlarge its capacity for useful work by multiplying its schools. In 1867 the School of Applied Mathematics with reference to Engineering was established.

In 1865 Charles S. Venable was appointed to the chair of mathematics, a position which he still occupies. He is a native of Virginia, and was born in 1827. After graduating at Hampden-Sidney College in 1842, he remained one year at the college as a resident graduate, pursuing mathematics under Col. B. L. Ewell (a West Point graduate, and afterward president of William and Mary College), and English literature and history under Maxwell. He then became tutor in mathematics, in which capacity he continued two years, devoting part of his time to the study of law. In 1845, he went to the University of Virginia and spent one session in the study of law, mathematics, and lan-

* Bledsoe's Philosophy of Mathematics, p. 130.

guages. Here he took the mathematical lectures of Professor Courtenay. He was then elected professor of mathematics at Hampden-Sidney College, to succeed Ewell. After remaining there one year he obtained leave of absence, returned to the University of Virginia, and studied mixed mathematics and engineering under Professor Courtenay. He returned to Hampden-Sidney in 1848, and filled the chair of mathematics till June, 1852. He then obtained leave of absence again, and visited Germany for the further prosecution of studies. In Berlin he studied astronomy under Encke, and mathematics with Dirichlet and Borchardt. He then went to Bonn, studying some months under Professor Argelander, the director of the observatory of Bonn. While in Germany astronomy was his chief branch of study. He then travelled in Southern Europe, studied for some time in Paris, visited England, and then returned to Hampden-Sidney College, in 1853. In 1856 he was elected to the chair of natural philosophy and chemistry at the University of Georgia, to succeed John Le Conte, and in 1857, professor of mathematics and astronomy in the South Carolina College. In 1858 he published an edition of Bourdon's Arithmetic. Venable took part in the attack upon Fort Sumter, and took active part in the War until its close.* Since his connection with the University of Virginia, Professor Venable has issued a series of text-books, consisting of First Lessons in Numbers, 1866, revised in 1870; Mental Arithmetic, 1866; Practical Arithmetic, 1867, revised in 1871; Intermediate Arithmetic, 1872; Elements of Algebra, 1869; Elements of Geometry, 1875; Notes on (analytical) Solid Geometry.

These rank among the best and most rigorously scientific school-books published in this country. In his arithmetics, the attempt is made "to render the reasoning of such arithmetics as those of Bourdon, Briot, DeMorgan, and Wrigley, easily accessible to the young." His Elements of Geometry is "after Legendre," but it differs from the original in the discussion of parallels, in the use of the methods of limits instead of the method of the *reductio ad absurdum*, in the fuller treatment of certain parts of the subject, and in giving, at the beginning, a chapter on the Theory of Proportion (in which the theory of limits is used for incommensurables) instead of presupposing a knowledge of proportion, as is done by Legendre. One feature is carried out in this geometry more extensively than in any other of our books, namely, the insertion of "hints to solutions of exercises." A teacher who does not make his pupils solve original problems in geometry, is a failure. But the exercises given in most books are not sufficiently graded, and the young beginner is very apt to get discouraged. The "hints" given in this book serve the excellent purpose of assisting and encouraging the pupil in his first attempts at original work. In 1887 Professor Venable published an Introduction to Modern Geometry, which serves as an

*Our sketch of the early career of Professor Venable is taken from La Borde's History of South Carolina College, 1874, p. 474.

appendix to his geometry. The treatment of the subject is metrical rather than descriptive.

The method of instruction under Professor Venable has been essentially the same as that followed by his predecessors. It consists of lectures, prelections on approved text-books, and exercises for testing and developing the power of the student in original solutions. Great stress is constantly laid on the solution by the student of original exercises. In this respect, each meeting of the class is a seminarium. In delivering their lectures, some professors of the university write condensed notes on the blackboard, others give syllabuses. The students very soon get up printed or lithographed notes on the lectures. The practice of reading the lectures does not prevail at the university.

One might suppose that in an institution where students have the privilege of attending whatever school they please, the enrollment in the school of mathematics would be comparatively small. This has, however, not been the case here. The attendance on this school is, as a rule, greater than on any other school of the academic department. In three or four sessions, since the War, the number of students in the school of Latin has been greater, but by not more than half a dozen students. The full attendance is in itself good evidence of the careful teaching and efficient work in the mathematical department. In order to present a fuller picture of the services of Professor Venable, we quote from a letter of R. H. Jesse, professor of Latin at the Tulane University of Louisiana, and a former student of the University of Virginia. "In my day Colonel Venable was absolutely the most popular among the students of all the professors in the University of Virginia. At the same time his control was perfect over all his classes, and indeed over any and all bodies of students with whom he came in contact. Doubtless his experience as an officer of rank in the Confederate service, his long practice in teaching, and his never failing kindness of heart and sympathy with young men, produced both the popularity and the power of control.

"Ever since I have known the institution well, now nearly twenty years, he has been, more than any other man, active and able in promoting her best interests. To him in large degree was due the increase by the State, in 1875 or 1876, of her annual contribution from fifteen thousand dollars to thirty thousand dollars. This increase was accompanied with the condition that all Virginia students able to pass the entrance examinations to the academical schools should be educated in those schools free of charge. To him *chiefly* was due the raising of the endowment fund whereby the McCormick telescope was gained for the university. To him *chiefly* has been due the large increase in attendance upon the university in late years. Twice he has been Chairman [of the faculty] and twice has he laid the office down voluntarily, when the university, guided safely by his wisdom and energy through some serious difficulties, had reached excellent condition again. He has had,

to my certain knowledge, many flattering calls to other fields, far more profitable in money, but he has immediately declined them all to stand fast by his *alma mater*."

The high and rigid standard inaugurated by Bonnycastle and Courtenay has been rigorously adhered to. The standard of graduation has always been high, in fact, very high in comparison with the standards in most other American colleges. The mathematical course has been broadened, as the preparation of students under the influence of the university upon the academies and colleges has become broader and better. "We have many excellent preparatory schools in Virginia," says Professor Venable, "which prepare students well, far into the differential and integral calculus in such works as Todhunter's and Courtenay's Calculus."

The course in mathematics, as stated in the catalogue for 1887–88, is as follows:

I. PURE MATHEMATICS.

JUNIOR CLASS.—This class meets three times a week (4½ hours) and studies theory of arithmetical notations and operations; algebra, through the binomial theorem; geometry, plane and solid; geometrical analysis, with numerous exercises for original solution; elementary plane trigonometry, embracing the solution of triangles, with the use of logarithms, and some applications to problems of "heights and distances." The preparation desirable for it is a good knowledge of arithmetic, of algebraic operations through equations of the second degree, and of the first three books of plane geometry.

Text-books.—Todhunter's Algebra; Venable's Legendre's Geometry, with collection of exercises; Todhunter's Trigonometry for Beginners.

INTERMEDIATE CLASS.—This class meets twice a week (3 hours) and studies geometrical analysis, with exercises for original solution; plane trigonometry, with applications; analytical geometry of two dimensions; spherical trigonometry, with applications; elements of the theory of equations. The preparation desirable for this class is a thorough knowledge of algebra through the binomial theorem, and logarithms; of synthetic geometry, plane and solid, with some training in the solution of geometrical problems; and a knowledge of the elements of plane trigonometry, including the use of logarithmic tables.

Text-books.—Snowball's Trigonometry, Puckle's Conic Sections, the Professor's Collection of Exercises in Plane Geometry.

SENIOR CLASS.—This class meets three times a week (4½ hours) and studies analytical geometry of three dimensions, through the discussion of the conicoids and some curves in space; differential and integral calculus, with various applications; a short course in the calculus of variations; the theory of equations, and lectures on the history of mathematics.

Text-books.—The Professor's Notes on Solid Geometry (Analytical); Todhunter's Differential Calculus; Williamson's Integral Calculus*; Todhunter's Theory of Equations.

Candidates for graduation in pure mathematics are required to pursue in the university the studies of both the Intermediate and Senior Classes.

II. MIXED MATHEMATICS.

This course is designed for those students who may desire to prosecute their studies beyond the limits of pure mathematics. It embraces an extended course of

*In former years Professor Venable used Courtenay's Integral Calculus, which was supplemented with notes which "nearly equalled the text." (Prof. R. H. Jesse.)

reading under the instruction and guidance of the professor on the applications of the differential and integral calculus to mechanics, physical astronomy, and selected portions of physics. The class in mixed mathematics meets twice a week (3 hours).

Text-books.—Price's Infinitesimal Calculus, Vols. II and III; Cheyne's Planetary Theory.

Mathematical physics and spherical astronomy are taught in the school of natural philosophy, in charge of Prof. F. H. Smith. Norton's Astronomy is one of the text-books. In this school, under practical physics, are studied also the method of least squares.

In addition to the under-graduate course in mathematics there is now a more extended course, occupying a large part of two sessions of nine months. It is given to graduates who are candidates for the degree of doctor of philosophy in the mathematical sciences. This course includes, in addition to the course in mixed mathematics, the study of modern higher algebra, modern higher geometry (Steiner's or some like work), a fuller study of the differential and integral calculus (Price and Hoüel), determinants (taught at the university for the last fifteen years), a fuller course in differential equations, probabilities, and other selections. If the candidate chooses astronomy for his secondary branch, then he studies Gauss's *Theoria Motus*, and enters into the practical computation of orbits. Should he choose physics, then he studies some of the advanced treatises in the line of mathematical physics.

In order to give a better idea of the course leading to the degree of doctor of philosophy, we quote from a letter of Dr. S. M. Barton:

"This doctorate course consisted of graduate studies in pure and mixed mathematics and mathematical and practical astronomy, and the text-books read, and on which I was examined, were as follows: Hoüel's *Calcul Infinitésimal*, four volumes; Chasles's *Traité de Géométrie Supérieure*; Price's Infinitesimal Calculus, Vol. III (Statics and Dynamics of Material Particles); Cheyne's Planetary Theory; Aldis's Rigid Dynamics; Notes and Examples selected by the Professor.

"The above were required in the mathematical department. In astronomy the text-books and requirements were: Gauss's *Theoria Motus;* Notes on the Computation of Orbits, by Prof. Ormond Stone; Notes on Least Squares, Perturbations, Variations of Constants, etc., by Professor Stone; Computation of the Orbit of Barbara (No. 234). This last was of course a work of several months.

"I was allowed to select my own subject for a thesis, which was accepted by the faculty and printed before I stood my last examinations.

"In the preparation for this thesis I was obliged to read, outside of the studies laid down in the course, the method of equipollences, and the principles of quaternions, and various articles bearing on the subject, in which I made use of the following works: *Exposition de la Méthode des Équipollences*, by Bellavitis, translated into French by Laisànt. *La Vraie Théorie des Quantités Négatives*, etc., by Mourey. Articles in the *Nouvelles Annales de Mathématiques*. Kelland and Tait's Introduction to Quaternions. Tait's Quaternions.

"In pursuing these doctorate studies I, of course, made use of many books for reference, among which I might mention Salmon's Conic Sections and Higher Plane Curves, and Geometry of Three Dimensions. Gregory's Examples. Vols. I and II of Price's Calculus. Some older works by Peacock and others, as well as some more elementary treatises. * * *

"I can not refrain * * * from alluding to one striking feature of the mathematical teaching at the University of Virginia, namely, independence in the student; and by independence I mean the *spirit* of *self-reliance* which enables the student to work out and elucidate for himself.

"The student is taught from the start to depend upon himself.

"This spirit of self-reliance pervades the mathematical department, and it promotes originality, as well as gives zest to the work.

"This would seem to be the only true way to teach mathematics, but many of our elementary teachers do little or nothing to inculcate this great principle."

The thesis referred to above is entitled "Bellavitis's Method of Equipollences" (1885). It contains an outline of the calculus of equipollences and of its relation to quaternions. It shows that while equipollences are more readily mastered, and yield on the whole more expeditious solutions of plane problems than quaternions, the latter are immeasurably superior in elegance, logical simplicity, and extent of application.

Since Professor Venable has been connected with the University of Virginia, the department of mathematics has graduated many students who have become prominent as teachers and scientists in their specialty. Chief among these are Prof. C. E. Vawter, professor of mathematics in Emory and Henry College for some years, now in charge of the Miller Manual Training School; Prof. G. Lanza, professor of mathematics at the Massachusetts Institute of Technology; Prof. W. M. Thornton, of the school of applied mathematics, University of Virginia; Professor Graves, professor of mathematics at the University of North Carolina; Professor Gore, professor of physics and astronomy at the University of North Carolina; Professor Bohannan, professor of mathematics at the University of Ohio (Columbus); Prof. H. A. Strode, principal of Kenmore University High School, Virginia; Prof. W. H. Echols, professor of engineering and president of the school of mines at the University of Missouri; Prof. W. H. Richancer, professor of mathematics at the school of mines, University of Missouri; Prof. T. U. Taylor, assistant professor of mathematics, University of Texas.

Applied mathematics, *i. e.*, mathematics applied to civil engineering, was taught in the school of mathematics almost at the beginning of the university. In 1832 a class in engineering was organized as a separate department under the professor of mathematics, and was maintained as an attachment to the school of mathematics until 1850. It was then left out of the catalogue from the fact, no doubt, that the successful

working of such a course imposed too heavy a burden upon the mathematical professor. In 1865 the department of civil engineering was revived and placed under the joint charge of the professors of mathematics, physics and chemistry. In 1867 Prof. Leopold J. Boeck was made assistant professor and placed in charge of the school of applied mathematics, comprising courses in civil and mining engineering. These led to the degrees of civil and mining engineer, respectively. In 1868 Professor Boeck was promoted to the full professorship of applied mathematics. He held the chair until 1875, when he resigned, and was succeeded by Wm. M. Thornton, as assistant professor. Professor Thornton was subsequently promoted to the full professorship of applied mathematics. This school has sent out a large number of engineers of sound training.

Mention should be made here of the school of practical astronomy, under the direction of Prof. Ormond Stone. He is also director of the McCormick Observatory, and editor of the Annals of Mathematics.

UNIVERSITY OF NORTH CAROLINA.*

Professor Mitchell's successor in the chair of mathematics was James Phillips, from 1826 to 1867. Professor Love speaks of him as follows: "He was born in England in 1792. It is not known at what school he received his early education. The greater portion of his mathematical education was gotten by private study. He came to America in 1818 and opened an academy in Harlem, N. Y. Here he won reputation as an instructor, and by contributions to the mathematical publications of the day. In 1826 he came to North Carolina as professor of mathematics and natural philosophy.

"He was a patient student of the masters in mathematics, of Ferguson, Newton, Delambre, Laplace, and others. He prepared a text-book on conic sections which was published and used as an introduction to analytic geometry. He left in manuscript the greater portion of a series of text-books on mathematics, including the calculus. These were most carefully prepared, but for some reason he never published any of them. Probably the War was the cause of his not publishing. He left directions when he died that all his MSS. should be burned. Among them were also many translations from French mathematical works.

"That Dr. Phillips never published more is very much to be regretted. He had great mathematical ability, and was an extremely careful and lucid writer. Like Dr. Mitchell, he divided his time and energy. Both of them were ministers and spent much time in the preparation of sermons. Dr. Phillips left hundreds of manuscript sermons; and these he directed to be burned with all his other MSS. He died suddenly of

* For all the information here given on the University of North Carolina, the writer is indebted to Prof. James L. Love, associate professor of mathematics at the university.

apoplexy in the college chapel, where he had gone to conduct morning prayers, on the 14th of March, 1867."

The requirements for admission were raised in 1835 so as to include all of arithmetic. It seems that in the same year a little of algebra—"Young's Algebra to simple equations"—was also required. The increase in the requisites for entering college were brought on at this time with excessive haste, and we are not surprised that, after three years' trial, algebra was withdrawn. It was not again required until 1855, when candidates were examined on "algebra through equations of the first degree." No alterations were made till 1868.

As regards the courses of study, Professor Love says: "In 1835 arithmetic was dropped, algebra was completed in the Freshman year, and conic sections and analytic geometry begun in the Sophomore year. In 1839 mechanics was introduced into the Sophomore and Junior years, civil engineering into the Senior year, and since that date analytic geometry has been completed in the Sophomore year. Calculus was begun in the Sophomore year in 1841, and from that date to 1868 it was sometimes in the Sophomore year and sometimes in the Junior year. For fifty years, from 1818 to 1868, first fluxions and then differential and integral calculus were required of all graduates. A three-years' course in engineering was introduced in 1854. It included in addition to the regular course required for graduation, descriptive geometry, drawing, shades and shadows, mechanics, civil engineering, and geodesy. This course was continued until 1862.

"An attempt was made in 1855 to offer some election of courses in the Sophomore and Junior years. Two courses were offered, the one analytical, the other geometrical. The latter embraced geometry, plane and spherical trigonometry, mensuration, surveying, navigation, natural philosophy, and astronomy. The analytical course included, in addition, analytical geometry, differential and integral calculus, statics and dynamics, acoustics and optics. During the Freshman year the two courses were identical, but for the Sophomore and Junior years different text-books were used, even for the same subjects, in the two courses. After two years' trial, these double courses were given up. From 1857 to 1868 the one mathematical course was as follows: *Freshman year*, algebra, geometry; *Sophomore year*, plane and spherical trigonometry with applications, analytical geometry, differential and integral calculus; *Junior year*, natural philosophy and astronomy."

Our list of books used by Professor Phillips is quite complete; Ryan's Algebra was used in 1827; Young's Algebra was introduced in 1836; Peirce's was studied from 1844 to 1868. In geometry, Legendre was used for a time. About 1843 Peirce's Geometry was introduced, and not dropped till 1868, except for the years 1855 to 1857, when Perkins and Loomis were used each one year. From 1857 to 1868, Munroe's "Geometry and Science of Form" was used in the Freshman class as an introduction to geometry. The idea of premising a course in demon-

strative geometry by a short one in empirical geometry is very commendable. In descriptive geometry, Davies' was introduced in 1854; also his Shades and Shadows. In 1844 Peirce's trigonometry was introduced; Perkins's was used from 1855 to 1856; Charles Phillips's from 1856 to 1860; Loomis's from 1860 to 1868. In conic sections James Phillips's was taught from 1830 to 1847, when Peirce's book was introduced. From 1851 to 1868 Loomis's was studied, except from 1853 to 1855, when Church's and Smith's Biot's were used, each a year.

In calculus the notation of Leibnitz was introduced in 1830. Hutton's work was supplanted in 1847 by Peirce's Curves, Functions, and Forces, which was followed in 1851 by Loomis's. This was used until 1868, except in 1853, when Church's was taught for one year. In astronomy, Peirce's book was introduced in 1847, Herschel's in 1855, and Norton's in 1857.

Before the Civil War the university was prosperous and popular. The courses in mathematics described above were certainly very creditable for their day. Hon. Wm. H. Battle spoke of the university as follows: "In the extent and variety of its studies, the number and ability of its instructors, and the number of its students, it surpassed nearly all similar institutions in our own section of the country, and was beginning to rival the old, time-honored establishments of Yale and Harvard. In the year 1858 its catalogue showed a larger number of under-graduates than that of any other college in the United States, except Yale. All this success was accomplished in a very short time. A glance at the rapidly increasing ratio of its graduates will illustrate the truth of my remark. For the first ten years after the date in which degrees were conferred by the university, the number of students who received the baccalaureate was 53; for the second decade it was 110; for the third, 259; for the fourth, 146; for the fifth, 308; for the sixth, 448; and for the seventh the annual number was going on at a rate which would have produced 882, nearly the double of that which immediately preceded it."*

During the Civil War nearly all Southern colleges closed their doors, but not so the University of North Carolina. It was the boast of its president that "during the four years of war the college bell never failed in its daily calls, that the faculty was ever in place for duty, and 'that all grew *fat* on sorghum and corn bread;' that the institution was maintained in full *working order*." The severest blow to the prosperity of the university came after the War. In 1868 the old faculty was turned out by the "reconstructed" State government, and from 1869 to 1871 a new faculty labored to make the university popular again. But political feeling was too high; the university was closed from 1871 to 1875.

*Address delivered before the two literary societies of the University of North Carolina, June 1, 1865, by Hon. Wm. H. Battle.

Charles Phillips became professor of mathematics in 1875. He had been tutor from 1844 to 1853, associate professor from 1855 to 1860, and professor of engineering from 1853 to 1860. In 1879 he was made professor emeritus of mathematics, and Ralph H. Graves, jr., who had been professor of engineering since 1875, became now professor of mathematics. Professor Graves is a graduate of the University of Virginia and a former pupil of Professor Venable. Since 1885 James Lee Love has been associate professor of mathematics. He graduated at the university at the head of his class, and then took a graduate course in mathematics at the Johns Hopkins University in the year 1884–85.

Under the present able corps of instructors, mathematical teaching is again flourishing. Since the re-opening, in 1875, the requirements in mathematics for admission have been: arithmetic, and algebra to quadratic equations. The course in mathematics has been as follows: *Freshmen*, algebra, geometry; *Sophmores*, plane and spherical trigonometry, logarithms, plane analytical geometry; *Juniors*, theory of equations, differential and integral calculus, natural philosophy; *Seniors*, mechanics, astronomy. The studies of the first and second years have been required of all graduates. The studies of the third year, except natural philosophy, have been elective. Mechanics and astronomy were required in all courses leading to degrees until 1885. Since that time mechanics is elective in all courses, and astronomy elective in the A. B. course. Since 1885 post-graduate electives have been offered in solid analytic geometry (Smith's), determinants, differential equations, modern algebra, and quaternions. From 1875 to 1879 a three-year course in engineering was offered. Since 1879 the course has been partially withdrawn; and at present (1888) it includes only a one-year course in surveying, descriptive geometry, and projective drawing.

Robinson's University Algebra was used from 1869 to 1871, and since 1875 Schuyler's, Venable's, Newcomb's, and Well's—Newcomb's most. In geometry the books have been, since 1875, those of Venable, Wentworth, Newcomb, and J. W. Wilson. In descriptive geometry and projective drawing Warren's is taught. Davies' Trigonometry was used from 1869 to 1871, Wheeler's since 1875, and Newcomb's since 1882. In calculus the works of Peck, Courtenay, Bowser, Byerly, and Todhunter have been in use. Since 1883 Williamson has been the textbook. Newcomb and Deschanel are the books in astronomy and physics.

In 1883 the Elisha Mitchell Scientific Society was organized. The professors of mathematics take part in its exercises. Meetings are held once each month for the presentation of papers on any scientific subject. The society publishes a Journal, with abstracts of the more important papers read, and the writer has before him Vol. V, Part I, in which appear two papers by Professor Graves on geometrical subjects. These have been published also in the Annals of Mathematics, to which Professor Graves is a frequent contributor.

UNIVERSITY OF SOUTH CAROLINA.*

The successor of Rev. Dr. Hanckel in the chair of mathematics was James Wallace. He entered upon his duties in 1820, and remained at the college for fourteen years. Some years previous to his coming to this institution he had been professor at Georgetown College, in West Washington. He possessed mathematical ability and fine attainments in his specialty. While at Columbia, S. C., he contributed to the Southern Review articles on "Geometry and Calculus," Vol. I; "Steam Engine and Railroad," Vol. VII; "Canal Navigation," Vol. VIII. In the first of the above articles a somewhat severe criticism of Hassler's Trigonometry is given. Wallace upholds the geometrical method and the line system. He contributed also to Silliman's Journal, in one number, giving an account of a new algebraic series of Stainville in Gergonne's Annals, but, by mistake, it was not duly accredited, and appeared like Wallace's work. This drew him into a controversy with Nathaniel Bowditch.

Wallace's ability is shown by his treatise on the Use of the Globes and Practical Astronomy (New York, 1812). This work was in advance of any other American treatise on astronomy of its day. The work had 512 pages, was printed closely, with lengthy notes in small type. Some parts required little or no knowledge of mathematics on the part of the reader; others assumed a knowledge of geometry, trigonometry, conic sections, and algebra, and the last part also of fluxions. The title page bears the motto, "*Quid munus Reipublicæ majus aut melius afferre possimus, quam si Inventutem bene erudiamus?*—Cicero."

M. La Borde says in his History that Wallace did not place very high value upon the above work. "He said the MS. of a work to which he had devoted twenty years of his life was destroyed by fire, and he thought that but for that accident he would have left something worthy of remembrance."

As a teacher Wallace was in some respects the opposite of Blackburn. The latter was somewhat hot-tempered, but Wallace was a patient and laborious teacher, who loved his art. "No obtuseness of perception, no degree of stolidity could provoke him to ill-temper." Upon leaving the college he retired to a small farm near Columbia, where he died in 1851.

After the departure of Wallace, Lewis R. Gibbs held a temporary appointment for one year, or part of one. In 1835 Thomas S. Twiss was appointed. He occupied the chair for eleven years. He was born in Troy, N. Y., graduated at West Point, and, before his election, was teaching a classical school at Augusta, Ga. He was remarkable for industry, punctuality, and "watching and waiting" to catch students in mischief. He enjoyed the reputation of arraigning more offenders than

* The material for this sketch was kindly furnished us by Prof. E. W. Davis, professor of mathematics and astronomy at the university.

any other two members of the faculty. Upon leaving the college he became president of some iron works in the Spartanburg district. From here he returned to his old home in New York.

The next mathematical professor, Matthew J. Williams, was likewise a West Point graduate (class of 1821). He was a native of Georgia, and had an early bent for arithmetic. At the Military Academy he was one of four to attain a maximum mark in mathematics. In 1825 he was stationed at Old Point Comfort, Va., then at Fort Howard, Wis. He resigned from the Army in 1828, and studied law in St. Louis. He practiced law in Georgia until 1835, when he received an appointment to the South Carolina Conference of the Methodist Episcopal Church, at Cokesbury, Abbeville County. Thence he was called to the South Carolina College. He resigned in 1853 on account of severe disease. His health failed during his last year at West Point, and he seems to have had a constant struggle with sickness from that time on. As a teacher, he was "zealous, industrious, and thorough." His enthusiasm knew no bounds. He was esteemed as a scholar, a man, and a Christian. When his health began to decline and there was fear he would have to give up his work, the president of the college wrote in his report to the trustees: "I can not express to you how much I value his services in the department which he fills, and I should regard it as a most deplorable calamity to the college to be deprived of his labors."

His successor, Charles F. McCay, a Pennsylvanian, was, at the time of his election, a professor at the University of Georgia, and a colleague of John and Joseph Le Conte. He was elected president of the South Carolina College in 1855. In an attempt to act as a "go-between" in a disagreement between faculty and students, he incurred the displeasure of both parties. After his resignation he went into business, and is now actuary of an insurance company in Baltimore. From what we can learn, he was a man of ability and a good teacher.

From 1857 to 1862 Charles S. Venable filled the chair of mathematics. Since the War he has been for nearly a quarter of a century professor at the University of Virginia, and has established for himself a lasting reputation as a teacher of mathematics. While professor at Columbia, he was, as yet, a young man, and was not so popular as a teacher.

We proceed to give the courses of study for the period preceding the War. In 1836 the terms for admission were, "arithmetic, including fractions and the extraction of roots." In 1848 was added, "algebra to equations of the first degree." In 1851 Davies' Bourdon was the algebra used. In 1853 the whole of Bourdon's Algebra was required for entrance. This requisition appears to have been excessive, and in 1859 it was reduced to "Bourdon's Algebra to Chapter IX" (thus omitting the general theory of equations and Sturm's theorem), or "Loomis's Algebra to Section XVII" (omitting permutations, combinations, series, logarithms, and general theory of equations). The catalogues, from 1857 to 1862, contain this: "A thorough knowledge of arithmetic being essential

to success in all classes of the college, applicants must be prepared for a full and searching examination in this study."

In 1836 the course of study was as follows:

"*Freshman year:* Bourdon's Algebra to equations of the third degree, ratios and proportions, summation of infinite series, nature and construction of logarithms, Legendre's plane geometry. *Sophomore year:* Legendre's solid geometry, constructions of determinate geometrical equations, Davies' mensuration and surveying, including methods of plotting and calculating surveys, measurement of heights and distances, and use of instruments in surveying. *Junior year:* Descriptive geometry and conic sections, principles of perspective, analytic geometry, fluxions—direct and inverse methods in their application to maxima, minima, quadrature, cubature, etc. *Senior year:* Natural philosophy and astronomy.

"There shall be daily recitations of each class, one after morning prayers, one at 11 A. M., one at 4 P. M. On Saturday morning there shall be one recitation."

In the introduction of descriptive geometry into the course, we notice West Point influences. The "fluxions" above mentioned must mean "differential and integral calculus." Mr. Twiss, the professor at this time, was a graduate of the Military Academy at West Point, and was not likely to teach fluxions and the Newtonian notation.

In 1838 the Freshmen finished the whole, both of algebra and geometry; the Sophomores had plane and spherical trigonometry in place of solid geometry.

In 1841 Davies' Calculus was studied in the Junior year. Three years later, the Sophomores were taught from Davies' works on Mensuration and Surveying, Analytical Geometry, and Descriptive Geometry. In 1848 (M. J. Williams, professor) Loomis's Conic Sections were studied. Descriptive geometry and calculus were taught by lectures. After completing the calculus, in the Junior year, Olmsted's Mechanical Philosophy was taken up. The Seniors had courses in astronomy and (Mahan's) civil engineering. Owing to a rise in the terms for admission Bourdon's algebra was omitted in the first year, the studies for the other classes remaining the same. In 1854 descriptive geometry was thrown out of the course. Professor McCay was not a West Point graduate, and attached, probably, less importance to this branch. In 1857 spherical geometry was transferred from the second to the third year.

In 1858 the Freshmen studied geometry (Legendre), reviewed algebra (applications of algebra to geometry); the Sophomores, mensuration, surveying and leveling, conic sections (Loomis), mechanics (gravity, laws of motion); the Juniors had lectures on calculus, spherical trigonometry, mechanical philosophy (Olmsted); the Seniors, astronomy, civil engineering, natural philosophy (Olmsted).

In 1860 Professor Venable introduced at the end of the first year theoretical arithmetic, using his own edition of Bourdon. He used also

Loomis's Geometry in place of Legendre. In 1861 Loomis's Geometry is mentioned, "with original problems." Algebra was reviewed and applied to "geometrical problems." We judge that extra efforts were made by Professor Venable to improve on the traditional methods of teaching, by requiring the student to do a great deal of original work in the line of solving problems.

In 1863 the buildings of the college "were taken possession of by the Confederate Government, and used as a hospital until the close of the War." Its charter was amended by the Legislature in 1865, and in the following year it was re-opened as the University of South Carolina.

The mathematical chair was given to E. P. Alexander, a graduate of West Point and a man of great ability. During the War he was a Confederate brigadier, distinguished himself at Gettysburg, and introduced "signalling" into the Confederate army. As a teacher he was much liked. He was very practical and to the point in his methods and illustrations. Since leaving the college, he has been connected with railroads, either as president or otherwise.

Prof. T. E. Hart, a graduate of Heidelberg, taught mathematics from 1870 to 1872. He was then and is now in very poor health, suffering from paralysis. While he was professor his classes had often to go to his house for recitation.

From 1873 to 1876 A. W. Cummings held the mathematical chair. At this time the college passed through the darkest period of its history. These were the unfortunate years of "reconstruction." In addition to the numerous obstacles which American colleges generally have had to encounter, the colleges in the South have had to contend with great political upheavals. Like the University of North Carolina, the University of South Carolina closed its doors. From 1876 to 1880 the institution was without faculty and without students.

When the institution opened, in 1866, its course of study was remodeled. In this reorganization the plan of the University of Virginia was followed. In the prospectus we read that "the university consists of eight schools;" that students are allowed to choose the departments which they wish to pursue, provided they enter at least three schools. In certain cases, however, students will be allowed to enter less than three schools."

The prospectus continues, as follows: "During the present year there will be no examinations or other requirements for admission, except that the applicant must be at least fifteen years of age; but in order to ensure uniformity of preparation in certain departments, a preparatory course has been prescribed, and after *this year* applicants (under eighteen years of age) will be required to bring a satisfactory certificate of proficiency, or to stand an examination. For applicants over eighteen years of age, no examination or certificate will be required during the next year."

"In all the different schools the method of instruction is by means of lectures and the study of text-books, accompanied in either case by rigid daily examinations."

In the "school of mathematics, and civil and military engineering and construction," the requirements for admission were: "Arithmetic in all its branches, including the extraction of square and cube roots;" "algebra, through equations of the second degree."

From 1867 to 1872 the terms were as above, together with "a knowledge of the first four books of geometry," which, "though not indispensable, is very desirable."

In 1872 the management of the university fell under the Reconstruction administration; negroes were admitted, and a four years' preparatory course was given. The catalogue of 1872–73 says:

"In arithmetic, attention should be paid to all the rules and calculations usually given in written arithmetic, and too much importance can not be paid to a thorough preliminary drill in mental arithmetic."

In the "college of literature, science, and the arts," the requirements are, in addition, for the *classical course*, "algebra, as far as equations of the second degree," and it is "recommended that they also master the first four books in Davies' Legendre, or the equivalent;" for the scientific course, "algebra, up to radical quantities."

In the catalogue for 1876, the requirements were "the whole of arithmetic," and "algebra as far as equations of the second degree."

The course of study in mathematics was, in 1866, algebra from equations of the second degree to general theory of equations and logarithms, geometry, plane and spherical trigonometry, surveying and the use of instruments, in the first year; in the second year, descriptive geometry, analytical geometry, calculus, mathematical drawing. Text-books: Loomis's books on algebra and geometry, Davies' Shades, Shadows, and Perspective, Church's Analytical Geometry and Calculus. In the "department of mechanical philosophy and astronomy," Prof. John Le Conte's Mechanics was taught, also Olmsted's Astronomy, with Herschel's Outlines and Norton's Astronomy for reference. In 1867 Loomis's Astronomy was used, as well as his series of mathematical text-books from his Algebra to his Calculus. In 1870 everything is the same as given above, except that mechanical philosophy and astronomy were temporarily taught by the professor of mathematics.

In 1872 Robinson's University Algebra and Loomis's Geometry were studied in the first year; Robinson's Trigonometry, Mensuration, Surveying, and Spherical Trigonometry in the second year; Robinson's Analytical Geometry and Conic Sections the third year. Later on Ficklin's Algebra was introduced.

"In 1879 the trustees of the university were empowered by act of the General Assembly to establish a College of Agriculture and Mechanics at Columbia, and to use the property and grounds of the college for this purpose. This was accordingly done in 1880."

"In 1881 the Legislature granted an annual appropriation for the support of the schools of the university, and in 1882 the South Carolina College was reorganized by the appointment of a full faculty. It went into active operation the fall of the same year."

From 1882 to 1888 Benjamin Sloan was the professor of mathematics. At present he is professor of physics and civil engineering. He is a South Carolinian, graduated at West Point in 1860, served in New Mexico before the War, and then entered into the Confederate service. The story goes that when he entered upon the duties of his chair at the college, he ordered a bookseller to get Courtenay's Calculus. "Calculus!" replied the bookseller, "what are you going to do with it?" "Teach it," was the reply. "You can't do that, no South Carolina boy ever studies calculus." Though this be merely the opinion of a jovial bookseller, it is, we fear, not without some truth when applied to the ten years preceding the reorganization and re-opening of the college in 1882. For four years it was under Reconstruction rule, and for six years its doors were closed to students.

Professor Sloan is a first-class teacher. He requires a great deal of original work of students, and inspires considerable enthusiasm. In his manner he is very quiet and easy. Among the students he is liked and popular.

In 1888 Dr. E. W. Davis was elected to the mathematical chair. He graduated at the University of Wisconsin in 1879, and after spending some time at the Washington Astronomical Observatory, went to the Johns Hopkins University, where for four years he studied mathematics under Professor Sylvester and his associates. As a subsidiary study Davis pursued physics under Professor Hastings. At this great university he soon caught the spirit and enthusiasm which is so contagious there. His mind was chiefly bent toward geometrical studies, and the papers from his pen, which are published in the Johns Hopkins University Circulars and the American Journal of Mathematics are evidences of his power as an original investigator. Before his appointment to his present position he was professor of mathematics for four years at the Florida Agricultural College in Lake City. In his teaching Professor Davis possesses great power in causing students to think. He is a bold advocate of greater freedom from formalism in mathematical instruction.

The terms for admission on the re-opening of the institution were, in mathematics, arithmetic, and algebra through equations of the first degree. Radicals were added in 1883. In 1884 the terms were, arithmetic, and algebra to equations of the second degree. No additions have been made since.

The mathematical course in 1882 consisted, in the first year, in the study of Newcomb's Algebra, Chauvenet's Geometry (six books); in the second year, in the study of Newcomb's Plane and Spherical Trigonometry, Puckle's Conic Sections; in the third year, in the further study

of conic sections (Puckle, Olney), and calculus (Olney, Todhunter). In applied mathematics courses were given in the second year on surveying (Gillespie) and drawing, Peck's Mechanics, Wood's Strength and Resistance of Materials, and Walton's Problems in Elementary Mechanics, astronomy (Loomis, Newcomb, and Holden), and Mahan's Civil Engineering. In 1884 Warren became the text-book in descriptive geometry. In 1885 Taylor's Calculus was introduced; in 1886 Watson's Descriptive Geometry and Merriman's Least Squares; in 1887 Newcomb's Analytic Geometry.

The mathematical text-books for 1888 are, in the first year, Todhunter's Algebra for Beginners, Byerly's Chauvenet's Geometry; in the second year, Blaklie's Plane and Spherical Trigonometry, Peirce's Tables; in the third year, Taylor's Calculus, Church's Descriptive Geometry; in the fourth year, Newcomb and Holden's Shorter Course in Astronomy.

This year (1888–89) a graduate department has been added. In mathematics it offers the following branches: Algebra (theory of equations, theory of determinants, etc.), geometry (projective geometry, higher plane curves, etc.), calculus (differential equations and finite differences), elliptic functions, astronomy, and quaternions.

UNIVERSITY OF ALABAMA.*

The University of Alabama was opened in 1831, with Gurdon Saltonstall in charge of the mathematical teaching. Two years later William W. Hudson became professor of mathematics, and held the position until 1837, when Frederick Augustus Porter Barnard became connected with the institution, and had charge of the mathematical department till 1849. The wonderful activity of this powerful man in the various departments of science gave a great stimulus to higher education in the State. He had previously been tutor at his *alma mater*, Yale. In 1849 he assumed the duties of the chair of chemistry at the University of Alabama. While connected with the institution as professor of mathematics and natural philosophy he wrote and published an arithmetic, which came for a time into pretty general use in Alabama. In 1846 he was appointed astronomer by the State, to settle a boundary dispute between Alabama and Florida. He was appointed astronomer for the State of Florida also, so that he represented both States in the settlement of the dispute. Professor Barnard was always fond of mathematics. He has written a number of valuable articles on mathematical subjects for Johnson's New Universal Cyclopædia.

By old students Professor Barnard is always spoken of in most laudable terms. Says Dr. B. Manly: "To me the study of physics, astronomy, etc., under Prof. F. A. P. Barnard, * * * and of chemistry and kindred sciences under Prof. R. H. Bramby, long deceased, were the

* Nearly all the material for this article was sent us by Prof. T. W. Palmer, professor of mathematics at the university.

most attractive parts of my college course." Mr. John A. Foster, now chancellor of the south-eastern chancery division of Alabama, was a student and then a tutor of mathematics at the university in the time that Barnard taught there. He says:

"I entered the Sophomore class of the University of Alabama at Tuscaloosa in the autumn of 1844, and received my diploma in August, 1847, in a class of eighteen. During my college course Prof. F. A. P. Barnard was the professor of mathematics and John G. Barr was the assistant professor of mathematics. Dr. Barnard afterward became the president of the University of Mississippi, and in 1861, being a Union man, resigned and went North, where he was for some time engaged in the scientific department of the Government, and afterward was president of Columbia College in the city of New York. A very short time ago I observed that he has retired from this work.

"Professor Barnard was not less distinguished as a scientist than as a mathematician. His reputation is world wide. I was a great friend of his, and up to 1858 I was a constant correspondent with him. I need hardly say that his instruction was thorough and far in advance of the methods which prevailed at that time. There has never been a better teacher of mathematics, and those now living still claim that the country is but now getting to the methods of teaching practiced by him more than forty years ago. Withal, he was a warm and generous friend, and was very popular with those who were his pupils. During the summer of 1844 or 1845 he went to Europe and spent some time in France, and on his return to the university he brought with him the newly discovered Daguerrean process, and took pictures experimentally before his class. He was hard of hearing and had a deep guttural voice, but no one had a happier faculty of making himself clearly understood. He married an English lady while I was his pupil.

"Capt. John G. Barr, the assistant professor, was worthy to occupy the position as second to this distinguished man. In 1847 he raised a company and went to the Mexican War, where he served with distinction until its close. Soon after he was appointed to a diplomatic position by the United States Government, and died at sea when on his way out to assume the duties of his official station. He was an able and successful teacher of mathematics."

Mr. Foster engaged in educational work till 1859 (being for some years president of a college in La Grange, Ga.), when he went to the practice of law.

The mathematical teaching at the university for the three years succeeding 1849 was in the hands of Prof. Landon Cabell Garland, now the honored chancellor and professor of natural philosophy and astronomy of Vanderbilt University. His successors as instructors of mathematics at the University of Alabama, before the War, were Profs. George Benagh (1852–60), Robert Kennon Hargrove (1855–57), James T. Murfee (1860–61), and William Jones Vaughn (1863–65).

Prof. R. K. Hargrove, after teaching mathematics for two years, joined the ministry of the M. E. Church South, and, a few years ago, was elected bishop by the general conference which met at Nashville, Tenn.

The terms for admission to the university were, 1833–56, arithmetic; 1857–59, arithmetic, and algebra through equations of the second degree; 1860–62, arithmetic, and algebra to equations of the first degree; the records for the next three years are lost.

Down to 1852 the professor of mathematics was at the same time professor of physics, according to the usual custom in American colleges at that day. In 1833 the Freshman class completed algebra (Colburn, Lacroix) and commenced geometry (Farrar's Legendre); the Sophomore class studied geometry, trigonometry, and conic sections. The Junior and Senior classes were taught mechanics, statics, heat, light, electricity, etc. The books used were the Cambridge Mathematics of Professor Farrar. This course continued without change until 1842, when surveying, mensuration, etc., were made an important part of the Sophomore work. In 1843 Davies' text-books were adopted. In 1845 Peirce's Algebra was introduced, but after two years it was displaced by Davies'. In 1849 the calculus was added to the Junior course. The text used was Church's until 1855, when Loomis's was adopted. From 1860 to 1865 the records are so incomplete that it is impossible to state whether or not any changes were made during that time.

Before the War, the university was prospering. "In the Junior and Senior classes," says Mr. Foster (class of 1847), "much attention was given to applied mathematics. Physics, astronomy, surveying, and navigation were taught. The university was but a college with a fine corps of professors, and presented advantages offered by very few other institutions of learning at that time."

The War naturally interfered with the successful working of the university. In 1865 the university buildings were destroyed by fire, and the institution was not opened again until 1869. The condition of the country at that time was not favorable for the advancement of education. In recent years, however, decided and encouraging progress has been made. A thrill of aspiration and enthusiasm has been running through Southern colleges.

The first year after the re-opening Prof. N. R. Chambliss taught the mathematics; the next year, Prof. J. D. F. Richards; and the year following, Prof. Hampton S. Whitfield, and the fourth year Prof. David L. Peck. In 1872–73 Prof. W. J. Vaughn held the mathematical chair; and from 1873 to 1878 Prof. H. S. Whitfield again. In 1878 Professor Vaughn assumed the duties of this chair for the third time, and discharged them for four years. Since 1882 Prof. Thomas Waverly Palmer has filled the chair and taught with marked success.

Vaughn is now professor of mathematics at Vanderbilt University. "Though he has never written text-books," says Professor Palmer, "still

he is justly regarded as one of the ablest mathematicians in our American colleges." Prof. J. K. Powers, president of the Alabama State Normal School, who studied at the university from 1871 to 1873, says that he had completed the course in pure mathematics before going there, and that he took applied mathematics there. "Prof. Wm. J. Vaughn at that time filled the chair of applied mathematics. He was (and is) an accomplished mathematician, an attractive instructor, a fine general scholar, and a charming gentleman. At that time the chair of pure mathematics was filled by Prof. D. L. Peck and Prof. H. S. Whitfield. I *knew* nothing of their methods, but *pure mathematics* was not popular in those days. In after years, when Prof. Vaughn assumed control of that work, no department of the university was more popular."

Of Professor Palmer, Chester Harding (class of '84, now a cadet at the U. S. Military Academy at West Point) says: "This gentleman, a graduate of the class of '81 of the university, had so satisfactorily filled the position of assistant professor during the preceding term, that his election was secured, as young as he was, against the claims of other applicants of extensive experience, reputation, and influence."

From 1869 to 1871 only the elements of arithmetic were required for admission. During the next two years, algebra to equations of the second degree was added. In 1873 the requirements were reduced to arithmetic alone. No change was made until 1878, when algebra through equations of the second degree was required. Gradual changes have been made every year since, and now the whole of algebra and three books of geometry are required.

The catalogue for 1887–88 states that the candidate for admission "must pass a satisfactory examination in arithmetic, in algebra through arithmetical and geometrical progression, and in the first two books of geometry. The examination in arithmetic will include the whole subject as embraced in such works as White's, Robinson's, Goff's, Greenleaf's, or Sandford's higher arithmetic. In algebra, particular stress will be placed upon the use of parentheses, factoring, highest common factor, lowest common multiple, simple and complex fractions, simple equations with one or more unknown quantities, involution, evolution, theory of exponents, radicals (including rationalization, imaginary quantities, properties of quadratic surds, square root of a binomial surd, and solution of equations containing radicals), quadratic equations, equations of the quadratic form, simultaneous quadratic equations, ratio and proportion, arithmetical and geometrical progression."

In 1874 the calculus was dropped from the university course, but was introduced again in 1878.

In 1881 there was a reorganization of the courses of study. Two courses of mathematics were arranged, one for classical and scientific students, and one for engineering students. The course for classical or scientific students embraced algebra, geometry, plane and spherical trigonometry, and analytic geometry. These subjects were completed

in the Sophomore class. Since 1881 no changes have been made in the classical and scientific courses.

The engineering course embraced all subjects that were taught in the classical and scientific, but to the Sophomore work was added descriptive geometry, and to the Junior class calculus. This course has been modified since. At present it consists of higher algebra and geometry for the Freshmen; plane and spherical trigonometry, analytical geometry, descriptive geometry, theory of equations, for the Sophomores; calculus, determinants, and quaternions for the Juniors.

Determinants and quaternions, which are regularly in the course since 1887, have been taught irregularly for several years. Quaternions are, according to catalogue, now taught in the third term of the Sophomore year, before the completion of analytic geometry. This is a somewhat new departure in the arrangement of mathematical studies, and one which is worthy of respectable and thoughtful consideration.

As to text-books, in 1871 Davies' Algebra and Geometry were used; also Church's Analytic Geometry and Calculus. In 1872 and 1873 the books were Robinson's Algebra and Geometry, and Loomis's Trigonometry and Analytic Geometry. In 1878 Peck's Analytic Geometry and Calculus were introduced. The books used at present are Well's Algebra, Wentworth's Geometry, Trigonometry, and Analytic Geometry, Bowser's Analytic Geometry, Taylor's Calculus, Church's Descriptive Geometry, Peck's Determinants, and Todhunter's Theory of Equations.

Cadet Chester Harding, who was a student at the University of Alabama from 1881 to 1884, gives the following reminiscences of the mathematical teaching there: "The training in mathematics was more extensive in scope and thoroughness in the engineering course than in the others, including in that course the elementary principles of descriptive geometry and calculus, while in the others the instruction ceased with the study of the conic sections and surfaces of the second order in analytical geometry.

"I chose the engineering course and began my instructions in the department of mathematics with trigonometry under Prof. W. J. Vaughn, who now fills the chair of mathematics at Vanderbilt University. Our text-book was Wheeler's Trigonometry. The trigonometric functions were taught as ratios, and stress was laid upon the circular system of measuring angles. * * *

"Analytical geometry came next, and our text-book was Professor Wood's Elements of Co-ordinate Geometry. Of the class of thirty in this branch, all were beginners but two or three who had been required to repeat the course because of their deficiency in the preceding year. Our progress was therefore slow at first, and much time was spent by the professor in explanations and illustrations. I see the first lessons still marked in the text-book I have before me now, and some were but two and a half of the quarto pages. These, however, were expected to

be thoroughly mastered, and many pains were taken to have the principles well absorbed by the students. Mere exercise of memory was little sought after in the mathematical department, and any originality on the part of a student in the deduction or application of a principle was highly commended.

"The course in analytical geometry closed with the end of the session, at which time a satisfactory written examination in the study was required of every member of the class. In the scientific and classical courses, mathematics terminated with the Sophomore year. In the Junior year the students of the engineering course, however, took up the study of calculus.

"At the end of my Sophomore year Professor Vaughn resigned his chair at the University of Alabama to accept a similar position at Vanderbilt. * * *

"In my Junior year the schedule of studies was so arranged that but three hours a week were devoted by my class to mathematics. This limited time permitted us to complete but one text-book, Prof. W. G. Peck's Elements of the Differential and Integral Calculus. From this text, however, we derived a knowledge of the practical utility of calculus, and became familiarized with the rules of differentiation and integration. I can hardly say that we acquired a more thorough knowledge than this; and indeed it seemed, from the time assigned to the study, to be without the purpose of the faculty that more than a groundwork should be acquired, for practical good in the understanding of the applications of calculus to mechanics and engineering. During my Junior year we also studied under Prof. R. A. Hardaway, in the department of engineering, the elements of descriptive geometry, using as a text-book Binn's Elements of Orthographic Projection.

"With the close of the Junior year the regular course in pure mathematics was ended."

As regards the conditions for graduation which have existed at various times, Professor Palmer says: "As a rule mathematics was required of every student for graduation, from 1831 to 1865. After the reorganization in 1869, mathematics was also required until 1875, when the elective system was adopted; it was entirely optional with the student then until 1880, when every student was required to take this subject through analytic geometry."

At present there are no electives, and all the mathematics in each course is required for a degree in that course.

UNIVERSITY OF MISSISSIPPI.

The educational record of Mississippi in the early period of her organized existence is quite honorable. Between 1798 and 1848 there had been established one hundred and ten institutions, under the various names of universities, colleges, academies, and schools. This proves that an entire obliviousness to the educational wants of the people did

not prevail. Our gratification is abated, however, by the consideration that these organizations proved inefficient, and that there was really but very little beneficial progress.

In 1848 was organized upon a firmer foundation the University of Mississippi. Considering the many difficulties that were encountered, the record of the university during its infant years before the War was honorable. Two names, both well known to the educational public, devoted their energies to promote its early growth—F. A. P. Barnard, now president of Columbia College, and A. T. Bledsoe, afterward professor at the University of Virginia and, still later, editor of the Southern Methodist Review.

From the beginning until 1854, Albert Taylor Bledsoe was professor of pure and applied mathematics, and astronomy. The mathematical requirements for admission were, at first, a knowledge of arithmetic. The catalogue of 1857–58 says: "Arithmetic—especially the subject of fractions, vulgar and decimal, proportion, and the extraction of roots;" the catalogue for 1859–60 adds to this, "algebra as far as simple equations." In the former catalogue we read also, "that, hereafter, no student will be admitted to any class in the university who shall fail to pass an entirely satisfactory examination on the subjects or authors required for admission to the class."

According to the catalogue of 1854, the *Freshmen* studied Davies' University Arithmetic, Davies' Bourdon, and Davies' Legendre; the *Sophomores* continued Davies' Bourdon and Legendre, and then took up Plane and Spherical Trigonometry and Surveying; the *Juniors* studied Descriptive Geometry, Shades and Shadows and Perspective, Davies' Analytical Geometry, and Descriptive Astronomy; the *Seniors*, Davies' Differential and Integral Calculus, and physical astronomy. In the introduction into the course of descriptive geometry, in the use throughout of Davies' text-books, and in the apparent thoroughness (for that time) of the mathematical course, we observe the influence of the U. S. Military Academy, through Professor Bledsoe, a West Point graduate.

When Bledsoe resigned to accept a professorship at the University of Virginia, Frederick Augustus Porter Barnard, a young man of remarkable mathematical talents, took his place. Barnard was a native of Massachusetts and entered Yale college in 1824. Before admittance to college he had given no time to mathematical study beyond the elements of arithmetic, but in college he began to exhibit decided mathematical talent and taste. His tutor, W. H. Holland, later professor of mathematics in Trinity College, Hartford, said of him: "I have never known any person except the late lamented Professor Fisher, who possessed so extraordinary natural aptitude." After graduation he was, for a time, tutor at Yale, then professor at the University of Alabama, and, in 1854, became Bledsoe's successor at the University of Mississippi. At the meeting of the board of trustees, in July, 1856, the chair of pure and applied mathematics and astronomy was divided into two,

the chair of pure mathematics, and the chair of natural science, civil engineering, and astronomy. Professor Barnard held the latter, though he continued to exercise supervision over the former, and was also elected president of the university. He filled these offices until the suspension of the exercises of the university, in 1861.

From 1856 to 1861 Jordan McCullogh Phipps was teacher of mathematics—at first adjunct professor, afterward full professor. Daniel B. Carr was tutor. The department of mathematics, physics, and engineering seems to have been the strongest at the institution. In consequence of frequent complaint that the general statement previously presented in the annual catalogues of the university had been unsatisfactory, a complete account of expenses and of the courses of instruction was given in the catalogues issued at this time. From the one of 1857–58 we quote the following:

"Instruction in pure mathematics commences with the beginning of the Freshman year, and is continued till the close of the Sophomore. In order to secure greater efficiency of instruction, the class will be divided into sections, which will be met by the instructor separately; and all operations in this and every other branch of mathematical science will be actually performed by the student in his presence, upon large wall-slates or blackboards. The instructor will also avail himself of the same means of illustrating processes, or principles, and explaining difficulties.

"The first subject attended to is algebra. It will be the instructor's endeavor to secure a thorough acquaintance with the elementary principles of the science, and a perfect familiarity with its practical operations. The subject of fractions will be especially dwelt on, after which will follow the resolution of simple equations, numerical and literal, involving one or more unknown quantities. In taking up, next in order, quadratic equations, the first object will be to secure on the part of the student a perfect understanding of the form of the binomial square; and this will be afterward applied to the completion of imperfect squares, in the several cases in which one of the terms of the root is a number, or a letter, or a numerical or literal fraction. The method being generalized, will then be applied to the reduction of abstract equations, and the statement and resolution of problems involving quadratics. Where the equation is denominate, the student will be required to interpret the result, to explain the ambiguous sign, and to distinguish cases in which the conditions of the problem involve an impossibility. * * *

"The subject of algebra will be completed by the discussion of the general theory of equations, their formation, their solution, and their properties, including in the course the ingenious theorem of Sturm.

"In all parts of this subject, encouragement will be held out to students to exercise their ingenuity in devising various modes of arriving at the same results; and special merit will be attached to the processes

which are the most succinct or elegant. As a stimulus to this species of ingenuity, problems not embraced in the text-book may from time to time be proposed by the instructor; and varieties in the mode of statement both of these and of those which occur in the regular course, will be called for from any who may choose to present them.

"Geometry, plane, solid, and spherical, will occupy the latter portion of the Freshman year. In this branch of science all demonstrations will be made from figures drawn upon the blackboards, or wall-slates, by the student reciting, and promptness and accuracy in this part of the business will be urgently inculcated and regarded as a merit. The student will, moreover, be advised to avoid a servile imitation of the exact forms of the diagrams given in the text-book, and will have his ingenuity exercised either in forming other figures to illustrate the same propositions, or in demonstrating the propositions from figures constructed for him. He will also be required to adopt a mode of lettering his figures different from that of the book; or to give the demonstrations without the use of letters at all, by pointing to the parts of the figure successively referred to in the demonstration.

"It will always be regarded as specially meritorious in a student to present a demonstration of any proposition founded on any legitimate method differing from that of the author; and the instructor will, himself, from time to time, illustrate this practice, by way of awakening the ingenuity of the student. For the purpose of still further encouraging originality of investigation, and exciting honorable emulation, the plan already described as to be pursued in algebra, will be continued here, of propounding propositions not contained in the text-book, of which demonstrations will be subsequently called for, and which will secure special distinction to such as satisfactorily solve them."

Equally full is the account of the mathematical work in the Sophomore year. The studies for that year were plane and spherical trigonometry, mensuration, surveying, leveling, navigation, and analytical geometry. Considerable field-work was done in surveying. The leveling rods employed had the common division to feet and fractions, and also the French metrical division.

The catalogue then proceeds as follows:

"The course of pure mathematics will conclude with the subject of the differential and integral calculus, which will be taught at the end of the Sophomore or the beginning of the Junior year. This will embrace the doctrine of functions, algebraic and transcendental, the differentiation of functions, successive differentials, theorems of Taylor and Maclaurin, logarithmic series, the development of a circular arc in terms of its functions, or of the functions in terms of the arc, partial differentials, differential equations of curves, principles of maxima and minima, expressions for tangents and normals, singular and multiple points, osculating circles, involutes and evolutes, transcendental curves, and spirals; the integration of regularly formed differentials, integra-

tion by series, integration of rational and irrational fractions, special methods of integration, the rectification of curves, the quadrature of curves and curved surfaces, the cubature of solids, and the integration of differentials of two or more variables."

In natural philosophy great efforts were made to secure a complete set of apparatus. In the catalogue for 1857–58 we read as follows: "It is probable that, with the opening of the ensuing session, the electrical apparatus of the University of Mississippi will be superior to any similar collection in the United States."

In astronomy the celestial motions were beautifully represented by Barlow's magnificent planetarium, eleven feet in diameter—"a piece of mechanism unrivaled in ingenuity, accuracy, and elegance." A portable transit instrument was also available for observations of meridian passages, and a sextant and a prismatic reflecting circle furnished means of making direct measurements of altitudes and arcs. The catalogue then says:

"The course of civil engineering, distinctly so called, falls entirely within the Senior year; but it is in considerable part only a further development and application of principles embraced in the sciences of pure mathematics and physics previously taught. The course will embrace geometrical and topographical drawing, the use of field instruments, such as the engineer's transit, the goniasmometer or pantometer, the leveling instrument, the theodolite, the sextant, the reflecting circle, and the plane table, descriptive geometry, trigonometrical surveying and geodesy, marine surveys, materials of structures, engineering statics, carpentry, masonry, bridge construction, surveys for location and construction of roads and railroads, laying out curves, staking out cuts and fills, hydraulic engineering, drainage, canals, locks, aqueducts, dams, sea walls, river improvements, and the dynamics and economy of transportation. * * *

"Throughout every part of the course, the student will be constantly encouraged and stimulated to consult other authorities on the subjects taught, besides the text-books: and the instructors will often refer them on special subjects, to such authorities. The following list embraces the text-books (first in order), and the authors to whom reference will most frequently be made:

"ALGEBRA: Perkins, Hackley, Peirce.
GEOMETRY: Perkins, Playfair (Euclid), Peirce.
TRIGONOMETRY: Perkins, Hackley, Peirce.
SURVEYING: Gillespie, Davies, Gummere.
ANALYTICAL GEOMETRY: Davies, Peirce.
CALCULUS: Davies, Peirce, Church, Jephson.
NATURAL PHILOSOPHY: Olmsted, Bartlett, Whewell, Brewster (Optics), Herschel (Light and Sound), Peirce.
ASTRONOMY: Olmsted, Gummere, Bartlett, Loomis.
CIVIL ENGINEERING: Mahan, Moseley, Wiesbach, Gillespie, Haupt, Bourne, Pambour."

The courses for the remaining years before the War were essentially the same as the one we have described. The constant use of the blackboard is emphasized throughout. The fact that pains are taken to explain the term as meaning "large wall-slates" rather tends to show that blackboards were then a novelty in Mississippi. As far as we can judge from the catalogues, the instruction was methodical and of high efficiency. A serious drawback to high scholarship was found, no doubt, in the lack of preliminary culture and training in students entering the university.

The attendance of students was good. The number of graduates from the department of arts from 1851 to 1859, inclusive, was 268. During the last year before the War the number of students in the college was 191, of whom 16 were "irregular" in grade.

Owing to the universal enlistment of males, even youths, in the Confederate States army, the university exercises were suspended in 1861, until October 1865. In 1865 General Claudius W. Sears, ex-brigadier-general of the Confederate States army, and a graduate of West Point, was elected professor of mathematics. This position he still holds.

The mathematical requirements for entering were, in 1866, "arithmetic and algebra, including equations of the first degree." The course of pure mathematics for the regular under-graduate curriculum was completed at the end of the Sophomore year, and consisted of Bourdon's Algebra, Legendre's Geometry, Trigonometry, Mensuration, Surveying, and Analytical Geometry.

A more extended course than was required for the degree of bachelor of arts could be obtained in the department of applied mathematics and civil engineering, which was in charge of General F. A. Shoup, a graduate of West Point, and now of the University of the South. The course of instruction in his department formed no necessary part of the under-graduate course. It was designed to meet the wants of such students as intended to make civil engineering or some other of the mechanic arts a profession. In this course analytical geometry and calculus were, of course, indispensable, and they could be studied while students were pursuing their branches in the department proper. The course could be completed by an ordinary student who came fairly well prepared in preliminary branches in about two years.

In 1870 the plan of instruction in the university was altered so as to include (1) a department of preparatory education, (2) a department of science, literature, and arts (leading, respectively, to the degrees of B. A., B. S., B. Ph., C. E.), and (3) a department of professional education (law).

The terms for admission into the bachelor of arts and bachelor of science courses were, in mathematics, arithmetic, and Davies' Elementary Algebra through equations of the second degree. Candidates for the bachelor of philosophy course and civil engineering were examined on the whole of Davies' Elementary Algebra. These requirements

have remained unchanged till the present time. The department of civil engineering was discontinued in 1876. In 1872 the first year's mathematical work in the course leading to the B. A., B. S., and B. Ph. degrees consisted in the study of Davies' Bourdon's Algebra, and Legendre's Geometry and Plane Trigonometry. During the first half of the Sophomore year Church's Analytical Geometry and Davies' Land Surveying (with use of instruments in the field) were studied. This completed the course in pure mathematics. A. B. students were taught Smith's Mechanics and Hydrostatics, Hydraulics, and Sound (Bartlett) in the Junior year, and Bartlett's Optics and Astronomy in the Senior year. B. S. students had Gummere's Astronomy in the second half of the third year. (The B. S. and B. Ph. were the only three years' courses.)

At the present time (1888) the mathematical course is decidedly stronger. Van Amringe's edition of Davies' text-books are used, except in analytical geometry and calculus, which are studied from the works of Church. The calculus is now studied during the latter part of the Sophomore year.

Prof. C. W. Sears has now occupied the mathematical chair for twenty-three years. One of his old pupils, Prof. Edward Mayes, says of him, "that he 'quizzes' 'like all possessed,' pretends that he does not know anything about it, and asks 'all sorts of impertinent questions.'" As Sydney Smith said of Alexander Pope, "I studied under him, and have lively recollections."

KENTUCKY UNIVERSITY.

The records of the Transylvania University for several years following 1817 appear to have been lost. In 1825 Thomas J. Matthews, the father of the late Justice Stanley Matthews, of the Supreme Court of the United States, is mentioned as being "professor of mathematics and natural philosophy." The subjects taught by him were "arithmetic, geometry, surveying, leveling, natural philosophy, and book-keeping." The entry for 1829 shows that Pestalozzian ideas had gained a foothold at the university, inasmuch as Colburn's Algebra is mentioned as the mathematical text-book for the *Freshmen*. The *Sophomores* studied Playfair's Geometry and Trigonometry; the *Juniors*, Day's Navigation, Surveying, Heights and Distances, Leveling; the *Seniors*, Bezout's Fluxions. Bezout's text-book had been translated from the French by Professor Farrar, of Harvard. It employed the notation of Leibnitz, and did not therefore teach "fluxions." The use of this term as a synonym for "differential and integral calculus" was, we believe, peculiarly American.

In 1832 John Lutz was elected "professor of mathematics and natural philosophy," and in 1837 Benjamin Moore. The latter resigned after one year's service.

The records from 1839 to 1865 can not be found. From old catalogues we glean the following: In 1844 R. T. P. Allen was professor, and the subjects taught were, in the *Freshman* year, Davies' Bourdon and Legendre; in the *Sophomore* year, plane and spherical trigonometry, heights and distances, mensuration of superficies and solids (Davies'), navigation (Day's), conic sections (Davies' Analytical Geometry), Surveying (Davies'), descriptive geometry (Davies'); in the *Junior* year, differential and integral calculus (Davies'); in the *Senior* year, Olmsted's Astronomy.

In 1848 James B. Dodd held the chair of mathematics and natural philosophy. At this time the course was as follows: *Freshman* year, arithmetic reviewed, Loomis's Algebra, five books of Legendre; *Sophomore* year, geometry completed, plane and spherical trigonometry and their applications, analytical geometry (Davies', 6 books); *Junior* year, Church's Calculus.

In 1850 the mathematics for the Junior and Senior classes consisted of descriptive geometry, analytical geometry, calculus, and analytical mechanics; but they were optional with the student.

Prof. James B. Dodd was the most prominent mathematical teacher that was connected with Transylvania University. He was a native of Virginia, and a self-made mathematician. In 1841 he became professor of mathematics at the Centenary College in Mississippi, and in 1846 was elected professor at the Transylvania University. He published several books, viz., an Elementary and Practical Arithmetic, High School Arithmetic, Elementary and Practical Algebra, Algebra for High Schools and Colleges, and Elements of Geometry and Mensuration. Some of these reached several editions. Professor Dodd contributed also to the Quarterly Review of the M. E. Church South. In 1849 he was appointed president *pro tempore* of the university.

In 1865 Transylvania University was merged into Kentucky University. The chair of mathematics in Kentucky University has been filled from 1859 to the present time by Henry H. White. From 1870 to 1876 James G. White acted as adjunct professor. From 1876 to 1878 he was professor. In mathematics the requirement for admission has been algebra through equations of the first degree. When Prof. Henry H. White first became connected with the university as professor, the course was as follows: Algebra completed, plane and solid geometry, application of algebra to geometry, plane and spherical trigonometry, surveying and navigation, analytical geometry, differential and integral calculus, mechanics, and astronomy, with original problems and exercises throughout the course when practicable. In 1804 the course was modified by dropping applications of algebra to geometry; in 1879, by the addition of conic sections (treated geometrically); and in 1884, by dropping conic sections and navigation.

The text-books used by Prof. Henry H. White at different times are as follows: In *Algebra*, Davies' Bourdon, Towne, Peck; in *Geometry*,

Davies' Legendre, Peck; in *Trigonometry*, Davies, Peck; in *Surveying* and *Navigation*, Davies, Loomis; in *Analytical Geometry*, Loomis, Peck; in *Calculus*, Loomis, Peck; in *Mechanics*, Olmsted, Snell's Olmsted, Peck; in *Astronomy*, Olmsted, Snell's Olmsted, Peck.

There have been no electives in mathematics up to this time, except that the student now has the choice between languages and calculus.

UNIVERSITY OF TENNESSEE.*

"The foundation of this university is connected with the earliest history of Tennessee.

"In 1794, by the first General Assembly of the 'Territory south of the Ohio,' was chartered Blount College, named in honor of William Blount, Governor of the Territory, and afterward one of the two United States Senators first chosen from the State of Tennessee.

"In 1807, under an act of Congress providing for the establishment of two colleges in Tennessee, East Tennessee College was chartered, and soon after the franchise and property of Blount College were transferred to the new institution. * * *

"In 1840 the name of East Tennessee College was changed, by act of Legislature, to East Tennessee University.

"In 1869 the Legislature gave in trust to the university the proceeds of the sale of public lands, donated by act of Congress of July 2, 1862, 'to the several States and Territories which may provide colleges for the benefit of agriculture and the mechanic arts.'

"In 1879 the name of East Tennessee University was changed, by an act of the Legislature, to the University of Tennessee."†

It is a source of regret to us that we have not been able to obtain any information whatever on the mathematical instruction at this institution during the first eighty years of its existence. Ever since it took the name of a university, it has been in an almost continual state of reorganization. These constant upheavals have resulted in the loss of almost all its records. "The requirements for admission and graduation," says Professor Carson, "have probably been changed, on an average, every two years." The terms for admission were not rigidly adhered to, and the standard for graduation has not always been high.

The catalogue of 1874–75, the earliest one that we have, gives John Kerr Payne as professor of mathematics and mechanical philosophy. The collegiate department comprised at this time three distinct courses viz., the agricultural course, the mechanical course, and the classical course. The standard for admission to the first two courses was, until 1874, lower than to the last course. In 1874–75 the mathematical studies in the agricultural course were according to catalogue, as fol-

* The writer is indebted to Prof. Wm. W. Carson, professor of mathematics and civil engineering at the University of Tennessee, for all the information herein contained.

† Catalogue of the University of Tennessee, 1885–86.

lows: *Freshmen*, Robinson's University Algebra, beginning with quadratic equations, Chauvenet's Geometry, beginning at the third book, Loomis's Conic Sections; *Sophomores*, Church's Descriptive Geometry, Loomis's Trigonometry and Surveying; *Juniors*, Olmsted's Natural Philosophy and Astronomy. In the mechanical and classical courses, the schedule was the same in mathematics, except that spherical trigonometry, Loomis's Analytical Geometry and Calculus, and civil engineering, were added.

The biennial report of the trustees for 1881 gives James Dinwiddie as professor of pure mathematics, and Samuel H. Lockett as professor of applied mathematics and mechanical philosophy. The report shows that the university was then organized into distinct schools, like the University of Virginia. These schools have existed, probably, since 1879. Of the school of pure mathematics, the report says:

"The subjects taught in the subcollegiate year of this school are elementary algebra, and four books of geometry. In the first collegiate year algebra and geometry are finished, and plane trigonometry is studied. In the second collegiate year are studied spherical trigonometry and analytical geometry of two dimensions, and in the third year differential and integral calculus."

The work in the school of applied mathematics is described as follows:

"Elementary experimental physics is taught in the first college year. The various subjects of statics and dynamics of solids, liquids, and gases; of acoustics, heat, light, electricity, and magnetism, are treated without the aid of the mathematics, and are illustrated by numerous experiments. The apparatus has been specially selected for that purpose.

"In the analytical mechanics, the power of the whole range of the mathematics is brought to bear upon the investigation of the laws of forces of nature, and the student is made familiar with the power and utility of mathematics by the solution of a large number of practical problems. Astronomy has thus far been taught without instruments, but the board of trustees has appropriated five hundred dollars for the purchase of a telescope. Surveying comprehends plane surveying, leveling, topographical surveying, and mining surveying; the use of the compass, transit, Y level, plane table, chain, and leveling rod; also plotting, making profiles and cross-sections, and topographical drawing with pen and brush. A large share of the student's time is given to field work and practice.

"Descriptive geometry is the foundation of both the science and art of drawing. It is followed by a course of problems in shades, shadows, and perspective—mechanical drawing.

"The course of engineering consists of the subjects treated in Professor Gillespie's Roads and Railroads and Professor Wood's revision of Mahan's Civil Engineering, and of a course of lectures by the instructor

on surface and thorough drainage, on agricultural, hydraulic, and marine engineering, and a brief outline of the science and art of military engineering. The engineering drawing consists of a course of instruction in the drawing of plans, sections, elevations, and details of bridges, tunnels, canal locks, etc.

"For the above engineering course students can substitute mechanism, machinery, and machine drawing."

The catalogue for 1883–84 mentions as text-books in the school of pure mathematics: "White's or Olney's Arithmetic; Davies' Bourdon, or Olney's Algebra; Olney's Trigonometry; Bowser's or Peck's Analytical Geometry; Bowser's or Peck's Calculus; Bledsoe's Philosophy of Mathematics.

"Extra examples, illustrating the different subjects taught, are given throughout the course."

This is the first time that we find Bledsoe's Philosophy of Mathematics named as one of the text-books in a college course. According to catalogue, it was used in the third collegiate class, which completed analytic geometry and then took up "differential and integral calculus, and the philosophy of mathematics." The idea of teaching the philosophy of mathematics is certainly a good one, but the subject is hardly presented by Bledsoe in a form suitable for a young student.

In the school of applied mathematics the books given in the catalogue for 1883–84 are, Gage's Physics; Loomis's Astronomy; Davies' New Surveying; Smith's Topographical Drawing; Church's Descriptive Geometry; Wood's, or Rankine's Mechanics; Mahan's Civil Engineering; Searles's Field Engineering.

In June, 1888, a reorganization and a re-classification of the various schools took place. The work of the "school of mathematics and civil engineering" for the year 1888–89 is as follows:

I. MATHEMATICS.

First class—(Sub-Freshman): Algebra (through surds and quadratics); Geometry (three books).

Second class—(Freshman): Geometry, Algebra.

Third class—(Sophomore): Trigonometry; Graphic Algebra; Analytical Geometry.

Fourth class—(Junior): Calculus.

Each class is taught in sections small enough to be well handled by the instructor. Great stress is laid, throughout the course, on the written solution of original problems—the aim being to induce clearness of thought by precision in expression. Each student is required to use the level, transit, and compass, from the beginning of his Freshman to the end of his Sophomore year. On entering the Freshman class the use and adjustments of the level are explained to him. He then practices with it, at times convenient to himself, until, by running such lines as may be required of him and submitting profiles and cross-sections, he shows his ability to handle the ordinary problems of drain-

age and irrigation. The graphical problems in geometry are solved, sometimes with drawing instruments on paper, and sometimes with engineering instruments on the ground. Thus habits of accuracy are enforced early in the course by the use of instruments of precision, and an elementary knowledge of surveying afforded.

For admission to the first class the applicant is examined in arithmetic only.

The text-books now in use are as follows: Hall and Knight's Algebra for the Sub-Freshman class, Wentworth's Algebra for the Freshman class, Wentworth's Geometry, Wells's Trigonometry, Puckle's Conic Sections (with lectures), Newcomb's Calculus. The Calculus is taught mainly by lectures, the text-book being used as a guide. As taught at present, it is based on the idea of fluxions, demonstrated by limits, and employs the notation of Leibnitz. In pure mathematics no higher branches than the calculus have been taught at the university, except during the session 1886–87, when a class in quaternions was taught. At present agricultural students must finish trigonometry, all others analytical geometry, while the engineering students must finish calculus.

II. CIVIL ENGINEERING.

1. (Sophomore): Descriptive Geometry; Land, City, and Mine Surveying.
2. (Junior): Stone Cutting; Astronomy.
3. (Junior): Elementary Mechanics; Analytical Mechanics.
4. (Junior): Surveys; Soundings; Maps; Profiles; Cross-sections; Estimates; Laying out Work; Engineering Materials and Methods.

The time of this class is mainly spent in practical work. It makes barometric reconnaissances; makes a map of some portion of the bed of the Tennessee River; does the field and office engineering work for a line of communications to join two selected points, etc.

5. (Senior): Analytical Mechanics; Applied Mechanics.
6. (Senior): Engineering Structures; Specifications and Contracts.
7. (Post-graduate): Economics of Roads; Sewerage; Water Supply; Hydraulics; Architecture.

The department is admirably equipped with the various engineering instruments. Of the more important (such as levels, transits, sextants, aneroids, etc.) it has a number of each. It has, with great care and expense, procured instruments of the finest workmanship and latest attachments, so that its students of engineering may see how much to expect the instrument-maker to contribute toward the attainment of accuracy and speed. Exercises requiring their use are continually required of every class.

The first *six* of these classes are required for the degree of bachelor of science in civil engineering—the *seven* for the degree of civil engineer.

At present the University of Tennessee is entering upon a career of remarkable prosperity. Like most of the higher institutions of learning in the South, it is experiencing a great revival. More thorough work and a higher standard of scholarship are everywhere perceivable.

The present prosperity of the University of Tennessee is due chiefly to the aggressive leadership of its President, Dr. C. W. Dabney, a graduate of the University of Virginia, and later of the University of Göttingen. He accepted the presidency in August, 1887, under conditions giving him great freedom to manage the institution according to his own ideas. In June, 1888, the professorships were declared vacant, and were then filled by men selected by the president. Prof. William W. Carson, who had been elected to the chair of mathematics in 1885, was now elected professor of mathematics and civil engineering. Professor Carson, a graduate of Washington and Lee, was civil engineer for a number of years. Of the other teachers of pure and applied mathematics, Prof. T. F. Burgdorff served about a dozen years in the U. S. Navy, and Prof. E. E. Gayle about an equal length of time in the U. S. Army. The three other instructors in this school are young men.

TULANE UNIVERSITY OF LOUISIANA.

The Tulane University came into existence as such in 1884, when, by a contract with the State of Louisiana, the administrators of the Tulane educational fund became the administrators of the University of Louisiana in perpetuity, agreeing to devote their income to its development.

The University of Louisiana had its origin in the Medical Department, which was established in 1834. This school has numbered among its professors and alumni the most distinguished medical men of Louisiana and the South. A law department was organized in 1847; and in 1878 the academic department of the University of Louisiana was opened. It existed under that name till 1884, when it was absorbed into Tulane University. Considering that the academic department of the University of Louisiana received from the State an annuity of only ten thousand dollars, it met with excellent success. A number of very earnest and well-trained young men were graduated during the six years of its existence. Its faculty consisted of only seven professors, but they were men of energy and ability. R. H. Jesse was dean of the faculty and professor of Latin. He was educated at the University of Virginia, and was a man of unusual executive ability. His individuality was strongly felt in the institution. He organized the department, taking the University of Virginia as his model. There was no curriculum or prescribed course of study. The parent or guardian had to choose, with the advice of the faculty, the branches to be pursued by the student. His cast of mind, as well as his future vocation, could thus receive due weight. In 1883 there were eight "schools." The student was required to attend at least three, but he was discouraged from electing more than four, in order to prevent superficial work.

The school of mathematics was in charge of J. L. Cross, the professor of mathematics. Professor Cross was, before the War, a student at the Virginia Military Institute, and a pupil of Prof. Francis H. Smith. The school of mathematics was organized into three regular

classes, the Junior, Intermediate, and Senior. During part of the time it was found necessary to establish also an introductory class for students deficient in preliminary studies. The requirements for admission to the Junior class were a knowledge of arithmetic and Loomis's Elements of Algebra. The Junior class studied Loomis's Treatise on Algebra, and Loomis's (later Wentworth's) Plane and Solid Geometry. The Intermediate class was taught Loomis's Plane and Spherical Trigonometry, and Loomis's Analytical Geometry. The Senior class completed the course in mathematics by the study of Church's Descriptive Geometry, and Loomis's Differential and Integral Calculus. Professor Cross is, we believe, the first teacher who ever carried classes in New Orleans through the calculus.

Very efficient work was done by students in the school of physics. This was in charge of Prof. Brown Ayres. Professor Ayres received his general education at the Washington and Lee University, and his training as a specialist at the Stevens Institute and the Johns Hopkins University. At the last institution he was honored with a fellowship in physics. He is a true lover of science, and, with great proficiency in the theoretical and mathematical parts of his subject, combines great mechanical ingenuity and skill. In his prelections on text-books he is extremely clear, and his experiments are always very successful and interesting. His great aim is to awaken in students a genuine love for pure science. In his school students had frequent opportunities of applying their knowledge of pure mathematics to physical problems. The theory of the combination of observations by the method of least squares was a study in his course. During several years he taught also analytical mechanics, using the work of De Volson Wood.

In 1884 the University of Louisiana was absorbed into the Tulane University of Louisiana. Paul Tulane, who had been in business in New Orleans for fifty years, donated the greater part of his large fortune for higher education in New Orleans. Owing to his munificence, Tulane University has the good fortune of being free from those pecuniary embarrassments with which the University of Louisiana had always to contend. Under the presidency of Col. William Preston Johnston, an educator of great ability and wide reputation, the courses of study as they had existed in the University of Louisiana were reorganized.* Not trusting in the ability of immature students, or even of parents unaccustomed to consider the due proportions and sequence of studies, to properly formulate their own ideals in education, Tulane College offered a series of six equivalent curricula with prescribed branches, all leading to the degree of bachelor of arts. These six courses of study were denominated, respectively, the Classical, Literary, Mathematical, Natural Science, Commercial, and Mechanical Courses. In the

* For further information regarding the plan and workings of Tulane University, see President Johnston's address on "Education in Louisiana," before the National Educational Convention, Topeka, Kan., July 15, 1885.

spring of 1880, the commercial course was discontinued, and the mathematical course had its name changed to physical science course.

All the professors of the University of Louisiana continued to hold their respective chairs under the new régime. Several new professors were added to the faculty.

The mathematical requirements for admission to Tulane College are a knowledge of algebra to quadratics and of plane geometry. The course in mathematics is the same for all Freshmen. After completing the algebra they take up solid geometry, plane and spherical trigonometry, surveying and leveling, and navigation. In the Sophomore year, classical and literary students pursue analytical geometry, three hours per week, before Christmas. This completes the mathematics for students in those two courses. In the three other courses mathematics is pursued six hours per week throughout the year, and consists in the study of analytical geometry and differential calculus. In the first half of the Junior year, students in the physical science course and mechanical course pursue the study of integral calculus. These branches are taught by Professor Cross from Loomis's text-books, excepting that Wentworth's book is used in geometry.

The mathematical teaching has, thus far, been strictly confined to the ordinary college branches. No work of university grade, as distinguished from college grade, has yet been attempted. "The end kept always in view is to impress the principles of mathematical truth clearly and deeply on the mind, by careful explanations, by daily examinations, and by constant application of these principles by the students themselves to numerous examples taken from the text-books and from other sources."* Professor Cross believes in making a clear presentation to the student of the principles of mathematics, without applying them to any great number of special cases. In his opinion, much valuable time is wasted in the solution of problems. If a student can give, for instance, the general solution of a quadratic equation, then there is no need of solving dozens of special exercises under this head. In geometry careful attention is given to the correct understanding of the demonstrations given in the book, but little or no effort is made to solve original exercises. In the class-room Professor Cross preserves strict discipline and is earnest in the discharge of his duties. When the routine work of the day is over, his mind finds relaxation and rest in a good game of chess or checkers.

Students in the mechanical and physical science courses study analytical mechanics under Professor Ayres six hours per week during the second half of the Junior year. This subject has been exceedingly well taught. The text-book used heretofore in connection with lectures has been Wood's Analytical Mechanics. This is a good text-book, inasmuch as the subject is taken up more or less inductively, and a large

*Catalogue of the Tulane University of Louisiana, 1888–89, p. 46.

number of special and well-graded problems is given to be worked by the student. Wood makes extensive use of the calculus in his Analytical Mechanics. The experience has been at this institution, as also at others, that students who have gone through Loomis's Calculus are hardly well enough prepared in that branch to pursue with ease a course in analytical mechanics. Some important parts of the integral calculus, particularly definite integrals, receive exceedingly meager treatment in this book. The course in analytical mechanics serves to impress more deeply and lastingly the principles of the calculus and displays to the student its wonderful power in the solution of all sorts of mechanical problems. This year (1888–89) Michie's Analytical Mechanics will be used as a text-book by Professor Ayres. It contains a beautiful chapter on graphical statics. In the Senior year students in the mechanical course take up the subject of applied mechanics. Professor Ayres is using, this year, Cotterill's Applied Mechanics, a standard work of great merit.

In 1883 a very fine collection of physical apparatus was purchased by the university at a great expense. In optics the collection is excellent. The university is fortunate in having a physicist who knows how to make use of delicate instruments. Since the above date Professor Ayres has devoted much of his time and energy toward building up a good laboratory. A practical physical laboratory is somewhat of a novelty in the South. Tulane University offers now as good and efficient courses in experimental physics to students of *college grade* as any university in the country.

Since Tulane University is dependent for its supply of students chiefly upon its own high school, wise provisions have been made for more thorough instruction in that department. With Professor Ashley D. Hurt as head-master the high school has been prosperous and thorough in its work. Both teachers and pupils are working with great earnestness, and it is gratifying to know that the number of students entering the college after graduating from the high school is decidedly on the increase.

The New Orleans Academy of Sciences holds its meetings at the Tulane University. The professors of the university are its leading members. There is a general meeting once every month for all members of the academy. In addition to this, there are section meetings. "Section A," the mathematical and physical section, meets the second Tuesday of every month. Professor Ayres has been the leading spirit in this section, and has contributed many an interesting paper on physics and mathematics. Two years ago the academy began publishing an annual volume, containing the principal papers read during the year. The publication for the year 1887–88 contains an article on the "History of Infinite Series," and an interesting article by Professor Ayres on "Physics and Psychology." During the last two years the

academy has been in a flourishing condition, and the quality of the work done has been improving continually.

In the fall of 1887 the H. Sophie Newcomb Memorial College for Young Women was opened as a branch of Tulane University. It was founded on an endowment made by Mrs. J. L. Newcomb, of New York. This institution is under the able management of President Brandt V. B. Dixon, who is also professor of metaphysics and mental science at the Tulane University. It is the aim to put the Newcomb College on an equal footing with the Tulane College. Young women will thus have the same facilities for higher education in New Orleans that young men have.

The first year (1887–88) was a year of organization. Many features of the school were of necessity only tentative. The great obstacle to high scholarship is the lack of proper preparation on the part of applicants. For this reason it has been necessary to establish a preparatory department. The Newcomb College offers four parallel and equivalent courses of study—the Classical, Literary, Scientific, and Industrial. In the two preparatory years, higher arithmetic and algebra are studied. It is the intention to introduce also a course on inventional geometry. The first year in college is devoted to geometry, the second to the completion of algebra and to trigonometry. To students taking the scientific and industrial courses, analytical geometry is offered in the Junior year, and calculus and astronomy in the Senior year. During the first year in the history of the college there were classes in algebra, geometry, and trigonometry. Wentworth's text-books were used. In the preparatory department there were two classes, one in arithmetic and algebra, and the other in algebra. The latter class did faithful and thorough work in Wentworth's Complete Algebra through quadratic equations. This division did as good work as any class of young men which the professor has taught. If not always quite as penetrating in the solution of problems as young men, the young ladies worked more faithfully and perseveringly. The lowest class of college grade finished plane geometry and then reviewed algebra as far as logarithms. The work in geometry was quite satisfactory. A great effort was made to induce students to solve original exercises. While paralogisms were very frequent, especially at first, the efforts were not without some success. The solving of original exercises in geometry is too much neglected in our schools; nor are our text-books always satisfactory on this subject. In the opinion of the writer, the number of exercises should be greatly increased, and very great care should be taken to either omit the difficult exercises or give "hints" as to their mode of solution. Students should not be permitted to get disheartened in this sort of work. "The inventive power grows best in the sunshine of encouragement." Wentworth has greatly improved his text-book in his revised edition of 1888, by the insertion of seven hundred additional exercises.

The professor has found that the interest which pupils take in their studies may be increased if the solution of problems and the cold logic of geometrical demonstrations are interspersed by historical remarks and anecdotes. A class in arithmetic will be pleased to hear about the Hindoos and their invention of the "Arabic notation;" they will marvel at the thousands of years which elapsed before people had even thought of introducing into the numeral notation that Columbus egg, the zero; they will find it astounding that it should have taken so long to *invent* a notation which they themselves can now *learn* in a few weeks. The class will take an interest in the history of decimal fractions and the various notations that were used once in place of our decimal point. After the pupils have learned how to bisect a given angle, surprise them by telling of the many futile attempts which have been made to solve by elementary geometry the apparently very simple problem of the trisection of an angle. When they know how to construct a square whose area is double the area of a given square, tell them about the duplication of the cube—how the wrath of Apollo could be appeased only by the construction of a cubical altar double the given altar, and how mathematicians long wrestled with this problem. After the class have exhausted their energies on the theorem of the right-angled triangle, tell them something about its discoverer—how Pythagoras, jubilant over his great accomplishment, [is said to have] sacrificed a hecatomb to the Muses who inspired him. When the value of mathematical training is called in question, quote the inscription over the entrance into the academy of Plato, the philosopher: "Let no one who is unacquainted with geometry enter here." To more advanced students the history of mathematics becomes instructive and profitable as well as interesting. It seems to me that students in analytical geometry should know something of Descartes, who originated this branch of geometry, that, taking up differential and integral calculus, they should become familiar with the parts which Newton, Leibnitz, and Lagrange played in creating the transcendental analysis. No one can claim to have a fair knowledge of this subject who knows not something about the three methods taught by these great analysts. In his historical talk it is possible for the teacher to make it plain to the student that mathematics is not a dead science in which no new discoveries are or can be made, but that it is a living science in which racing progress is being made all the time.

UNIVERSITY OF TEXAS.

The University of Texas opened its doors to students for the first time in 1882. The first professor of mathematics was Leroy Brown, who served one year. He was succeeded by G. B. Halsted as professor of pure and applied mathematics. At the same time with Halsted, A. V. Lane was elected assistant instructor in mathematics. He was advanced to the position of assistant professor of applied mathematics in 1885.

Prof. G. B. Halsted was graduated in Princeton in 1875, and received the degree of doctor of philosophy at the Johns Hopkins University in 1879, where he had studied for two years under Professor Sylvester, and had held a fellowship in mathematics. Before taking his degree he spent some time in Berlin, prosecuting mathematical studies. In 1878 he was appointed tutor in mathematics at Princeton College, and three years later instructor in post-graduate mathematics.

Dr. Halsted has established a wide reputation as a mathematician and logician. He has contributed to the American Journal of Mathematics, the Annals of Mathematics, the Mathematical Magazine, the English Philosophical Magazine, and several other scientific journals. He has published two books, An Elementary Treatise on Mensuration (Boston, 1881), and The Elements of Geometry (New York, 1885). His books and scientific articles have been favorably reviewed in leading foreign journals. His Metrical Geometry (mensuration) is the best book of its kind that has been published in this country. It contains many new and interesting features. Of these we would mention his treatment of solid angles (the words *steregon* and *steradian*, now quite generally adopted, were manufactured by him and first used here) and his discussion of the prismatoid, deriving a general formula for its volume. He introduced a distinction between the words *sphere* and *globe* (making one to mean a surface and the other a solid), which is worthy of general adoption.

The distinguishing feature of the two works of Halsted is their scientific rigor. Teachers who favor a rigid treatment of geometry will find it in his Elements. The book rejects the "directional method" as wholly unscientific; also the use of the word "distance" as a fundamental geometric concept. The word *sect*, first used in his Mensuration, is introduced here, meaning "the part of a line between two definite points." Many teachers do not endorse the introduction of this new technical term in elementary geometry, as they think that there is no particular call for it. The author is certainly right in protesting against the use of the word "distance" in two different senses. That there has really been a want for some of the other new technical terms first introduced by Halsted is evident by the fact that they have been adopted in standard works, such as the Encyclopædia Britannica.

Like his Mensuration, his Elements of Geometry possesses many novelties. In his book on Rectangles he introduces a strictly geometric algebra, where a and b mean sects, and, by definition, ab means their rectangle, thus avoiding measurement and the use of numbers. Ratio and proportion are strictly treated, but without limits. The book on two-dimensional spherics gives a novel method of treating spherics. His demonstration of the two-term prismoidal formula has been translated into French by the editors of a mathematical journal published in Belgium. Halsted is the first writer in this country to preface a geometry by a preliminary chapter on logic. Judged from a scientific point

of view, we believe Halsted's Geometry to be the peer of any geometry published in America.

Professor Lane has contributed one article on "Roulettes" to the American Journal of Mathematics, and has written a neat little book on Adjustments of the Compass, Transit and Level. Professor Lane taught chiefly the applied mathematics, *i. e.*, mathematics applied to engineering, and reached good results in his work. In June, 1888, he resigned his professorship, and his place was filled by the selection of a native Texan, T. U. Taylor. Professor Taylor is a graduate of the University of Virginia, and before accepting the present position was professor of pure and applied mathematics in the Miller Manual Labor School of Virginia.

The mathematical requirements for admission have been from the beginning the same as they are now, except that Prof. L. Brown examined students in Wentworth's Geometry instead of Halsted's. As stated in the catalogue of 1887–88, the terms for admission are as follows: "Arithmetic, including proportion, decimals, interest, discount, and the metric system; algebra, including theory of exponents, radicals, simple and quadratic equations; and the elements of plane geometry (corresponding to the first six books of Halsted's Geometry).

"Passing these examinations, a student will be admitted to the Freshman class in the course of science, or to the Junior class of the law department."

Great efforts are being made to cause the high schools in the State to work in line with the university. High schools desiring the privilege of sending their graduates to the university without examination are inspected by committees from the faculty of the university, and if the work of a school be found satisfactory the school is "approved." Thus far the number of irregular students in the academical department of the university has been large, but as the institution grows older, the students entering with a view of taking a four-years' course and graduating will doubtless rapidly increase.

During the first year of the university there were, naturally, no classes formed in the higher mathematics. At the beginning of the second year, in addition to the lower classes, there was a Sophomore class in analytic geometry, and a Junior class in differential and integral calculus. At the beginning of the third year, in addition to these, there was a Senior class in quaternions, and since then there have always been Freshman, Sophomore, Junior, and Senior classes in mathematics.

At the beginning, Wentworth's Algebra and Geometry were used by Professor Brown. When Professor Halsted entered upon his duties at this university he "found that the lack of rigor in Wentworth's Geometry was so exasperating" that he "could not continue to use it with comfort or a clear conscience," and so he put in form for the printer his own manuscript on geometry. His geometry has been used since its

issue, supplemented by Halsted's Mensuration. The analytic geometry used is Puckle's Conic Sections. Until the present year (1888–89) Byerly's Calculus has been taught. Post-graduate courses in mathematics are now offered to students.

The present mathematical course is as follows (catalogue 1887–88):

The *Freshman class* will study algebra, solid geometry, spherics, mensuration, plane and spherical trigonometry, with their applications to surveying, navigation, etc.

The *Sophomore class* will study analytical geometry, graphic algebra, and theory of equations.

The *Junior class* will study analytical geometry of three dimensions, differential and integral calculus. This course of study will embrace the applications of the calculus to mechanics and physics.

The *Senior class* will study determinants, quaternions, invariants, and quantics. * * *

In the higher classes will be discussed the history and logical structure of the mathematical sciences, and the logical theory of the calculus, the theory of limits, and the infinitesimal method.

Text-books.—Wentworth's Complete Algebra; Halsted's Geometry (John Wiley & Sons, New York); Halsted's Mensuration, 3d Ed. (Ginn & Co.); Wentworth's Trigonometry, Surveying, and Navigation; Graphic Algebra, by Phillips & Beebe; Puckle's Conic Sections, 5th Ed.; Smith's Solid Geometry; Newcomb's Differential and Integral Calculus; Theory of Equations, by Burnside and Panton, 2d Ed.; Muir's Determinants; Scott's Determinants; Salmon's Modern Higher Algebra, 4th Ed.; Hardy's Quaternions.

Engineering students are required to take the four-years' course; science students, the studies for the first three years; arts students, those of the first two years; and letters students, those of the first year.

Two post-graduate courses are offered:

I. A course preparatory to original investigation in the objective sciences. This will include infinitesimal calculus, the method of least squares, kinematic, linkage, differential equations, the calculus of finite differences.

Text books.—Williamson's Differential Calculus, Williamson's Integral Calculus, Clifford's Kinematic, Forsyth's Differential Equations, Boole's Differential Equations, Boole's Calculus of Finite Differences, Merriman's Method of Least Squares.

II. A course preparatory to original investigation in the subjective sciences. This will include projective geometry, the theory of numbers, the algebra of logic, the theory of probability, non-Euclidian geometry.

Text-books.—Cremona's Projective Geometry; Lejeune Dirichlet's *Zahlentheorie*, 3d Ed.; Macfarlane's Algebra of Logic; Boole's Laws of Thought; Todhunter's History of the Theory of Probability; Frischauf's *Absolute Geometrie.*

The catalogue for 1887–88 gives one student taking post-graduate studies in mathematics.

The university is open to both sexes. "A number of young ladies still show that they are capable of mastering even the abstruse modern developments of this oldest of the sciences." (Professor Halsted, June, 1888.)

WASHINGTON UNIVERSITY.

Up to the date of writing we have not been able to secure the information desirable for a sketch of the mathematical teaching at this university, but an excellent biographical notice of Professor William

Chauvenet, the first professor of mathematics at Washington University, has been written for us by his son, Regis Chauvenet, now president of the State School of Mines, at Golden, Colo. Professor William Chauvenet ranks among the coryphæi of science in America. He and Benjamin Peirce have done more for the advancement of mathematical and astronomical science, and for the *raising to a higher level* of the instruction in these subjects, than any other two Americans. It is our wish, on that account, to place before the reader a somewhat full sketch of the life and works of Professor William Chauvenet. The biographical notice above referred to is as follows:

"William Marc Chauvenet, father of the subject of this sketch, was born at Narbonne, France, in 1790, and came to the United States in 1816. He was the youngest of four brothers, another of whom also came to this country but has left no descendants. William Marc was a man of education and culture, versed in several languages, and a constant reader. He came to America, however, in connection with a manufacturing enterprise which had its headquarters in New York, with a branch at Boston. The latter department was under Mr. Chauvenet's charge, and here he married, in 1819, Miss Mary B. Kerr, of Roxbury. This was before a heavy defalcation in the New York house, which broke up the enterprise so badly that all investments in it proved to be total losses. Mr. Chauvenet having an idea that rural life would suit his taste, bought a small farm close to Milford, Pike County, Pa., and it was here that his only child, William Chauvenet, was born, May 24, 1820.

"By the advice of friends Mr. Chauvenet soon gave up his attempt at farming, and settled in Philadelphia, where his son grew to manhood. His rapid progress at school attracted such attention from his instructors, especially in mathematics, that his father easily yielded to their advice, and sent him to Yale College, where he graduated in 1840, '*facile princeps*' in mathematics, and high in standing in all other branches. The honorary societies, 'Phi Delta Kappa' and 'Chi Delta Theta,' denoting respectively the fifteen of highest standing and the fifteen best writers of the class, each claimed him as a member.

"Upon his return to his home he was, after a brief incumbency in a subordinate position, appointed professor of mathematics in the Navy. Late in 1841 he married Miss Catherine Hemple, of Philadelphia. Shortly after this he served a brief term on a United States vessel, as instructor to midshipmen, but did not go upon a foreign cruise, and was soon detailed to the 'Naval Asylum,' then situated at Philadelphia. Here midshipmen were sent at that time, to receive instruction and examinations, principally in mathematics and the theory of navigation. The young professor was struck with the imperfections in the education of naval officers, and it was very largely through his efforts, aided by such influences as he could bring to bear on the matter, that a commission was appointed to draft a plan for a fixed 'Naval Academy,' corre-

sponding to the Military Academy at West Point. Six naval officers constituted this commission, Professor Chauvenet being of the number. The appointment of so young a man (he was but twenty-four at the time) on a commission of such importance indicates what must have been his record, and the impression he made upon his seniors in years and rank.

"The Naval Academy was formally called into existence in the year 1845, being located at Annapolis, Md. Professor Chauvenet was appointed to the chair of mathematics, and resided at the academy until his resignation from the Navy in 1859.

"It was not long after this change of residence that he began to plan his work on trigonometry, which was published in 1850. Its title, 'A Treatise on Plane and Spherical Trigonometry,' partly indicated that it was not a students' class-book merely, but that it took up most of the more advanced applications of the subject. It soon assumed the position it still retains as the standard reference work in its line.

"Some time before this publication, Professor Chauvenet had persuaded his father to retire from business and accept a position at the academy. He came as instructor in the French language, and remained at his post until his death in 1855.

"It having been decided to erect an astronomical observatory at the academy, Professor Chauvenet was made professor of astronomy and put in charge of the observatory. As he became more and more interested in his work, the idea of his next treatise, 'Spherical and Practical Astronomy,' grew upon him, and, just previous to his resignation, had assumed such form that he issued a prospectus for its publication as a subscription work. This was never carried out.

"In 1859 he was notified that his application for the professorship of mathematics at Yale College would be followed by his election to that position. Almost simultaneously with this came a call to St. Louis, Mo., where he was offered the same chair in the then newly-established Washington University. After much deliberation he accepted the latter, and removed with his family (including at that time his mother) to St. Louis, in the fall of 1859.

"Chancellor Hoyt, who was at the head of the 'Washington' at this time, died early in the 'sixties,' and Professor Chauvenet was elected to the vacancy. He still continued his duties as professor of mathematics, however, and now resumed his work on the 'Astronomy.' The risks of publication were great, and his means did not enable him to guarantee the publishers against loss. The Civil War was in progress, and the time seemed inopportune for such an undertaking. It was to the liberality of certain friends, chiefly to the initiative of Mr. (afterward Judge) Thomas T. Gantt, of the St. Louis bar, that a guarantee fund was raised, sufficient in the opinion of the publishers to prevent any loss to them. The work, in two octavo volumes, was published in 1863.

"Few works of a scientific nature, by American authors, have been

received with such universal favor, by those competent to judge of its merits, as was this. Its reputation was quite as great in Europe as here, while of course it is not (as it was never intended to be) a treatise much known outside of scientific, and more especially astronomical, circles. Its scope, and the rigorous methods adopted, are sufficiently indicated in the author's preface. It retains to-day its standard character, as fully as when this was first recognized by the scientific world upon its publication.

"Professor Chauvenet's mother died in St. Louis, not long after the appearance of the Astronomy, and it was but a few months later that the first symptoms of the disease that proved finally fatal to him, made their appearance. Partial recovery and resumption of his duties was followed by a long period of alternating hopes and fears, during which time he tried in vain different parts of the United States, from South Carolina to Minnesota. During this illness he worked at his only elementary publication, the 'Geometry,' which he undertook, partly because he had long thought that the popular texts of the day were marked by too strict an adherence to strictly 'Euclidian' methods, and partly because he wished to provide an income for his family, by the publication of a text for which he had reason to suppose there would be a larger sale than was possible with advanced treatises. The publication of this work shortly antedated his death, which occurred at St. Paul, Minn., December 13, 1870.

"Professor Chauvenet left, so to speak, two distinct impressions behind him. By far the larger circle, in numbers, of those who knew him, were of those to whom his scientific attainments, though known, were as traditions merely, since they were in a field whose extent was to them only a matter of vague conjecture. To these he left the impression of a man of wide and varied culture, and keen critical taste. Probably few scientists of distinction were more keenly interested in lines outside of their own specialties. He was not only a critic in music, but to his latest day a pianist of no mean ability, always expressing a preference, in his own playing, for the works of Beethoven, which he rendered with an interpretation which never failed to excite the admiration of musicians whose execution surpassed his own. His knowledge of English literature was extensive, but he read and re-read a few authors, at least in the latter part of his life, and his great familiarity with many of these gave point to the old adage, 'fear the man of few books,' though perhaps not in the sense in which these words were originally intended. He was a ready writer, and contributed at times reviews, partly scientific, to various journals. His style was clear and unaffected, while, in the review of a pretentious or ignorant author, he had the gift of a delicate sarcasm, so light at times as only to be visible to one reading between the lines. For other pretenders he could drop this mask, and write with severity; but only twice in his life, to the knowledge of the present writer, did he ever do so. In addition to his more important writings,

he was the author of a 'Lunar Method,' still used in the Navy, and invented a device called the 'great circle protractor,' by which the navigator is enabled (knowing his position) to lay down his course on a 'great circle' of the globe, without further calculation. This invention was purchased by the United States Government not long after the close of the Civil War.

"Professor Chauvenet's scientific reputation needs little comment on the part of the present writer. He was one of a group of scientists in his own or cognate lines, who were the first to secure recognition abroad, as well as at home, for the position of the exact sciences in the United States. Among his more intimate scientific friends were Benjamin Peirce and Wolcott Gibbs (Harvard), Dr. B. A. Gould, and many others whose names are as household words in the history of scientific progress in this country. At the formation of the National Academy of Sciences he was one of the prominent members. But while his scientific reputation will outlast his personal memory, it is doubtful if to those who knew him, even of his scientific associates, it will ever be as present as his strong personal attractiveness, the result at once of an easy and varied culture, and of a simple dignity of character, which impressed alike his family, his friends, and his pupils. His family, consisting at the time of his death of his wife, four sons, and a daughter, are all still living (1889)."

The only mathematical book written by Chauvenet and not mentioned in the above sketch is a little book entitled Binomial Theorem and Logarithms, published in 1843 for the use of midshipmen at the Naval School, Philadelphia.

As regards the quality of Professor Chauvenet's books, Prof. T. H. Safford, of Williams College, says: "This excellent man and lucid writer was admirably adapted to promote mathematical study in this country. His father, a Frenchman of much culture, trained him very thoroughly in the knowledge of the French language, even in its niceties. They habitually corresponded in that language; and the son was enabled to study the mathematical writings of his ancestral country in a way which enabled him to reproduce in English their ease and grace of style, as well as their matter. In these respects his works are far more attractive than those of ordinary English writers; his Trigonometry is much the best work on the subject which I know of in any language; his Spherical and Practical Astronomy is frequently quoted by eminent continental astronomers; and his Geometry has raised the standard of our ordinary text-books, of which it is by far the best existing."*

Chauvenet's books, especially his Geometry, have been used in the best of our schools. Recently a revised edition of his Geometry has been brought out by Professor Byerly, of Harvard. Among the chief modifications made by him are the following: (1) The "exercises," which

* Mathematical Teaching, by Prof. T. H. Safford, 1887, p. 9.

in the original are at the end of the book, are most of them placed in direct connection with the theorems which they serve to illustrate. (2) The admirable little chapter in the original edition on "Modern Geometry" is omitted. (3) The "directional method" is introduced. The first is, no doubt, a change for the better; the second and third are, we think, to be regretted. It seems to us that the day has come when a college course should set aside some little time to the study of modern methods in elementary geometry, and not confine itself to the ancient. The introduction of the "directional method," in our opinion, robs the book of some of that admirable rigor for which the original work of Chauvenet is so justly celebrated.

His Trigonometry and Astronomy are the first American works to introduce the consideration of the *general spherical triangle*, in which the six parts of the triangle are not subjected to the condition that they shall each be less than 180°, but may have any values less than 360°. This feature is mainly due to Gauss. The methods of investigation followed in these two books are chiefly those of the German school, of which Bessel was the head.

UNIVERSITY OF MICHIGAN.*

The University of Michigan opened in 1841. In its organization Prussian ideas predominated. But the régime which existed during the first ten years in the history of the university did not prove efficient. A re-organization was therefore effected in 1852. The board of regents were, from that time on, rendered independent of the Legislature by intrusting their election to the people. The German method of governing the faculty by an annual president elected by that body was abandoned in 1852, and it was henceforth the duty of the board of regents to appoint a chancellor for the university.

The first appointment to a professorship at the University of Michigan was that of George Palmer Williams, in 1841. He was first assigned to the chair of ancient languages. On the work of this department, however, he did not enter, but exchanged it for that of mathematics and natural philosophy.

Professor Williams was born in Woodstock, Vt., in 1802. After graduating at the University of Vermont he studied theology at Andover, then became tutor at Kenyon College, and later professor of languages in the Western University of Pennsylvania. Thence he returned to Kenyon College, where he remained until 1837, when he entered upon the services of the board of regents of the University of Michigan, as principal of the Pontiac Branch.

At the University of Michigan he was professor of mathematics and natural philosophy until 1854, professor of mathematics from 1854 to

* For part of the information herein contained we are indebted to Prof. W. W. Beman, of Ann Arbor. The writer is also under obligation to Charles E. Lowrey, Ph. D., for interesting oral communications.

1863, and professor of physics from 1863 to 1875. Williams was a man of culture and refinement, and understood well the branches which he attempted to teach. As an instructor he lacked thoroughness. "Though he never felt himself called upon to force the reluctant mind into a thorough understanding of that for which it had no liking, he helped those who desired to study in attaining to the established standard, and, in a private way, he loved to aid those who desired his help in transcending that limit. Astronomy, though not nominally in his professorship, he taught until the revision of the course in 1854, and a great enthusiasm was annually awakened among the students as they came to the calculation of eclipses."*

The mathematical requirements for admission were, in 1847, arithmetic, and algebra through simple equations. The college course for that year included algebra, geometry, conic sections, plane and spherical geometry, and calculus. In 1848 it was the same, save calculus *or* analytical geometry, and in 1849 calculus *and* analytical geometry. The text-books were those of Professor Davies, of West Point.

Before its reorganization, in 1852, "the institution had flagged somewhat in popular interest; the number of its students had fallen off; a more vigorous and aggressive leadership was imperatively needed."† In the year just named, Dr. Henry P. Tappan, of New York, was inaugurated first chancellor. His connection with the university marks a new era in its history. During the reconstruction, German ideals were constantly kept in view. He thoroughly understood the workings of German universities and was a recognized champion among us of university education, as distinguished from college education. In the first catalogue (1852–53) issued by him, we read: "An institution can not deserve the name of a university which does not aim, in all the material of learning, in the professorships which it establishes, and in the whole scope of its provisions, to make it possible for every student to study what he pleases and to any extent he pleases. It is proposed, therefore, at as early a day as practicable, to open courses of lectures for those who have graduated at this or other institutions, and for those who in other ways have made such preparation as may enable them to attend upon them with advantage. These lectures, in accordance with the educational systems of Germany and France, will form the proper development of the university, in distinction to the college or gymnasium now in operation." The university system has been growing at Ann Arbor, though at first very slowly.

The first fruits of the plan laid down in the catalogue just named was the appointment to the chair of astronomy, in 1854, of Dr. Francis Brünnow, of Leipsic, a favorite pupil and assistant of the celebrated astronomer Encke. Brünnow remained at the university until 1863, when he resigned to take charge of the Dudley Observatory. Later, he

* University of Michigan, by Andrew T. Brook, 1875, p. 298.

† The Study of History in American Colleges, by Herbert B. Adams, p. 90.

became director of the Royal Observatory in Dublin, Ireland. Under his able management the observatory at the University of Michigan (called the Detroit Observatory, in recognition of the liberality of citizens of Detroit who founded it) soon rose to high rank. Besides the "Tables of Flora" and the "Tables of Victoria," published at Ann Arbor, Dr. Brünnow contributed to science his large work on Spherical Astronomy and many papers on astronomical subjects. But the influence of its renowned scholar was felt also in the department of pure mathematics. It is he who gave the university its start mathematically. When Professor Olney became a member of the faculty, then the university had already made a respectable beginning in the study of exact science.

The year 1856 marks the earliest dawn of the "elective system" at the University of Michigan. One of the elective studies offered to Seniors in that year was astronomy. Professor Brünnow lectured on this subject to an elective class of one—James C. Watson.* With reference to this class Dr. White happily said, that "that was the best audience that any professor in Michigan University ever had." Brünnow, with his pupil Watson, reminds us of Gauss, of Göttingen, who lectured at that great university to less than half a dozen students, while Thibaut, a mathematician of no scientific standing, presented the elements of mathematics to audiences of hundreds. "If I had the choice," said Hankel, "I should prefer being Gauss to Thibaut." If we had the choice, we should prefer being a Brünnow lecturing to one or two Watsons, rather than being very ordinary teachers lecturing to large classes of easy-going students.

Watson was born in upper Canada in 1838. He early exhibited extraordinary mental power and activity. When the lad was twelve his parents were anxiously casting about to secure for him the privileges of a liberal education. They looked eastward to Toronto and westward to Michigan. Being in humble circumstances, they chose the latter place, *because education there was free.* Young Watson entered at the Ann Arbor High School, but after an attendance of one day and a half he was graduated, for it was found that in the sciences he was altogether beyond anything which his teachers had thought of. The poverty of his parents made it necessary for him to partly rely upon his own support. At this time the future astronomer could be seen going about sawing wood for boys in college, while his mother took in washing to support herself and boy. At the university Watson displayed as much talent for languages as he did for mathematics. The story goes that he decided between mathematics and Greek, as his specialty, by throwing a penny. "There slips the penny, for which?" A noticeable exploit in the Junior year was his reading the entire Mécanique

* Our remarks on Professor Watson are drawn chiefly from an address delivered by Prof. J. C. Freeman, of the University of Wisconsin, and printed in the Ægis, Vol. I, No. 37, June 24, 1887.

Céleste of La Place. In the Senior year he took the course of lectures under Brünnow, spoken of above.

While yet very young, Watson contributed numerous astronomical and mathematical articles to foreign journals. He published in 1867, at the age of twenty-nine, his great work on Theoretical Astronomy. Its design appears from these prefatory words: "Having carefully read the works of the great masters, my plan was to prepare a complete work on the subject, commencing with the fundamental principles of dynamics and systematically treating, from one point of view, all the problems presented." The book gives a systematic derivation of the formulæ for calculating the geocentric and heliocentric places, and determining orbits, and for computing special perturbations, including also the method of least squares, together with a collection of auxiliary tables, etc. The work was translated into continental languages and became the text-book in many observatories in Germany, France, and England.

When Brünnow left Ann Arbor, in 1863, Watson became his successor. Watson discovered a considerable number of Asteroids. Twenty-three times, says Professor Freeman, he knew the joy felt by

> "Some watcher of the skies
> When a new planet swims into his ken."

He was led to believe that there existed between Mercury and the sun a planet hitherto unknown. During his observation of the eclipse in 1878, at Denver, he caught sight, as he thought, of this new planet.

Watson's genius made the University of Michigan known in scientific circles throughout the world. His mind was pre-eminently fitted for his specialty. With a powerful memory and great mechanical genius, he combined the ability to grasp abstruse problems by a kind of intuition. He was a man of wonderful activity. Says Professor Freeman: "There was a tireless energy in the man that impressed every beholder. Some of you recall the feeling you had when Grant or Sherman joined the army in the field, or when you saw Sheridan making his last mile from Winchester to Cedar Creek. Something of the same inspiration Watson gave his associates."

During his directorship of the observatory, Watson generally delivered every year to the student community a course of popular lectures, but was otherwise relieved from further duties of giving instruction, excepting to pupils intending to make astronomy their specialty. He had little patience with the average boy, but his interest in his special students never flagged. He took great pains to secure for them suitable positions. Old pupils of his may be found holding responsible positions in the U. S. Navy, Patent Office, and Coast Survey. His two most favorite pupils were George C. Comstock and John Martin Schaeberly. Watson took the former with him when he left Ann Arbor, in 1879, to take charge of the Washburn Observatory at the University of Wisconsin. Mr. Schaeberly remained at the Detroit Observatory until

1888, when he accepted a place at the Lick Observatory. He was succeeded at Ann Arbor by W. W. Campbell. After Watson left Ann Arbor, Prof. Mark W. Harrington became director of the observatory there.

During his first years after graduation, Watson taught, besides astronomy, mathematics and physics. Thus, from 1859 to 1860 he was professor of astronomy and instructor in mathematics; from 1860 to 1863, instructor in physics and mathematics. Other young instructors in mathematics of this time were W. P. Trowbridge, 1856 to 1857, a graduate of the U. S. Military Academy; and John Emory Clark, 1857 to 1859. Both of them became connected, later, with Yale College, the former as professor of mechanical engineering, the latter as professor of mathematics. These young men did much, no doubt, to supply that thoroughness which was wanting in the teaching of Professor Williams, the regular professor of mathematics. A beneficial stimulus to the study of pure mathematics was exerted by the department of engineering; for good work in that department was impossible without good preliminary instruction in pure mathematics. Connected with the department of civil engineering, from 1855 to 1857, was William Guy Peck, a graduate of West Point. He was succeeded by De Volson Wood, who had just graduated at the Rensselaer Polytechnic Institute. After leaving the University of Michigan, in 1872, Wood became professor of mathematics and mechanics in the Stevens Institute of Technology. He is the author of Resistance of Materials, Roofs and Bridges, Elementary Mechanics, Analytical Mechanics, revised edition of Mahan's Civil Engineering, and Elements of Coördinate Geometry (including Cartesian Geometry, Quaternions, and Modern Geometry). Professor Wood's text-books contain numerous examples to be worked by the student. These books possess many good features, and have been used quite extensively in our colleges and technical schools. Professor Wood has been a very diligent contributor to a large number of mathematical and scientific periodicals, and has thereby done much toward stimulating interest and activity in applied mathematics.

The year 1863 is marked in the history of the University of Michigan by the departure of Brünnow and the arrival of Olney. Prof. Edward Olney occupied the chair of mathematics until his death, in 1887. He was born in Moreau, Saratoga County, N. Y., in 1827. With slender opportunities for early education, he achieved through his own energy distinction as a teacher and scholar. He began his career as a teacher in elementary schools. Though he had himself never studied Latin, he began teaching it and he kept ahead of his class, "because he had more application." He thus educated himself in languages as well as in mathematics. He acquired great teaching power, and it is to this that his great success is chiefly due. During the ten years preceding his appointment at Ann Arbor, he was professor at Kalamazoo College, Michigan.

At the University of Michigan his teaching was marked by great thoroughness. He was a rather slow man, and took great pains with the poorer students. He had the happy faculty of inducing all students to perform faithful work. It is related that the son of a certain prominent Congressman once labored under the conceit that his father's reputation would exempt him from the necessity of studying whenever he felt disinclined to do so. Once, when being called upon to recite, he answered, "not prepared." Professor Olney assured him that the lesson was easy, asked him to rise from his seat, and then proceeded, much to the amusement of the rest of the class, to develop with him the entire lesson of the day by asking him questions. In that way was spent the whole hour. The class was made to assist him in some of the more difficult points. The Congressman's son concluded, on that occasion, that it was, after all, more agreeable to his feelings to prepare his mathematics carefully in his own room than to expose his ignorance before the whole class by being kept reciting for a whole hour. At times Professor Olney enjoyed joking at the expense of those who would not be injured by it. The result of his teaching was a high average standing among students. The first important step toward reaching good results consisted in a strict adherence to the requirements laid down for admission. If a student failed in his entrance examination, then Professor Olney took much pains to see that the deficiencies would be made up under a competent private teacher who was personally known to him. The rigid requirements for admission gave the mathematical department great leverage.

Professor Olney was an active promoter of various humanitarian enterprises, and was much interested in the educational work of the Baptist denomination, of which he was a member. He was interested in the progress of Kalamazoo College (Baptist) quite as much as in that of Michigan University. His library is now the property of that college. At the time of his death he was engaged in the revision of his series of text-books to meet the increased demands of the times.

In 1860, before Olney was connected with the university, the terms for admission were—to the classical course, arithmetic, and algebra through simple equations; to the scientific course, arithmetic, algebra through quadratic equations and radicals, and the first and third books of Davies' Legendre. In 1864 quadratic equations were added to the classical course, and to the scientific course the fourth book of Legendre. In 1867 the requirements for the classical course were raised so as to equal those in the scientific course, but in the following year quadratic equations were temporarily withdrawn. The fifth book of Legendre was added in the scientific course in 1869. In 1870 all of Legendre was required, and five books in the classical course. In the next year arithmetic, Olney's Complete Algebra, and Parts I and II of Olney's Geometry (including plane, solid, and spherical geometry), were the requirements in both courses. No changes have been made since.

The college curriculum in 1854 was, for both courses, algebra, geometry, trigonometry, analytical geometry, and calculus. The next year calculus was withdrawn from the classical course, but was re-instated in 1864, and in 1868 was made elective. In 1878 all courses except those for the degree of B. L. (English) embraced calculus. In 1881 the B. L. course included trigonometry. Since then calculus has been elective in all courses except the scientific. Analytical geometry has been added to the B. L. course.

During the last eight or ten years the "university system" has been growing rapidly at Ann Arbor. Mathematical studies of university grade have been offered. Determinants, quaternions, and modern analytical geometry were first announced in 1878; higher algebra in 1879; synthetic geometry and elliptic functions in 1885; theory of functions in 1886; differential equations (advanced) in 1887. The calculus of variations (probably as much as is contained in Church's or Courtenay's Calculus) was announced first in 1866.

The text-books which have been used at the University of Michigan, at different periods, are as follows:

Algebra.—Davies' Bourdon, Ray's—Part II, Olney's University Algebra, Newcomb's College Algebra, Chas. Smith's Treatise on Algebra, Salmon's Higher Algebra, Burnside and Panton.

Determinants.—Muir, Scott, Dostor, Peck.

Geometry.—Davies' Legendre, Olney, Ray.

Trigonometry.—Davies' Legendre, Loomis, Olney.

Synthetic Geometry.—Reye, Steiner.

Analytic Geometry.—Davies, Loomis, Church, Olney, Peirce's Curves, Functions, and Forces, Chas. Smith, Salmon, Frost, Aldis, Whitworth, Ciebsch.

Calculus.—Davies, Church, Loomis, Courtenay, Olney, Price, Todhunter, Williamson, Jordan.

Differential Equations.—Boole, Forsyth.

Calculus of Variations.—Todhunter, Carll.

Quaternions.—Kelland & Tait, Hardy, Tait.

Elliptic Functions.—Durége, Bobek, Jordan.

Prof. G. C. Comstock, of the Washburn Observatory, gives the following reminiscences of the mathematical instruction at Ann Arbor:*

"I entered the University of Michigan in the fall of 1873, with a preparation in mathematics consisting of arithmetic, elementary algebra through quadratic equations and including a very hurried view of logarithms, and plane, solid, and spherical geometry. The mathematical course given in the university at that time comprised, in the Freshman year, Olney's University Algebra, inventive geometry (consisting of an assignment of theorems for which the student was expected to find demonstrations), and plane and spherical trigonometry. In the Sophomore year, general geometry and differential and integral calculus. Descriptive geometry was required of engineering students, and was occasionally taught to others.

* Letter to the writer, November 6, 1888.

"The Freshmen were taught by instructors, usually young men of not much experience in teaching, but once a week they (the students) went up to Professor Olney for a review of the week's work, and these occasions were the trials of a Freshman's life. Olney's stern and rigid discipline had won for him among students the sobriquet "Old Toughy." He was not, however, a harsh man, and although the students stood in awe of him I think that he was generally liked by them. One feature of the weekly reviews may serve to illustrate his discipline and his power of enforcing it. He insisted upon the attention of each student being given to the demonstrations and explanations which the person reciting was engaged upon, and given so closely that the latter might be stopped at any point and any other student required to take up the demonstration at that point and carry it on without duplicating anything which had already been given.

"The University Algebra given the Freshman class contained an elementary view of infinitesimals, extending to the differentiation of algebraic functions and the use of Taylor's formula; and also a presentation of loci of equations, by which the student became familiar with the geometrical representation of an equation. The Sophomore thus came to this study of general geometry and calculus with some preliminary notions of these subjects. The study of the calculus was elective, but every Sophomore was required to take an elementary course in general geometry, and to make use here of the principles of the calculus which he had learned as a Freshman.

"Professor Olney's tastes were decidedly geometrical in character, and he constantly sought to translate analytical expressions into their geometrical equivalents, and much of his success as a teacher is probably due to this.

"Professor Beman, on the other hand, is an analyst, a 'lightning mathematician' in the student vernacular, and, in my day, the facility with which he handled mathematical expressions dazed and discouraged the student, who usually felt that he did not get much from Professor Beman.

"The criticism which I should now make upon the mathematical teaching which I received, is that little or no attempt was made to point out the applications of mathematics, and to encourage the student to apply it to those numerous problems of physical science, of engineering, and of navigation, which serve as powerful stimulants to the interest. The student was taught how to solve a spherical triangle, and how to look out logarithms from a table, but was never required to solve such a triangle and obtain numerical results.

"The text-books in use were those written by Professor Olney, none other being employed even for reference. There were no mathematical clubs or seminaries, and no facilities offered for the study of mathematics beyond the prescribed curriculum."

Professor Olney is the author of a complete set of mathematical text-books, which have displaced the works of Davies, Loomis, and Robinson in many schools, both in the East and in the West. His works are quite distinctive in the arrangement of subjects, and mark a decided advance over the other books just named. In the explanatory notes added here and there, in the tabular views at the end of chapters, in the judicious selection of examples, we see the fruits of long experience in the class-room. His books exhibit him in the light of a great teacher rather than a great mathematician. He was greatly aided in his work by Professor Beman, who prepared all the "keys" to the mathematical books, and did a great deal of critical work. It has been stated that Professor Olney could never get his publishers to print the books in the form which seemed the most perfect to him. He considered the traditional classification of mathematical subjects very defective, and wished to write a System of Mathematics in which he could embody his own ideals on this point. He thought, for example, that a considerable part of algebra should be taught before taking up the advanced parts of arithmetic, such as percentage and its applications, and that plane geometry should precede mensuration in arithmetic. By discarding the usual division of mathematics into separate volumes on arithmetic, algebra, geometry, etc., and by writing a system of mathematics he hoped to introduce great improvements. The publishers, on the other hand, preferred the traditional classification, as the books would then meet with larger sale. Professor Olney was thus hampered, to some extent, in the execution of his ideal scheme.

In his published works, the science of geometry is brought under two great heads, Special or Elementary Geometry, and General Geometry. The former consists of four parts: The *First Part* is an empirical geometry, designed as an introduction, in which the fundamental facts are illustrated but not demonstrated. The *Second Part* contains the elements of demonstrative geometry, designed for schools of lower grade. The *Third Part* was written to meet the special needs at the University of Michigan. It was studied in the Freshman class by students who had mastered the Second Part. The effort is made here to encourage original research. This part contains also applications of algebra to geometry, and an introduction to modern geometry. The *Fourth Part* consists of plane and spherical trigonometry, treated geometrically. The old "line-system" is still retained here.

General Geometry was intended to be developed by him in two separate volumes, but only the first was published. The first treats of plane loci, the second was intended for loci in space. This first volume may be very roughly described as covering the field generally occupied by analytical geometry and calculus. Olney favored the infinitesimal method, which he used also in his Elementary Geometry, where he permits the number of sides of a regular polygon circumscribed about a circle to become "infinite," and to coincide with the

circle. We are glad that this method is at the present time being more and more eliminated from *elementary* text-books. It is worthy of note that in his calculus Olney gives the elegant method, discovered by Prof. James C. Watson, of demonstrating the rule for differentiating a logarithm without the use of series.

In some courses the subjects have been taught exclusively by lectures, but the present tendency is to use the best text-book available, and supplement it with lectures as may be found advisable. Of late years a good deal of attention has been given to the careful and critical reading of such works as Salmon's Conic Sections, Higher Algebra, Geometry of Three Dimensions, Frost's Solid Geometry, Jordan's *Cours d'Analyse*, Forsyth's Differential Equations, Price's Calculus, Carll's Calculus of Variations, Burnside and Panton's Theory of Equations, Reye's *Geometrie der Lage*, Steiner's *Vorlesungen über synthetische Geometrie*, Clebsch's *Vorlesungen über Geometrie der Ebene*. It is thought that better results have been secured in this way than when the student's attention is largely given to the taking of notes.

Since the death of Professor Olney, Professor Beman has been filling the professorship of mathematics. He graduated at the University of Michigan in 1870. Excepting the first year after graduation (when he was instructor in Greek at another institution), he has been teaching continually at his *alma mater*—from 1871 to 1874 as instructor in mathematics, then as assistant professor and as associate professor of mathematics, and, finally, as full professor. He has done much toward introducing the "university system" in his department, and has been a contributor to our mathematical journals, particularly to the Analyst and the Annals of Mathematics.

For several years Charles N. Jones has been professor of applied mathematics. He has been a very successful teacher of mechanics. Professor Beman has two or three assistants in the department of pure mathematics.

A mathematical club was organized in 1887. It is under the control of the students, but an active interest is continually shown by the various instructors. Papers of some length are presented, problems discussed, etc.

UNIVERSITY OF WISCONSIN.

The University of Wisconsin was organized in 1848, and formally opened in 1850. A preparatory department was established in 1849, and it was not till 1851 that regular college classes were formed. Like most other State universities, the University of Wisconsin had a hard struggle for existence during its early years. Our State Legislatures did not always pursue a wise course toward their higher institutions of learning. The lands which were granted to the States by the General Government for the support of higher education were disposed of in a manner intended to "encourage immigration," rather than to foster a great

university. But in later years, say since 1875, the policy of the Wisconsin Legislature has been much more liberal, and the university has been advancing with prodigious strides.

The first professor connected with the institution was John W. Sterling. He was the teacher in the preparatory department, which was started in 1849, in a small building, before the university had any large buildings of its own. After the college department was organized, Sterling became professor of mathematics and natural philosophy, which position he retained until about 1867, when a separate chair was created for physics. From that time on until June, 1881, he was professor of mathematics.

Professor Sterling was born July 17, 1816, in Wyoming County, Pa., and died in March, 1885, at Madison, Wis. He was graduated at the College of New Jersey in 1840, and at the Princeton Theological Seminary in 1844. His mathematical and astronomical instruction at Princeton must have been received from Prof. A. B. Dod and Prof. Stephen Alexander. He went to Wisconsin in 1846, and became professor in Carroll College, Waukesha. Three years later he entered upon his long career as professor at the University of Wisconsin. For one-third of a century he was connected with that institution. Never did man work more faithfully than did he for its advancement. When the university was passing through the "*Sturm und Drang Periode*," and when it was without a head, he more than once, as dean of the faculty, assumed the duties of president. He was a man of industry and energy, and was ready to teach any branch, on an emergency. Among the students he was popular. He encouraged faint-hearted students, took them to his table, lent or gave them money when he had little himself. He invariably treated students like gentlemen of mature judgment and common sense. The great mass of students appreciated this, but occasionally there were some too young to do so and who should have received severer treatment and more summary action. In his prime Professor Sterling was a man of great physical strength. During even his last years he walked as erect as a young man of twenty.

He took a living interest in mathematics even during the last days of his life. Though he may not have kept pace with recent advances in this science, he had a good knowledge of such subjects as were treated in our ordinary American text-books. He never published any works of his own. When Professor Watson, the astronomer, came from Ann Arbor to the University of Wisconsin, in order to take charge of the magnificent new observatory erected by the munificence of Governor Washburn, an agreement was contemplated or reached between Watson and Sterling to prepare jointly a series of mathematical text-books. Watson's wonderful mathematical talent and Sterling's long experience in teaching would, indeed, have made a strong combination, but the scheme was frustrated by the untimely death, in 1880, of the great *astronomer*.

In the class-room Sterling's discipline was characterized by great mildness. He would carefully explain to the class the principal parts of each lesson. Even in the last year of his teaching his prelections were always very clear, and any student who felt a desire to understand the subjects which he taught, could certainly do so by following the exposition given in the class. While Professor Sterling always explained well, he was, in his last years at least, not sufficiently exacting; he would not compel a boy to study. The consequence was that some got from him a good knowledge of elementary mathematics, while others took advantage of the professor's leniency. In calculus he taught both the method of limits and the infinitesimal method. The text-book was based on the former, but Professor Sterling rather favored the latter. The principles of the calculus were not always unfolded with desired rigor, and not unfrequently some of the best scholars in the class shook their heads at the unceremonial rejection of quantities, simply because they were very, very small.

Among students Professor Sterling went by the familiar name of "Johnnie." In 1881 he was made professor emeritus of mathematics. Though his active duties in the class-room ceased at that time, he continued to take a living interest in all matters pertaining to the university to the end of his days.

We are not able to give the courses in mathematics during the early days of the university. During the last years of his teaching Professor Sterling used Loomis's works throughout. From a communication received by the writer from Prof. James D. Butler, it would appear that the works of Loomis were the first ones taught in pure mathematics by Professor Sterling. In algebra he used Loomis, afterward Davies, and then again Loomis. In conic sections he used at one time the work of Coffin. "Smith's Analytic Geometry" is also one of the books mentioned. This was most likely F. H. Smith's translation of Biot. Other books mentioned are Peck's Mechanics, Robinson's Astronomy, Snell's Olmsted's Astronomy, Snell's Olmsted in optics and pneumatics, and Loomis's Calculus.

In 1876–77 the Freshmen studied Loomis's Algebra, beginning with quadratics, Loomis's Geometry, Loomis's Plane Trigonometry. The Sophomores were instructed in Loomis's Conic Sections and Analytic Geometry, Practical Surveying (six weeks), and Calculus. This ended the course in pure mathematics. The Juniors were offered Peck's Mechanics and "lectures." The mathematical course for engineering students embraced also descriptive geometry (Church). All students were required to pursue mathematics through analytic geometry; the calculus was elective except to students in civil and mechanical engineering. Until about the year 1878, William J. L. Nicodemus was professor of military science and civil and mechanical engineering. He was spoken of by students as a man of great ability in his line. On the death of Nicodemus one of his pupils, Allan Darst Conover, assumed

charge of the department of civil engineering. In mechanical engineering the instruction fell into the hands of Storm Bull, a relative of the celebrated Ole Bull. Prof. Storm Bull studied at the Polytechnicum in Zurich, Switzerland, and is a thorough master of the subjects which he teaches. Descriptive geometry has been taught by him ever since his connection with the university.

On Sterling's retirement the management of the mathematical department was entrusted to Charles A. Van Velzer, a young man who for three years had listened to the inspiring words of Professor Sylvester at the Johns Hopkins University. Van Velzer graduated at Cornell University in 1876. After having been instructor at his *alma mater* for one year, he went to the Johns Hopkins University, where he was honored with a fellowship in mathematics. His power for original research is exhibited in his contributions to the American Journal of Mathematics (on "Compound Determinants"), the Johns Hopkins University Circulars, the Analyst, and the Mathematical Magazine.

In the fall of 1888 appeared, in two separate volumes, a preliminary edition of Van Velzer and Slichter's "Course in Algebra." Slichter is assistant professor of mathematics at the university. This book is now used in the Freshman class. The preliminary edition was gotten up for the purpose of being tested in the class-room. After the test, such revisions will be made as experience may seem to require. In the regular edition the two parts will be placed together in one volume. The work is not intended for beginners, but for students entering the Freshman class of our colleges, who already possess a fair knowledge of the elements. It impresses the progressive teacher as being different from most other works, and of great excellence. Many an antiquated and traditional notion has been thrown overboard, and many new features have been introduced. They have not been adopted simply for the sake of producing a book different from others; on the contrary, the authors have profited by what seemed good in other algebras.

The first volume of Van Velzer and Slichter's Algebra embraces, in addition to the usual subjects, the theory of limits and derivatives. In the treatment of series the authors not only state, but emphasize the fact that infinite series must be convergent in order to be used with safety. Some teachers might doubt the expediency of introducing Taylor's Formula into a book on algebra on account of the difficulty encountered in a complete and rigorous proof of it.

The second volume contains chapters on imaginaries, the discussion of the rational integral function of x, the solution of numerical equations of higher degree, graphic representation of equations, and determinants. These five chapters cover 75 pages. The treatment of these subjects appears to us admirable. Not more is given on each subject than can be conveniently taught in any college whose pupils possess a thorough *knowledge of* algebra through quadratics before entering the Freshman

class. A pleasant feature of the work is the occasional "historical notes." This is the first American work on algebra, as far as we know, which states explicitly that the logarithms invented by Napier are different from the natural logarithms.

The strongest feature of this algebra is its style. Students who have been in Professor Van Velzer's class-room will perceive that his great power of oral explanation and elucidation has been happily transferred to the printed page. Nowhere is the language of the book above the comprehension of ordinary students. The objective method of explanation is adopted throughout.

To Professor Van Velzer belongs the credit of introducing the modern higher mathematics into the University of Wisconsin. The writer knows of a student of great taste for mathematics, who studied Loomis's Calculus in the year preceding the arrival of Van Velzer, in Madison, and who labored under the impression that he had mastered about all that was to be known in *pure* mathematics. He was no little surprised when the new professor, fresh from the Johns Hopkins University, began to talk about determinants, quaternions, theory of functions, theory of numbers, and multiple algebra. The student's pride was wounded when he learned that Loomis's Calculus could convey only a very meagre knowledge of the transcendental analysis.

In 1883 some alterations were made in the mathematical requirements for admission. During the years immediately preceding, the requirements for all the courses of the university had been, arithmetic, algebra through quadratic equations, and plane geometry. At the time named above, solid geometry was added to the requisitions for all regular courses in the university except the "ancient classical."

The university has established close and friendly relations with the high schools in the State, and the number of "accredited high schools" is now fifty-six. Of these only six, however, prepare students for *all* courses in the university. This intimate relation with the high schools has had a wholesome influence upon both the university and the high schools.

As regards the regular classes of mathematics in the college, we may say that since the retirement of Professor Sterling, Loomis's Algebra has been retained until 1888. Now, Van Velzer and Slichter's Algebra is used. In trigonometry, Wheeler's work was introduced in 1882. Before that time Loomis's was taught. In solid geometry, Wentworth's has been used lately, in place of Loomis's. In analytical geometry, Loomis's work was superseded some years ago by the English work of Smith. In calculus, Professor Van Velzer has taught Byerly's, but this year (1888–89) he is using Newcomb's.

In the Sophomore year, and more especially in the Junior and Senior years, the elective system has been in operation, with some restrictions. Since 1881 elective studies in pure mathematics, covering the calculus and other branches, have been offered every year. There have always

been students with a taste for the higher branches of mathematics. In later years the attendance upon these branches has been on the increase. In the winter term of 1881–82 determinants were taught for the first time at the University of Wisconsin. In the spring term was organized a class of five or six students in quaternions. Hardy's text-book was used. Lately Professor Van Velzer has preferred the work by Kelland and Tait. During the year 1882–83 there was an elective class of about the same number as the preceding, studying Boole's Differential Equations. Professor Van Velzer's constant aim was to induce students to do independent work. He was always glad to listen to such modified treatment of the lesson in the book as the student might think of. This method of conducting the recitation gave rise to many interesting and profitable discussions. Considerable time was given to the subject of singular solutions. The work of Boole was still used in 1887–88, but from now on, that of Forsyth will be used, the former being out of print.

The special courses in pure mathematics during the last two years have been as follows: Class of two students in Boole's Differential Equations in the fall term of 1886–87, three hours per week; class of two in the same text-book, winter term of 1886–87, two hours per week; class of six in modern algebra (no text-book), winter term of 1886–87, three hours per week; class of six in Boole's Differential Equations, winter term of 1887–88, three hours per week; class of three in Boole's Differential Equations, spring term of 1887–88, three hours per week; class of six in Kelland and Tait's Quaternions, spring term of 1887–88, three hours per week; class of seven in Smith's Analytical Geometry of Three Dimensions, fall term of 1887–88, three hours per week; class of three in quantics (Salmon), fall term of 1887–88, two hours per week.

It is the practice at the University of Wisconsin to give special honors, upon the recommendation of the professors in the several departments, to the candidates for the bachelor's degree who have done special work under the direction of the professor of any department and prepared an acceptable thesis; but the amount of work required for a special honor must be at least the equivalent of a full study for one term, and in case of those branches in which there are longer or shorter elective courses, the student must have taken the longer course. It has been specified, furthermore, that candidates for special honors must have a general average standing of 85, and one of 93 per cent. in the department of which the application is made.

The number of special-honor students in mathematics in late years has been quite as great, if not greater, than in any other department, though the studies of this department are, to say the least, as difficult as those of any other. In the class of '83 there were three special-honor men in mathematics. The titles of their theses were as follows: "Singular Solutions of Differential Equations," "Pole and Polar and Reciprocal

Polars in Curves and Surfaces of the Second Order," and "Development and Dissection of Riemann's Surfaces." Should it be claimed that these theses are the work of immature students, then we may answer that for candidates for the bachelor's degree they are nevertheless creditable. The writer of the first thesis (L. M. Hoskins) is now doing excellent work as instructor in engineering at the university. The writer of the second thesis (L. S. Hulburt) is now professor of mathematics at the university of Dakota. M. Updegraff, of the class of '84, wrote a thesis on "Resultants." He holds now a responsible position at the National Observatory at Cordoba, Argentine Republic, South America. Titles of later "special-honor theses" are, "Approximation to the Roots of Numerical Equations," "Maxima and Minima," "On the Equation $\sin my. \cos ny = \sin mx. \cos nx$," "Different Systems of Co-ordinates." These theses are certainly indicative of a healthful activity in the under-graduate mathematical department.

For special studies pursued after graduation and the presentation of an acceptable thesis, the degree of Master is conferred. The following are the titles of two theses written to secure the degree of "master of science in mathematics:" "The Hodograph," "On a Quadratic Form" (in the theory of numbers).

The present courses in mathematics offered at the University of Wisconsin (catalogue 1887–88) are as follows:

Subcourse I, *Algebra*. Five exercises a week during the fall term. (Professor Van Velzer and Mr. Slichter.)
Required of Freshmen in all courses.

Subcourse II, *Theory of Equations*, including the elements of determinants, and graphic algebra. Five exercises a week during the winter term. (Professor Van Velzer and Mr. Slichter.)
Required of Freshmen in the Modern Classical, English, General Science, and Engineering Courses.

Subcourse III, *Solid Geometry*. Five exercises a week during the winter term. (Professor Van Velzer or Mr. Slichter.)
Required of Freshmen in the Ancient Classical Course.

Subcourse IV, *Trigonometry*. Five exercises a week during the spring term. (Professor Van Velzer and Mr. Slichter.)
Required of Freshmen in all courses.

Subcourse V, *Descriptive Geometry*. The topics taught embrace the projection of lines, planes, surfaces, and solids, the intersection of each of these with any one of the others, tangent lines to curves and surfaces and tangent planes to surfaces, problems in shades and shadows, of lines and surfaces, linear perspective and isometric projection. The class-room exercises are accompanied by work in the draughting room. The text-book used is Church's Descriptive Geometry. Full study during the spring term, Freshman year, and three-fifths study during the fall term, Sophomore year. (Professor Bull.)
Required of Freshmen in Civil and Mechanical Engineering. Elective for other students.

Subcourse VI, *Analytic Geometry*. Five exercises a week during the fall term. (Professor Van Velzer.)
Required of engineering Sophomores and scientific Sophomores who pursue mathematical, physical, or astronomical studies. Elective for other students.

Subcourse VII, *Differential Calculus.* Five exercises a week during the winter term. (Professor Van Velzer.)

Required of engineering Sophomores and scientific Sophomores who pursue mathematical, physical, or astronomical studies. Elective for other students.

Subcourse VIII, *Integral Calculus.* Five exercises a week during the spring term. (Professor Van Velzer.)

Required of engineering Sophomores and scientific Sophomores who pursue mathematical, physical, or astronomical studies. Elective for other students.

Subcourse XIX, *Method of Least Squares.* This is a course in the theory of probabilities as applied to the adjustment of errors of observation. It will be first given in 1889. Must be preceded by subcourses VI, VII, and VIII, three-fifths study during the winter term. (Mr. Hoskins.)

Required of Seniors in Civil Engineering.

Subcourses IX to XVIII, *special advanced electives.* Courses varying from year to year are offered in the following subjects: IX, *Modern Analytic Geometry;* X, *Higher Plane Curves;* XI, *Geometry of Three Dimensions;* XII, *Differential Equations;* XIII, *Spherical Harmonics;* XIV, *Elliptic Functions;* XV, *Theory of Functions;* XVI, *Theory of Numbers;* XVII, *Quantics;* and XVIII, *Quaternions.*

Very good work has been done, at times, by students in the department of mathematical physics. Prof. John E. Davies, the professor of physics, takes a living interest in pure as well as applied mathematics. His reading in pure mathematics has, indeed, been very extensive. Mathematical reading is a recreation to him. He would not unfrequently take with him some mathematical work—as, for instance, Tait's Quaternions—to faculty meetings, that he might pass pleasantly the otherwise tedious sessions of that august assembly. Many years ago he made, for his own use, a complete translation of Koenigsberger's work on Elliptic Functions.

The university offers excellent facilities for the study of astronomy. The Washburn Observatory has a large equatorial for use in original work, and also a smaller one for the use of students. After the death of Professor Watson, Professor Holden became director of the Observatory. He held this position until his appointment as director of the Lick Observatory. Prof. George C. Comstock is now professor of astronomy and associate director of the Washburn Observatory. Professor Comstock is a pupil of Watson, and came to Wisconsin from Ann Arbor with Watson. Before assuming the duties of his present position he was for two or three years professor of mathematics and astronomy at the University of Ohio.

The instruction in analytical mechanics is in charge of Mr. L. M. Hoskins, a young man of very marked mathematical talent. He graduated in 1883 at the head of a class of sixty-five, and was afterward appointed fellow in mathematics in Harvard University. Through his influence, the study of analytical mechanics had been made much more prominent in the engineering courses than it had been formerly. Two terms are now devoted to it instead of only one. Bowser's Elements of Analytical Mechanics is the text-book used.

Mr. Hoskins has contributed to the Annals of Mathematics, the *Mathematical* Magazine, and Van Nostrand's Engineering Magazine.

JOHNS HOPKINS UNIVERSITY.

President Daniel C. Gilman once said to the trustees of the Johns Hopkins University, when the question of "How to begin a university" was upon their minds, "Enlist a great mathematician and a distinguished Grecian; your problem will be solved. Such men can teach in a dwelling-house as well as in a palace. Part of the apparatus they will bring, part we will furnish. Other teachers will follow them."* So it came to pass that, before there were any buildings for classes, a professor of mathematics and a professor of Greek were secured for the new university.

When President Gilman was engaged in the all-important work of selecting men for the above positions, he may have been actuated in his choice by thoughts similar to those of Prof. G. Chrystal, who, before a learned body of English scientists, once expressed himself as follows:† "Science can not live among the people, and scientific education can not be more than a wordy rehearsal of dead text-books, unless we have living contact with the working minds of living men. It takes the hand of God to make a great mind, but contact with a great mind will make a little mind greater. The most valuable instruction in any art or science is to sit at the feet of a master, and the next best, to have contact of another who has himself been so instructed."

Is there a student among us who has studied with Sylvester and who will deny the truth of the above? Is there a mathematician, who has sat as a pupil at the feet of Benjamin Peirce, who will deny it? It is a fortunate circumstance for the progress of the exact sciences in this country that, at a time when the "Father of American Mathematics" was approaching his grave, there came among us another master who gave the study of mathematics a fresh and powerful impulse. Professor Sylvester is a mathematical genius, who has no superior in England, except, perhaps, Professor Cayley.

James Joseph Sylvester was born in London in 1814, and was educated at the University of Cambridge. He came to this country to fill the professorship at the University of Virginia when he was a very young man, but his stay among us then was very short. He became a member of the Royal Society at the age of twenty-five. For some time he was professor of natural philosophy in University College, London. In 1855 he became professor of mathematics in the Royal Military Academy at Woolwich, and in 1876 was elected for the position at the Johns Hopkins University.

Sylvester's activity has been wonderful. Prior to 1863 he published 112 scientific memoirs, which are recorded in the Royal Society's Index of Scientific Papers. A most important paper, printed in the Philo-

* Annual report of the president of the Johns Hopkins University, 1888, p. 29.

† *Nature*, September 10, 1885, Section A of Brit. Assocation, opening address by Prof. G. Chrystal, president of the section.

sophical Transactions of 1864, is Sylvester's Theorem on Newton's Rule for discovering the number of real and imaginary roots of an equation. Of this Todhunter says:* "If we consider the intrinsic beauty of the theorem, * * * the interest which belongs to the rule associated with the great name of Newton, and the long lapse of years during which the reason and extent of that rule remained undiscovered by mathematicians, among whom Maclaurin, Waring, and Euler are explicitly included, we must regard Professor Sylvester's investigations made to the theory of equations in modern times justly to be ranked with those of Fourier, Sturm, and Cauchy." A few of his numerous other investigations, made before coming to Baltimore, are on the Rotation of a Rigid Body; on the Analytical Development of Fresnel's Optical Theory of Crystals; on Reversion of Series; on the Involution of Six Lines in Space, "culminating in the result that if these six lines represent forces in equilibrium they must lie on a ruled cubic surface;" on a general theorem by which, for instance, the quintic can be expressed as the sum of three fifth powers. In 1859 he gave a course of lectures at King's College, London, on the subject of The Partitions of Numbers and the Solution of Simultaneous Equations in Integers, in which it fell to his lot "to show how the difficulties might be overcome which had previously baffled the efforts of mathematicians, and especially of one bearing no less venerable a name than that of Leonard Euler," and also laid the basis of a method which has since been carried out to a much greater extent in his "Constructive Theory of Partitions," published in the American Journal of Mathematics, in writing which he "received much valuable co-operation and material contributions" from his "pupils in the Johns Hopkins University."†

Professor Sylvester's most celebrated work has been in modern higher algebra. A very large portion of the theory of determinants is due to him, and the epoch-making theory of invariants owes its origin and early development almost exclusively to his genius and that of Professor Cayley.

The Johns Hopkins University offered to Professor Sylvester every facility for original work that could be desired. By the system of "fellowships" a number of talented young men were drawn to Baltimore, who were capable not only of understanding the teachings of their great master, but, in many cases, also of aiding him in his researches. The university, moreover, started the American Journal of Mathematics, in which all investigations in mathematics could be published and thereby brought before the mathematical public. Professor Sylvester's time was not taken up by the usual routine work in school, but was almost wholly given to the pursuit of his favorite subjects. He lectured, perhaps, two or three times per week, but these lectures generally disclosed some new discovery in algebra.

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* Theory of Equations, page 250.

† *Inaugural Lecture* delivered by Professor Sylvester before the University of Oxford, December 12, 1885, published in Nature, January 7, 1886.

Though he had passed his sixtieth year before he came to the Johns Hopkins University, his mind seemed to be as strong and active as ever. The group of students he had gathered about him were almost constantly made to feel the glow of new ideas or of old ones in a new form. From 1877 to 1882, Professor Sylvester contributed thirty articles and notes to the American Journal of Mathematics; twenty-two to the *Comptes Rendus de l'Académie des Sciences de l'Institut de France;* one paper to the Proceedings of the Royal Society, "On the Limits to the Order and Degree of the Fundamental Invariants of Binary Quantics" (1878); four to the Messenger of Mathematics; four to the London, Edinburgh, and Dublin Philosophical Magazine; six to the *Journal für reine und angewandte Mathematik*, Berlin.* If this list be complete, the number of original papers published by him while at the Johns Hopkins University was sixty-seven. Special mention may be made here of a proof by Professor Sylvester, printed in the Philosophical Magazine for 1878, of a theorem on the number of linearly independent differentiants, which had been awaiting proof for over a quarter of a century. He was led to undertake the investigation of this subject by a question put to him by one of his students in connection with a footnote given at one place in Faà de Bruno's *Théorie des Formes Binaires.*

Since his return to England, Sylvester has been developing a new subject, which he calls the "Method of Reciprocants." The lectures which he delivered on this subject at the University of Oxford have been reported by Mr. Hammond and published in the American Journal of Mathematics.

Sylvester has manufactured a large number of technical terms in mathematics. He himself speaks on this point as follows: "Perhaps I may, without immodesty, lay claim to the appellation of the mathematical Adam, as I believe that I have given more names (passed into general circulation) to the creatures of the mathematical reason than all the other mathematicians of the age combined."†

In his writings, Professor Sylvester is often very eloquent. His style is peculiarly flowery, and indicative of very powerful imagination. His articles are frequently interspersed with short pieces of poetry, either quoted or of his own composition. Thus, in his article in Nature, January, 1886, is given a short poem, "On a Missing Member of a Family Group of Terms in an Algebraical Formula;" followed by this sentence: "Having now refreshed ourselves and bathed the tips of our fingers in the Pierian spring, let us turn back for a few brief moments to a light banquet of the reason."

Since the beginning of the Johns Hopkins University, twenty fellowships have been open annually to competition, each yielding five hundred dollars and exempting the holder from all charges for tuition. This system was instituted for the purpose of affording to young men

* U. S. Bureau of Education, Circular of Information No. 1, 1888, p. 220.

† *Nature*, Dec. 15, 1887, p. 152, note.

of talent an opportunity of continuing their studies in the university, while looking forward to positions as professors, teachers, and investigators. They have been given to graduate students who showed particular aptitude for advanced work in their chosen specialty. During the time when Sylvester was connected with the university there were nearly always three or four fellowships granted to mathematical students, but, in recent years the number has been reduced to two, in consequence of an increase in the number of departments in the university, among which the fellowships must be divided. Among the first holders of fellowships in mathematics were Thomas Craig (1876–78), George B. Halsted (1876–78), Fabian Franklin (1877–79), W. I. Stringham (1878–80), C. A. Van Velzer (1878–81), all holding leading and responsible positions now, as professors of mathematics.

Professor Sylvester's first high class at the new university consisted of only one student, G. B. Halsted, who had persisted in urging Sylvester to lecture on the modern algebra. The attempt to lecture on this subject led him into new investigations in quantics. In his address on Commemoration Day at the Johns Hopkins, he spoke about this work as follows:

"This is the kind of investigation in which I have for the last month or two been immersed, and which I entertain great hopes of bringing to a successful issue. Why do I mention it here? It is to illustrate my opinion as to the invaluable aid of teaching to the teacher, in throwing him back upon his own thoughts and leading him to evolve new results from ideas that would have otherwise remained passive or dormant in his mind.

"But for the persistence of a student of this university in urging upon me his desire to study with me the modern algebra I should never have been led into this investigation; and the new facts and principles which I have discovered in regard to it (important facts, I believe), would, so far as I am concerned, have remained still hidden in the womb of time. In vain I represented to this inquisitive student that he would do better to take up some other subject lying less off the beaten track of study, such as the higher parts of the calculus or elliptic functions, or the theory of substitutions, or I wot not what besides. He stuck with perfect respectfulness, but with invincible pertinacity, to his point. He would have the new algebra (Heaven knows where he had heard about it, for it is almost unknown in this continent), that or nothing. I was obliged to yield, and what was the consequence? In trying to throw light upon an obscure explanation in our text-book, my brain took fire, I plunged with re-quickened zeal into a subject which I had for years abandoned, and found food for thoughts which have engaged my attention for a considerable time past, and will probably occupy all my powers of contemplation advantageously for several months to come."

This extract describes the beginning of his scientific activity and productiveness in the New World.

It may not be without interest to learn what some of his former pupils at the Johns Hopkins University have to say about him. Says Dr. G. B. Halsted:

"Young Americans could hardly realize that the great Sylvester, who with Cayley outranks all English-speaking mathematicians, was actually at work in our land. All young men who felt within themselves the divine longing of creative power hastened to Baltimore, made at once by this Euclid a new Alexandria. It was this great awakening and concentration of mathematical promise, and the subsequent facilities offered for publication of original work, which, rather than any teaching, made the American renaissance in mathematics. * * *

"A short, broad man of tremendous vitality, the physical type of Hereward, the Last of the English, and his brother-in-arms, Winter, Sylvester's capacious head was ever lost in the highest cloud-lands of pure mathematics. Often in the dead of night he would get his favorite pupil, that he might communicate the very last product of his creative thought. Everything he saw suggested to him something new in the higher algebra. This transmutation of everything into new mathematics was a revelation to those who knew him intimately. They began to do it themselves. His ease and fertility of invention proved a constant encouragement, while his contempt for provincial stupidities, such as the American hieroglyphics for π and e, which have even found their way into Webster's Dictionary, made each young worker apply to himself the strictest tests.

"To know him was to know one of the historic figures of all time, one of the immortals; and when he was really moved to speak, his eloquence equalled his genius. I never saw a more astonished man than James Russell Lowell listening to the impassioned oratory of Sylvester's address upon the bigotry of Christians.

"That the presence of such a man in America was epoch-making is not to be wondered at. His loss to us was a national misfortune."*

In answer to an inquiry about Sylvester's methods of teaching, Dr. E. W. Davis (fellow from 1882 to 1884) writes hurriedly as follows: "Sylvester's *methods!* He had none. 'Three lectures will be delivered on a New Universal Algebra,' he would say; then, 'The course must be extended to twelve.' It did last all the rest of that year. The following year the course was to be *Substitutions-Théorie*, by Netto. We all got the text. He lectured about three times, following the text closely and stopping sharp at the end of the hour. Then he began to think about matrices again. 'I must give one lecture a week on those,' he said. He could not confine himself to the hour, nor to the one lecture a week. Two weeks were passed, and Netto was forgotten entirely and never mentioned again. Statements like the following were not unfrequent in his lectures: 'I haven't proved this, but I am as sure as I can

* Letter to the writer, December 25, 1888.

be of anything that it must be so. From this it will follow, etc.' At the next lecture it turned out that what he was so sure of was false. Never mind, he kept on forever guessing and trying, and presently a wonderful discovery followed, then another and another. Afterward he would go back and work it all over again, and surprise us with all sorts of side lights. He then made another leap in the dark, more treasures were discovered, and so on forever."

Let us now listen to another of his old pupils, Mr. A. S. Hathaway (fellow from 1882 to 1884):

"I can see him now, with his white beard and few locks of gray hair, his forehead wrinkled o'er with thoughts, writing rapidly his figures and formulæ on the board, sometimes explaining as he wrote, while we, his listeners, caught the reflected sounds from the board. But stop, something is not right, he pauses, his hand goes to his forehead to help his thought, he goes over the work again, emphasizes the leading points, and finally discovers his difficulty. Perhaps it is some error in his figures, perhaps an oversight in the reasoning. Sometimes, however, the difficulty is not elucidated, and then there is not much to the rest of the lecture. But at the next lecture we would hear of some new discovery that was the outcome of that difficulty, and of some article for the Journal, which he had begun. If a text-book had been taken up at the beginning, with the intention of following it, that text-book was most likely doomed to oblivion for the rest of the term, or until the class had been made listeners to every new thought and principle that had sprung from the laboratory of his mind, in consequence of that first difficulty. Other difficulties would soon appear, so that no text-book could last more than half of the term. In this way his class listened to almost all of the work that subsequently appeared in the Journal. It seemed to be the quality of his mind that he must adhere to one subject. He would think about it, talk about it to his class, and finally write about it for the Journal. The merest accident might start him, but once started, every moment, every thought was given to it, and, as much as possible, he read what others had done in the same direction; but this last seemed to be his weak point; he could not read without meeting difficulties in the way of understanding the author. Thus, often his own work reproduced what others had done, and he did not find it out until too late.

"A notable example of this is his theory of cyclotomic functions, which he had reproduced in several foreign journals, only to find that he had been greatly anticipated by foreign authors. It was manifest, one of the critics said, that the learned professor had not read Kummer's elementary results in the theory of ideal primes. Yet Professor Smith's report on the theory of numbers, which contained a full synopsis of Kummer's theory, was Professor Sylvester's constant companion.

"This weakness of Professor Sylvester, in not being able to read what others had done, is perhaps a concomitant of his peculiar genius. Other *minds* could pass over little difficulties and not be troubled by them,

and so go on to a final understanding of the results of the author. But not so with him. A difficulty, however small, worried him, and he was sure to have difficulties until the subject had been worked over in his own way, to correspond with his own mode of thought. To read the work of others, meant therefore to him an almost independent development of it. Like the man whose pleasure in life is to pioneer the way for society into the forests, his rugged mind could derive satisfaction only in hewing out its own paths; and only when his efforts brought him into the uncleared fields of mathematics did he find his place in the Universe."

These reminiscences are extremely interesting, inasmuch as they show the workings of a great mind. The mathematical reader will surely enjoy the following reminiscences of "Silly," by one of his favorite pupils, Dr. W. P. Durfee, professor of mathematics at Hobart College, Geneva, N. Y. He was a fellow in mathematics from 1881 to 1883. Speaking of his recollections of Sylvester, he says:

"I don't know that I can do better than preface them by an account, as far as my memory serves me, of the work we did while I was at the Johns Hopkins University. I say *we*, as I always think of the whole staff as working together, so thoroughly did Sylvester inspire us all with the subject which was immediately interesting him. I went to Baltimore in October, 1881, as a fellow, and, though my previous mathematical training had been of the scantiest, I had the courage of ignorance and immediately began to attend Sylvester's lectures, while Mr. Davis and some others thought they would wait for a year and prepare themselves to profit by them. Sylvester began to lecture on the Theory of Numbers, and promised to follow Lejeune Dirichlet's book; he did so for, perhaps, six or eight lectures, when some discussion which came up led him off, and he interpolated lectures on the subject of frequency, and after some weeks interpolated something else in the midst of these. After some further interpolations he was led to the consideration of his Universal Algebra, and never finished any of the previous subjects. This finished the first year, and, although we had not received a systematic course of lectures on any subject, we had been led to take a living interest in several subjects, and, to my mind, were greatly gainers thereby. The second year, 1882–83, he started off on the subject of substitutions, but our experience was similar to that of the preceding year, and I can not now, after the six years which have intervened, disentangle the various topics that engaged his attention. Amongst others were Turey's series, partitions, and universal algebra. He could not lecture on a subject which was not at the same time engaging his attention. His lectures were generally the result of his thought for the preceding day or two, and often were suggested by ideas that came to him while talking. The one great advantage that this method had for his students was that everything was fresh, and we saw, as it were, the very genesis of his ideas. One could not help being inspired by such

teaching, and many of us were led to investigate on lines which he touched upon. He was always pleased at what one had to suggest, and generally bore interruptions with patience. He would often stop to discuss points that arose, and accepted our opinions as of some worth. I must qualify these latter statements somewhat, as he was apt to be partial, and it made all the difference in the world who it was that interrupted him.

"His manner of lecturing was highly rhetorical and elocutionary. When about to enunciate an important or remarkable statement he would draw himself up till he stood on the very tips of his toes, and in deep tones thunder out his sentences. He preached at us at such times, and not infrequently he wound up by quoting a few lines of poetry to impress on us the importance of what he had been declaring. I remember distinctly an incident that occurred when he was at work on his Universal Algebra. He had jumped to a conclusion which he was unable to prove by logical deduction. He stated this fact to us in the lecture, and then went on, "GENTLEMEN" [here he raised himself on his toes], "I am *certain* that my conclusion is correct. I will WAGER a hundred pounds to *one;* yes, I will WAGER my *life* on it." The capitals indicate when he rose on his toes and the italics when he rocked back on to his heels. In such bursts as these he always held his hands tightly clenched and close to his side, while his elbows stuck out in the plane of his body, so that his bended arm made an angle of about 140°.

"Personally I had considerable contact with him, as I did work under his direction that made it necessary for me to see him at his rooms. On such occasions I always made an engagement with him two or three days beforehand, and then at his request dropped him a postal, which reached him an hour or two before I went and reminded him that I was coming. I always found him interested in my work and full of suggestions.

"He had one very remarkable peculiarity. He seldom remembered theorems, propositions, etc., but had always to deduce them when he wished to use them. In this he was the very antithesis of Cayley, who was thoroughly conversant with everything that had been done in every branch of mathematics.

"I remember once submitting to Sylvester some investigations that I had been engaged on, and he immediately denied my first statement, saying that such a proposition had never been heard of, let alone proved. To his astonishment, I showed him a paper of his own in which he had proved the proposition; in fact, I believe the object of his paper had been the very proof which was so strange to him."

By request of Professor Sylvester, Professor Cayley, the Sadlerian professor of pure mathematics in Cambridge, England, was associated in the mathematical work of the Johns Hopkins University from January *to* June, 1882. Of him Dr. Durfee says: "His subject was Abel-

ian and Theta Functions, and he stuck closely to his text. While his work was of great interest and importance, he did not arouse enthusiasm and diffuse inspiration, as Sylvester did."

These were proud days for the Johns Hopkins University, when the two greatest living English mathematicians were lecturing within her walls.

The first associate appointed in mathematics, when the university first opened in 1876, was Dr. W. E. Story. He graduated at Harvard University in 1871, then studied for some years in Germany, receiving the degree of doctor of philosophy at the University of Leipsic in 1875. For one year preceding the opening of the Johns Hopkins University he was tutor of mathematics at Harvard. At the Johns Hopkins University his lectures and his original researches have been chiefly in geometry. He was for several years associate editor of the American Journal of Mathematics.

Dr. Story is not only an eminent mathematician, but also a good teacher. He is ever ready to give private interviews to students and to explain to them difficult points, or offer criticisms and suggestions upon original inquiries which the student may be engaged in. Dr. Story is an admirable lecturer, clear, logical, deliberate, proceeding step by step, so that the student may be sure to follow his reasoning. His work on the blackboard is written in an elegant hand, and is always scrupulously accurate. In 1884 the university secured a magnificent set of geometrical models for the study of surfaces. Some of these are often brought by Dr. Story into the lecture-room to illustrate his subject. In his lectures Dr. Story generally follows some particular text-book, such as Clebsch on Conic Sections, Salmon on Analytic Geometry of Three Dimensions, or Steiner on Synthetic Geometry, but he often brings in researches of more recent date, and also inquiries of his own.

Another member of the mathematical staff is Dr. Thomas Craig. He graduated with the degree of civil engineer at Lafayette College in 1875, was one of the first persons elected to a fellowship at the Johns Hopkins University, and in 1878 received the degree of doctor of philosophy. He began lecturing at the university when he was a student. After graduation he was connected for a short period with the U. S. Coast and Geodetic Survey, for which he prepared in 1879 a Treatise on the Mathematical Theory of Projections. During his stay in Washington he studied also Theory of Functions from the work of Königsberger, under the direction of Professor Newcomb, of the Nautical Almanac. Dr. Craig has made the theory of functions and differential equations his specialty. He has not only kept pace with the most recent rapid advances of these broad and deep subjects, but has added numerous contributions of his own. Most of them have appeared in the American Journal of Mathematics, while some have been published in foreign journals. He is working on subjects which are receiving extensive development in the hands of Fuchs, Hermite, Poincaré, Appel,

Darboux, Picard, and others. There seem to be altogether too few Americans interested in this line of work and prepared to participate in its advancement. The mind of Dr. Craig moves with great rapidity. A quick and brilliant student finds his lectures profitable and inspiring. Some of his courses on differential equations and the theory of functions are very advanced and difficult, and can be followed only by the maturest of students.

Dr. Craig associates with the students familiarly. It has been his practice to invite occasionally students to his house to spend a mathematical evening, when all sorts of subjects would be discussed in a free and easy style.

A somewhat more recent appointment as associate in mathematics is that of Dr. Fabian Franklin. He graduated at the Columbian University in 1869, was fellow in mathematics from 1877 to 1879, and received the degree of doctor of philosophy in 1880. He was appointed assistant in mathematics before taking his degree. Franklin always took great interest in Professor Sylvester's researches while the latter was at the Johns Hopkins University, and generally was at work on similar lines, while Dr. Story and Dr. Craig followed more generally lines of investigation of their own. Some of the articles printed in the American Journal of Mathematics have appeared under the joint authorship of Sylvester and Franklin. Professor Sylvester entertained the highest opinion of Dr. Franklin.

Dr. Franklin has done more teaching in the under-graduate department than the other members of the mathematical staff, for the reason that he excels them all in his power of imparting instruction. His teaching power is indeed great. It is seldom that a person of so high mathematical talent is as good an instructor of younger pupils. Dr. Franklin possesses a remarkably quick eye for short methods. The student seldom listens to one of his lectures in which proofs are not given in a shorter, simpler manner than in the book; seldom is a paper read in the Mathematical Society which is not followed, in the ensuing discussion, by suggestions by Dr. Franklin of a shorter method. His papers published in the American Journal of Mathematics display the same power. As a teacher Dr. Franklin is extremely popular among the students.

In Dr. Story, Dr. Craig, and Dr. Franklin, Professor Sylvester had an eminently efficient corps of fellow-laborers. Their mathematical researches have made their names favorably known wherever advanced mathematics finds a votary.

The instruction for graduates during the time that Professor Sylvester was connected with the university was as follows:*

Courses of Instruction, Hours per Week, and Attendance, 1876–83.

Determinants and Modern Algebra: Professor Sylvester, 1876–77, 2d half-year, 2 hrs. (7); 1877–78, 2 hrs. (5); 1878–79, 2 hrs. (8).

* *Eleventh Annual Report* of the President of the Johns Hopkins University, p. 49.

Theory of Numbers: Professor Sylvester, 1879–80, 2 hrs. (8); 1880–81, 2 hrs. (6); 1881–82, 1st half-year, 2 hrs. (7).
Theory of Partitions: Professor Sylvester, 1882–83, 2d half-year, 2 hrs. (10).
Algebra of Multiple Quantity: Professor Sylvester, 1881–82, 2d half-year, 2 hrs. (12); 1883–84, 1st half-year, 2 hrs. (6).
Theory of Substitutions: Professor Sylvester, 1882–83, 1st half-year, 2 hrs. (9).
Algebraical Geometry and Abelian and Theta Functions: Professor Cayley, 1881–82, 2d half-year, 2 hrs. (14).
Quaternions: Dr. Story, 1877–78, 2 hrs. (2); 1879–80, 3 hrs. (4); 1881–82, 3 hrs. (7); 1882–83, 2d half-year, 3 hrs. (4).
Higher Plane Curves: Dr. Story, 1880–81, 2 hrs. (5); 1881–82, 1st half year, 3 hrs. (1); 1883–84, 2 hrs. (2).
Solid Analytic Geometry (General Theory of Surfaces and Curves): Dr. Story, 1881–82, 2d half-year, 3 hrs. (1); 1882–83, 1st half-year, 3 hrs. (6).
Theory of Geometrical Congruences: Dr. Story, 1882–83, 2d half-year, 2 hrs. (4).
Modern Synthetic Geometry: Dr. Franklin, 1877–78, 2 hrs. (2).
Theory of Invariants: Dr. Story, 1882–83, 10 lectures (8); 1883–84, 3 hrs. (6).
Determinants: Dr. Franklin, 1880–81, 1st half-year, 2 hrs. (9); 1882–83, 20 lectures (9).
Modern Algebra: Dr. Franklin, 1880–81, 2d half-year, 2 hrs. (6); 1881–82, 2d half-year, 2 hrs. (6).
Elliptic Functions: Dr. Story, 1878–79, 2 hrs. (2); 1879–80 (continuation of the previous year's course), 3 hrs. (4); Dr. Craig, 1881–82, 3 hrs. (8); 1883–84, 3 hrs. (4).
Elliptic and Theta Functions: Dr. Craig, 1882–83, 3 hrs. (10); 1883–84, 3 hrs. (2).
General Theory of Functions, including Riemann's Theory: Dr. Craig, 1879–80, 30 lectures (2); 1880–81, 1st half-year, 3 hrs. (3).
Spherical Harmonics: Dr. Craig, 1878–79, 10 lectures (6); 1879–80, 20 lectures (6); 1881–82, 1st half-year, 2 hrs. (4); 1883–84, 2d half-year, 1 hr. (4).
Cylindric or Bessel's Functions: Dr. Craig, 1879–80, 10 lectures (2).
Partial Differential Equations: Dr. Craig, 1880–81, 2d half-year, 2 hrs. (5); 1881–82, 2d half-year, 3 hrs. (9); 1882–83, 2d half-year, 2 hrs. (2); 1883–84, 2d half-year, 2 hrs. (4).
Calculus of Variations: Dr. Craig, 1879–80, 12 lectures (9); 1881–82, 1st half-year, 2 hrs. (8); 1882–83, 1st half-year, 2 hrs. (6).
Definite Integrals: Dr. Craig, 1876–77, 1st half-year, 3 hrs. (5); 1882–83, 1st half-year, 2 hrs. (2).
Mathematical Astronomy: Dr. Story, 1877–78, 3 hrs. (2); 1882–83, 3 hrs. (2); 1883–84, 3 hrs. (2).
Elementary Mechanics: Dr. Craig, 1876–77, 2d half-year (8).
Statics: Dr. Franklin, 1882–83, 2d half-year, 3 hrs. (5).
Analytic Mechanics: Dr. Craig, 1877–78, 1st half-year (6); Dr. Story, 1880–81, 2d half-year, 2 hrs. (6); Dr. Craig, 1881–82, 1st half-year, 3 hrs. (8); 1882–83, 1st half-year, 3 hrs. (4); Dr. Franklin, 1883–84, 3 hrs (6).
Theoretical Dynamics: Dr. Craig, 1878–79, 15 lectures (6); 1883–84, 2 hrs (5).
Mathematical Theory of Elasticity: Dr. Story, 1876–77, 2d half-year, 2 hrs. (4); 1877–78, 2 hrs. (2); Dr. Craig, 1881–82, 3 hrs. (4); 1883–84, 2d half-year, 2 hrs. (3).
Hydrodynamics: Dr. Craig, 1878–79, 24 lectures (7); 1880–81, 1st half-year, 2 hrs. (6); 2d half-year, 4 hrs. (3); 1882–83, 2d half-year, 3 hrs. (5).
Mathematical Theory of Sound: Dr. Craig, 1883–84, 3 hrs. (5).

It may be of interest to give a list of the advanced students of mathematics during the seven years that Sylvester was connected with the Johns Hopkins University, and their present occupation. Dr. Craig and Dr. Franklin are, as we have seen, instructors in mathematics at the Johns Hopkins. The list continues as follows: G. B. Halsted,

professor of mathematics, University of Texas; W. I. Stringham, professor of mathematics, University of California; C. A. Van Velzer, professor of mathematics, University of Wisconsin; O. H. Mitchell, professor of mathematics, Marietta College, Ohio; R. W. Prentiss, in the office of the U. S. Nautical Almanac, Washington; H. M. Perry, instructor in mathematics, Cascadilla School, Ithaca, N. Y.; W. P. Durfee, professor of mathematics, Hobart College, N. Y.; G. S. Ely, examiner, U. S. Patent Office; E. W. Davis, professor of mathematics, University of South Carolina; A. S. Hathaway, instructor in mathematics, Cornell University; G. Bissing, examiner, U. S. Patent Office; A. L. Daniels, instructor in mathematics, Princeton College, 1883–84.

The success in training students for independent research has been very great. To convince himself of this, the reader need only examine the abstracts of papers prepared by students, which have been published in the Johns Hopkins Circulars and in the American Journal of Mathematics. Each one of the names given above will be found to appear repeatedly in those publications, as a contributor.

In December, 1883, Professor Sylvester started for England to enter upon his new duties as Savilian professor of geometry in the University of Oxford. The robe of the departing prophet dropped upon the shoulders of Professor Newcomb. No American would have been more worthy of succeeding Sylvester. As an astronomer his name has long shone with a luster which fills with pride every American breast.

Simon Newcomb was born in Wallace, Nova Scotia, in 1835. After being educated by his father he engaged for some time in teaching. He came to the United States at the age of eighteen, and was engaged for two years as teacher in Maryland. There he became acquainted with Joseph Henry, of the Smithsonian Institution, and Julius E. Hilgard, of the U. S. Coast Survey. Recognizing his talent for mathematics, they secured for him, in 1857, a position as computer on the Nautical Almanac, which was then published in Cambridge, Mass. In Cambridge he came under the influence of Prof. Benjamin Peirce. In the catalogues of 1856 and 1857 his name appears as a student of mathematics in the Sheffield Scientific School. He graduated in 1858, and continued as a graduate student for three years thereafter. He was then appointed professor of mathematics in the U. S. Navy, and stationed at the Naval Observatory. He was chief director of a commission created by Congress to observe the transit of Venus in 1874. In that year the Royal Society of England awarded him a gold medal for his Tables of Uranus and Neptune. In 1870 he undertook to investigate the errors of Hansen's Lunar Tables as compared with observations prior to 1850. The results of this onerous task were published in 1878. In the years 1880 to 1882 he and Michelson measured the velocity of light by operations on such a large scale and such refined methods as to throw in the shade all earlier efforts of a similar kind. For the purpose of this measurement they set up fixed and revolving mirrors on opposite shores of the Potomac, at a distance of nearly 4 kilometers.

Since 1877 he has been in charge of the office of the American Ephemeris and Nautical Almanac. Since 1867 that office has been in Washington, instead of Cambridge. Professor Newcomb's predecessor in this office was J. H. C. Coffin, who in 1877 was placed on the retired list, having been senior professor of mathematics in the Navy since 1848.

Professor Newcomb has quite a large corps of assistants in Washington. His researches in astronomy during the last ten or twelve years have been described in the Nation of September 6, 1888, as follows:

"The general object of this work is the determination of the form, size, and position of the orbits of all the large planets of the solar system, from the best and most recent observations, and the preparation of entirely new and uniform tables for predicting the future positions of these objects. The first of the four sections of the work relates to the general perturbations of the planets by each other, and the part already in hand comprises the four inner planets, Mercury, Venus, the Earth, and Mars, in which fourteen pairs of planets come into play. Twelve of these were completed some months since, and only the action of Jupiter on Venus and Mars remained undetermined. In the next place, the older observations of the planets must be recalculated, and thus problems constantly arise which can not be met by general rules. All the observations at Greenwich from 1765 to 1811 have been completely reduced with modern data. Earlier Greenwich observations were similarly treated by Dr. Auwers, of Berlin, who liberally presented the complete calculations as his contribution to the work of the Nautical Almanac Office. In a recent report of this work Professor Newcomb gives further details of his progress in the treatment of other classes of planetary observations.

"Following this collation of all the available observations of each planet, comes the theoretical preparation of their corresponding positions at the time of observation. This forms the most laborious and difficult part of the work; and had Leverrier's tables, the best hitherto, been used without modification, Professor Newcomb would have found it impractiable to complete it with the number of computers at his command; but he has skillfully avoided the difficulty by a reconstruction of Leverrier's work in such form that it should be much less laborious to use, while sufficiently accurate for the purpose required. These theoretical positions must next be compared directly with the observations, one by one, and the differences between the two are then, by suitable mathematical processes, construed as implying the nature and amount of certain corrections to the planet's motion in its orbit. More than a full year must still elapse, says Professor Newcomb, before the work on the four inner planets will have advanced to the stage where this direct comparison is ready to be made. There remain the four outer planets, on the two more important of which, Jupiter and Saturn, Mr. Hill, of the same office, has been engaged for many years, and his new theory

of their complicated motion is already in the printer's hands. The two outer planets, Uranus and Neptune, have not yet been begun."

Of Professor Newcomb's labors Professor Cayley has said: "Professor Newcomb's writings exhibit, all of them, a combination on the one hand of mathematical skill and power, and on the other of good hard work, devoted to the furtherance of astronomical science."

His book on Popular Astronomy (1877) is well known. It has been republished in England and translated into German. The treatise on Astronomy by Newcomb and Holden, and their "Shorter Course" on Astronomy, are works which have been introduced as text-books into our colleges almost universally.

Professor Newcomb's scientific work has not been confined to astronomy. He has carried on investigations on subjects purely mathematical. One of the most important is his article on "Elementary Theorems Relating to the Geometry of a Space of Three Dimensions and of Uniform Positive Curvature in the Fourth Dimension," published in Borchardt's Journal, Bd. 83, Berlin, 1877. Full extracts of this very important contribution to non-Euclidian geometry are given in the Enclyclopædia Britannica, article "Measurement." It is gratifying to know that through Professor Newcomb America has done something toward developing the far-reaching generalizations of non-Euclidian geometry and hyper-space. In Volume I of the American Journal of Mathematics he has a note "On a Class of Transformations which Surfaces may Undergo in Space of more than Three Dimensions," in which he shows, for instance, that if a fourth dimension were added to space, a closed material surface (or shell) could be turned inside out by simple flexure without either stretching or tearing. Later articles have been on the theory of errors in observations. In former years he also contributed to the Mathematical Monthly and the Analyst.

Professor Newcomb has written a series of college text-books on mathematics. In 1881 appeared his Algebra for Colleges and his Elements of Geometry; in 1882 his Trigonometry and Logarithms, and School Algebra; in 1884 his Analytical Geometry and Essentials of Trigonometry; in 1887 his Differential and Integral Calculus. These works have been favorably reviewed by the press, and are everywhere highly respected. Professor Newcomb's fundamental idea has been to lead up to new and strange conceptions by slow and gradual steps. "All mathematical conceptions require time to become engrafted upon the mind, and the more time the greater their abstruseness." The student is gradually made familiar in these books with the conceptions of variables, functions, increments, infinitesimals, and limits, long before he takes up the calculus, so in the study of the calculus he is not confronted, at the outset and all at once, by a host of new and strange ideas, but possesses already a considerable degree of familiarity with them. With the publication of Newcomb's Algebra has begun a considerable "shaking" of the "dry bones" in this science, and we now possess works on this subject that are of considerable merit.

Professor Newcomb studies political economy as a recreation, and every now and then there is a commotion in the camp of political economists, caused by a bomb thrown into their midst by Professor Newcomb, in the form of some magazine article or book.

In 1884 Professor Newcomb added to his duties as superintendent of the Nautical Almanac that of professor of mathematics and astronomy at the Johns Hopkins University. He generally delivers at that institution two lectures per week. The effect of his connection with the mathematical department has been that the mathematical course is more thoroughly systematized and more carefully graded than formerly, and that the attention of students is drawn also to higher astronomy, theoretical and practical. An observatory for instruction is now provided by the university. Besides a telescope of 9½ inches aperture there is a meridian circle with collimators, mercury-basin, and other appliances. Professor Newcomb entered upon his duties at the Johns Hopkins University in 1884 by giving a course of lectures on celestial mechanics. Among other things it embraced his own paper on the "Development of the Perturbative Function and its Derivative in Sines and Co-sines of the Eccentric Anomaly and in Powers of the Eccentricities and Inclinations." The lectures were well attended by the graduate students. At the blackboard Professor Newcomb does not manipulate the crayon with so great dexterity as do his associates, who have been in the lecture-room all their lives, but his lectures are clear, instructive, original, and popular among the students.

Since the departure of Professor Sylvester the following courses of lectures have been given to graduate students:

Courses of Instruction, Hours per Week, and Attendance, 1884–'88.

Analytical and Celestial Mechanics: Prof. Newcomb, 1884–'85, 2 hrs. (11).
Practical and Theoretical Astronomy: Prof. Newcomb, 1885–'86, 2 hrs. (9); 1886–'87, 2 hrs. (7).
Theory of Special Perturbations: Prof. Newcomb, 1887–'88, 1st half-year, 2 hrs.
History of Astronomy: Prof. Newcomb, 1887–'88, March and April, 2 hrs.
Computation of Orbits: Prof. Newcomb, 1887–'88, May, 2 hrs.
Theory of Numbers: Dr. Story, 1884–'85, 1st half-year, 2 hrs. (9).
Modern Synthetic Geometry: Dr. Story, 1884–'85, 1st half-year, 3 hrs. (8).
Introductory Course for Graduates: Dr. Story, 1884–'85, 5 hrs. (10); 1885–'86, 5 hrs. (7); 1886–'87, 5 hrs. (10); 1887–'88, 5 hrs.
Modern Algebra: Dr. Story, 1884–'85, 2d half-year, 2 hrs. (9).
Quaternions: Dr. Story, 1884–'85, 2d half-year, 3 hrs. (8); 1886–'87. 3 hrs. (5); 1887–'88, 3 hrs.
Finite Differences and Interpolation: Dr. Story, 1885–'86, 1st half-year, 2 hrs. (5).
Advanced Analytic Geometry: Dr. Story, 1885–'86, 3 hrs. (4); 1886–'87, 2 hrs. (8); 1887–'88, 2 hrs.
Theory of Probabilities: Dr. Story, 1885–'86, 2d half-year, 2 hrs. (5).
Calculus of Variations: Dr. Craig, 1884–'85, 1st half-year, 2 hrs. (5).
Theory of Functions: Dr. Craig, 1884–'85, 3 hrs. (5); 1885–'86, 1st half-year, 3 hrs. (4); 1886–'87, 3 hrs. (6); 1887–'88, 1st half-year, 3 hrs.
Hydrodynamics: Dr. Craig, 1884–'85, 1st half-year, 3 hrs. (6); 1885–'86, 1st half-year, 3 hrs. (4); 1886–'87, 1st half-year, 3 hrs. (4); 1887–'88, 1st half-year, 3 hrs.

Linear Differential Equations: Dr. Craig, 1884–'85, 2d half-year, 3 hrs. (3); 1885–'86, 2 hrs. (4); 1887–'88, 2d half-year, 2 hrs.
Theoretical Dynamics: Dr. Craig, 1887–'88, 2d half-year, 2 hrs.
Differential Equations: Dr. Craig, 1887–'88, 2 hrs.
Mathematical Theory of Elasticity: Dr. Craig, 1885–'86, 2d half-year, 3 hrs. (4).
Elliptic and Abelian Functions: Dr. Craig, 1885–'86, 2d half-year, 3 hrs. (4); 1886–'87, 1st half-year, 2 hrs. (6).
Abelian Functions: Dr. Craig, 1887–'88, 2 hrs.
Problems in Mechanics: Dr. Franklin, 1884–'85, 2 hrs. (5); 1885–'86, 2 hrs. (6); 1886–'87, 2 hrs. (8); 1887–'88, 2 hrs.

Since the fall of 1884 Dr. Story has been giving every year an Introductory Course to graduate students, which consists of short courses of lectures on the leading branches of higher mathematics. They are intended to give the student a general view of the whole field, which afterward he is to enter upon and study in its details.

The Johns Hopkins University went into operation primarily as a *University*, giving instruction to students who had graduated from college. A regular college course was, however, organized, and it has been growing rapidly from year to year. In the college the student has the choice between several parallel curricula, which are assumed to be equally honorable, liberal, and difficult, and which therefore lead to the same degree of bachelor of arts. Seven groups have been arranged. Some of them embrace no mathematics at all; but, in those courses where it does enter, the instruction is very thorough. Take, for instance, Dr. Story's lectures on conic sections; the method of treatment is entirely modern, and presupposes a knowledge of determinants. A syllabus has been prepared for the use of the students. The lectures resemble the course given in the work of Clebsch. The student who may have studied such books as Loomis's Analytical Geometry, and who may labor with the conceit that he has mastered analytical geometry and conic sections, will soon discover that he has learned only the A B C, and that he is wholly ignorant of the more elegant methods of modern times.

Connected with the mathematical department of the university has always been a mathematical seminary, which during the time of Sylvester constituted in fact the mathematical society of the university. The meetings were held monthly. In it the instructors and more advanced students would present and discuss their original researches. Care was taken to eliminate papers of little or no value by immature students. Professor Sylvester generally presided. "If you were fortunate," says Dr. E. W. Davis, "you had your paper first on the program. Short it must be and to the point. Sylvester would be pleased. Then came his paper, or two of them. After him came the rest, but no show did theystand; Sylvester was dreaming of his own higher flights and where they would yet carry him."

Since the time of Newcomb this mathematical seminary has been called the Mathematical Society. It is carried on in the same way as *before. Three* mathematical seminaries proper have since existed, one

conducted by Professor Newcomb, another by Dr. Story, and the third by Dr. Craig. The meetings are held in the evening, and weekly. Each instructor selects for his seminary topics from his special studies; Newcomb, astronomical subjects; Story, geometrical subjects or quaternions; Craig, theory of functions or differential equations. Professor Newcomb's seminary work is closely connected with his lectures. The student elaborates some particular points of the lectures or makes practical application of the principles involved. In one case the computation of the orbit of a comet was taken up. Dr. Story, in the year 1885–86, took up the subject of plane curves for his seminary, and dwelt considerably on quartic and quintic curves, giving matter from Möbius and Zeuthen, and the result of his own study on quintics. The student was expected, if possible, to begin where he had left off and carry on investigations along lines pointed out by him. Dr. Story's talk on this subject in this seminary suggested to one of the students a subject of a thesis for the doctor's degree. In the fall of 1888 Dr. Story began his seminary work with the seventeenth example, p. 103, in Tait's Quaternions. Dr. Craig's seminary has generally been upon subjects in continuation and extension of those upon which he is lecturing at the time. If, for instance, he is lecturing on functions, following the "*Cours de M. Hermite*," he may in his seminary bring up matter from Briot and Bouquet. At other times he has introduced work into his seminary intended to be preparatory to certain advanced courses which he expected to offer.

MATHEMATICAL JOURNALS.

The mathematical journals which we are about to discuss were of a much higher grade than those of preceding years. First in order of time is the *Mathematical Miscellany*, a semi-annual publication, edited by Charles Gill. He was teacher of mathematics at the St. Paul's Collegiate Institute at Flushing, Long Island. Eight numbers were published; the first in February, 1836, and the last in November, 1839. Like many other journals of this kind, it had a Junior and Senior department—the former for young students, the latter for those more advanced. The first number was entirely the work of the editor, excepting two or three new problems. Mr. Gill was much interested in Diophantine analysis. In 1848 he published a little book on the Application of Angular Analysis to the Solution of Indeterminate Problems of the Second Degree, which contains some of his investigations on this subject.

Another enthusiastic worker in the field of Diophantine analysis, and a frequent contributor to Gill's journal, was William Lenhart, a favorite pupil of Robert Adrain. Having been afflicted for twenty-eight years with a spasmodic affection of the limbs, occasioned by a fall in early life, which confined him in a measure to his room, he had devoted a considerable portion of his time to Diophantine analysis. To him is

attributed the solution of the problem, to divide unity into six parts such that, if unity be added to each, the sums will be cubes.

The evident defect in Lenhart's processes was their tentative character. In fact, this criticism applies to all work done in Diophantine analysis by American computers, down to the present time. It is true even of old Diophantus himself. To this ancient Alexandrian algebraist, who is the author of the earliest treatise on algebra extant, as well as to his American followers of modern times, general methods were quite unknown. Each problem has its own distinct method, which is often useless for the most closely related problems. It has been remarked by H. Hankel that, after having studied one hundred solutions of Diophantus, it is difficult to solve the one hundred and first. It is to be regretted that American students should have wasted so much time over Diophantine analysis, instead of falling in line with European workers in the theory of numbers as developed by Gauss and others. Previous to the publication of the American Journal of Mathematics, our journals contained no contributions whatever on the theory of numbers, excepting the Mathematical Miscellany, which had some few articles by Benjamin Peirce and Theodore Strong, which involved Gaussian methods. Among the contributors to the Mathematical Miscellany were Theodore Strong, Benjamin Peirce, Charles Avery, Marcus Catlin of Hamilton College, George R. Perkins, O. Root, William Lenhart, Lyman Abbott, jr., B. Docharty, and others.

The next mathematical periodical was the *Cambridge Miscellany of Mathematics, Physics, and Astronomy*, edited by Benjamin Peirce and Joseph Lovering, of Harvard, and published quarterly. The last problems proposed in Gill's journal were solved here. Four numbers only were published, the first in 1842. The list of contributors to this journal was about the same as to the preceding. The most valuable articles were those written by the editors.

During the next fifteen years America was without a mathematical journal; but in 1858, J. D. Runkle, of the Nautical Almanac office in Boston, started the *Mathematical Monthly*. He has since held the distinguished position of professor of mathematics at (and, for a time, president of) the Massachusetts Institute of Technology, where he has been especially interested in developing the department of manual training. As will be seen presently, the time for beginning the publication of a long-lived mathematical journal was not opportune. Three volumes only appeared. On a fly-leaf the editor announced to his subscribers that over one third of the subscribers to Volume I discontinued their subscriptions at the close. "I have supposed," he says, "that those who continued their subscription to the second volume would not be so likely to discontinue it to the third volume, and I have made my arrangements accordingly. If, however, any considerable number should discontinue now, it will be subject to very serious loss. * * * *I ask as a favor* for all to continue *to* Volume III, and notify me during

the year if they intend to discontinue at its close. I shall then know whether to begin the fourth volume. I shall not realize a dollar." This announcement discloses obstacles which all our journals that have been dependent entirely upon their subscribers for financial support have had to encounter, and which none except the more recent could long resist. Moreover, the Civil War was now at hand. "On account of the present disturbed state of public affairs, the publication of the Mathematical Monthly will be discontinued until further notice." This was the end of the Monthly, in 1861.

The salient features in the plan upon which the periodical was conducted, as stated by David S. Hart,* were: "The publication of five problems in each number, adapted to the capacities of the young students, to be answered in the third succeeding number. The insertion of notes and queries, short discussions and articles of a fragmentary character, too valuable to be lost; and, lastly, essays not exceeding eight pages, on various subjects, in all departments of mathematics. Besides, there were notices and reviews of the mathematical works issued, both old and new. Among the most interesting articles are the account of the comet of Donati, with elegant descriptive plates, written by the astronomical professor of Harvard University (Vol. I, Nos. 2 and 3); a complete catalogue of the writings of John Herschel (Vol. III, No. 7); articles on indeterminate analysis, by Rev. A. D. Wheeler, of Brunswick, Me. (Vol. II, Nos. 1, 6, and 12), and the Diophantine analysis (Vol. III, No. 11). Other articles on the Diophantine analysis by Mr. Wheeler would have been inserted, if the Mathematical Monthly had been continued. 'The Economy and Symmetry of the Honey-bees' Cells,' by Chauncey Wright (Vol. II, No. 9). Simon Newcomb gives several interesting 'Notes on Probabilities.' In Vol. II, No. 2, there is an article containing a complete list of the writings of Nathaniel Bowditch, accompanied with short sketches of the same, which is extremely interesting. * * * The periodical is embellished by portraits of N. Bowditch, Prof. Benjamin Peirce, and Sir John Herschel, which are finely executed." The Monthly presented a very neat appearance to the eye. In the mathematical notation employed and in the treatment of mathematical subjects, Benjamin Peirce's influence was clearly perceptible. From a scientific point of view, the Monthly excelled any of its predecessors.

Since 1861, we had no mathematical periodical in the United States for thirteen years. In January, 1874, was published in Des Moines, Iowa, *The Analyst: A Monthly Journal of Pure and Applied Mathematics*, edited and published by Joel E. Hendricks, a self-taught mathematician. Mr. Hendricks did the printing of the journal himself. It continued until November, 1883. No previous journal of mathematics in this country had been published regularly for so long a time as this. Its long life and beneficial influence are due to a very great extent to

* Analyst, Vol. II, No. 5, p. 131, Des Moines, Iowa.

the untiring energy and self-sacrificing interest of its editor. Its discontinuance, after nine years, was not due to want of support, but to the failing health and strength of Mr. Hendricks. At first it appeared monthly, afterward bi-monthly.

The list of contributors included the most prominent teachers of mathematics in this country. The names were no longer those found in the Mathematical Miscellany or Cambridge Miscellany. A new generation of workers had come.

As in previous periodicals, so in this, a great part of each number was devoted to problems. Though the solution of problems is the lowest form of mathematical research, it is, nevertheless, important, not for its scientific, but for its educational value. It induced teachers to look beyond the text-book and to attempt work of their own. The Analyst bears evidence, moreover, of an approaching departure from antiquated views and methods, of a tendency among teachers to look into the history and philosophy of mathematics and to familiarize themselves with the researches of foreign investigators of this century. Thus, discussions regarding the fundamental principles of the differential calculus were carried on. Levi W. Meech gave an "Educational Testimony Concerning the Calculus;" W. D. Wilson, of Cornell, gave "A New Method of Finding Differentials;" Joseph Ficklin, of Missouri, showed how one might "find the differential of a variable quantity without the use of infinitesimals or limits;" C. H. Judson, of South Carolina, gave a valuable "investigation of the mathematical relations of zero and infinity," which displayed the wholesome effects of the study of such authors as De Morgan. Judson dealt powerful blows against the reckless reasoning that had been in vogue so long, but, during an occasional unguarded moment, he was hit by his opponents in return. De Volson Wood, of the Stevens Institute, and Simon Newcomb, of Washington, discussed the doctrine of limits.

Another subject considered in the Analyst was the possibility of an algebraic solution of equations of the fifth degree. A. B. Nelson, professor of mathematics in Centre College, Danville, Ky., translated from the German an article written in 1861 by Adolph Von Der Schulenburg, entitled, "Solution of the General Equation of the Fifth Degree." The translation and publication of it seem to have been called forth by a statement of W. D. Henkle in the Educational Notes and Queries, to the effect that proofs of the impossibility of such a solution had been given by Abel and Wantzel. Nelson's paper was followed by a translation from Serret's *Cours d'Algèbre Supérieure*, by Alexander Evans, of Elkton, Md., of Wantzel's "Demonstration of the Impossibility of Resolving Algebraically General Equations of a Degree Higher than the Fourth." Evans also contributed the (non-algebraic) "Solution of the Equation of the Fifth Degree," translated from the Theory of Elliptic Functions of Briot and Bouquet. W. E. Heal, of Wheeling, Ind., followed with an article pointing out the error in Schulenburg's

solution. One might have supposed that this question had now come to a rest, but not so. About two years later T. S. E. Dixon, of Chicago, thought he had found a solution, and he published it in the Analyst, but, in the next number, he stated that he had discovered "the weak link in the chain" of its logic.

Of the articles on modern higher mathematics, we mention the "Brief Account of the Essential Features of Grassmann's Extensive Algebra," by W. W. Beman, of Ann Arbor; "Symmetrical Functions, etc.," and "Recent Results in the Study of Linkages," by W. W. Johnson, and an article on determinants by C. A. Van Velzer, of the University of Wisconsin.

Among the historical papers is the very complete and interesting "Historical Sketch of American Mathematical Periodicals," by David S. Hart, of Stonington, Conn.; a "History of the Method of Least Squares," by M. Merriman. Merriman also published Robert Adrain's second proof of the principle.

Among other articles of interest are "Multisection of Angles," and "A Singular Value of π," by J. W. Nicholson, of The Louisiana State University, at Baton Rouge. The latter article was commented upon by W. W. Johnson, then professor of mathematics in St. John's College, Annapolis, Md., who was a frequent and most gifted contributor to the Analyst. Asaph Hall wrote on comets and meteors, George R. Perkins on indeterminate problems, E. B. Seitz on probability. Other important contributors were Daniel Kirkwood, David Trowbridge, Artemas Martin, and G. W. Hill.

Well known among the mathematical public of America is Artemas Martin. Before speaking of his two periodicals we shall briefly sketch his life. This gives us at the same time an opportunity of mentioning many publications which, though not purely mathematical, contained a mathematical department. We can think of few American periodicals of the last thirty years, paying any considerable attention to elementary mathematics, for which Dr. Martin has not been a contributor. Dr. Martin was born in 1835. In 1869 he moved to Erie County, Pa., where he lived on a farm for fifteen years, engaged as a market-gardener. He is almost wholly self-taught. His leisure moments were devoted to the study of the "bewitching science." Through the influence of the Hon. W. L. Scott, Member of Congress from Erie, Martin was appointed, in 1885, librarian in the office of the U. S. Coast and Geodetic Survey. He has a large private library containing a very fine collection of American books on mathematics. When the writer was in Washington he enjoyed the great privilege of examining this collection and of seeing many a quaint and curious volume of great rarity.

Martin began his mathematical career when in his eighteenth year, by contributing solutions to the Pittsburg *Almanac* and soon afterward contributed problems to the "Riddler Column" of the Philadelphia *Saturday Evening Post*, and was one of the leading contributors for

twenty years. In 1864 he began contributing problems and solutions to *Clark's School Visitor*, afterward the *School-day Magazine*, published in Philadelphia. In June, 1870, he took charge of the "Stairway Department" as editor, the mathematical part of which he had conducted for some years before. In 1875 he was chosen editor of the department of higher mathematics in the *Normal Monthly*, published at Millersville, Pa., by Edward Brooks. The Monthly was discontinued in 1876. In this journal he published a series of sixteen articles on Diophantine analysis. He contributed to the mathematical department of the *Illinois Teacher* (1865–67); the *Iowa Instructor*, 1865; the *National Educator*, Kutztown, Pa.; the *Yates County Chronicle*, a weekly paper of New York, the mathematical department of which was edited by Samuel H. Wright; *Barnes's Educational Monthly;* the *Maine Farmers' Almanac; Educational Notes and Queries*, edited and published by W. D. Henkle, of Ohio. Dr. Martin published thirteen articles on "average" in *Wittenberger*, from 1876 to 1880 inclusive. The mathematical department of this was edited by William Hoover, afterward professor of mathematics in the Ohio University at Athens. Martin's name is familiar also to the readers of the *School Visitor*, a journal started in 1880, and edited and published monthly by John S. Royer in Gettysburg, Darke County, Ohio; of the *Davenport Monthly*, Davenport, Iowa; and of *The Bizarre*, conducted by S. C. and L. M. Gould, in Manchester, N. H. All these journals devoted a portion of their space to mathematics, and to all these Dr. Martin contributed. The mathematics they contained were of course of an elementary kind. He contributed also to English journals on elementary mathematics. Besides the above periodicals we mention the *Railroad Gazette* (New York and Chicago), which contained problems in applied mathematics; the *Mathematician*, edited by Royal Cooper, 1877, and utterly worthless; and the *Wheel*, New York, 1868, of which only one number ever appeared, in which the question was discussed how many revolutions upon its own axis a wheel will make in rolling once around a fixed wheel of the same size.*

In the spring of 1877 Artemas Martin issued the first number of his *Mathematical Visitor*, which he still continues to publish annually. "Although he has never served an hour as apprentice in a printing office to learn the art preservative, he has done all the type-setting for his publications, except that for the first three numbers of the Visitor, and has printed all the numbers of the Visitor except the first five on a self-inking lever press only $6\frac{1}{2}$ x 10 inches inside of chase. The numbers of the Visitor he has printed himself have been pronounced by competent judges to be as fine specimens of mathematical printing as have ever been executed. The *Magazine* he puts in type and gets the presswork

* For a more complete list of journals containing mathematical departments, see *The Bizarre, Notes and Queries*, Volume V, No. 12, December, 1888.

done at a printing office, as his press is too small to safely print it, although he printed one number on it."*

Of the Visitor generally six hundred copies have been printed. The list of contributors exceeds one hundred. In the introduction Dr. Martin says: "It was stated nearly three-quarters of a century ago that the learned Dr. Hutton declared that the Ladies' Diary had produced more mathematicians in England than all the mathematical authors in that kingdom." The aim of the Visitor is, if possible, to reach similar results in this country. It is devoted to the solution of problems. They deal more particularly in Diophantine analysis, average, and probability.

In January, 1882, Dr. Martin issued the first number of the *Mathematical Magazine*, which is published quarterly. It was intentionally made more elementary than the Analyst of Mr. Hendricks or the Annals of Mathematics. It was devoted mainly to arithmetic, algebra, geometry, and trigonometry. One of the features is the solution and discussion "of such of the problems found in the various text-books in use as are of special interest, or present some difficulty." Many of the articles found in the Magazine and Visitor came from the pen of the editor himself. Numerous different proofs of the Pythagorean proposition were given in the former, of which we may mention one by James A. Garfield. It was taken from a magazine of 1876 or 1877, and was found pasted on a fly-leaf of an old geometry. It resembles somewhat the old Hindoo proof. Dr. G. B. Halsted contributed several articles on the prismoidal formula. J. W. Nicholson gave a "universal demonstration" of the binomial theorem, without, however, giving a moment's thought to the question of convergency, whenever the series is infinite. William Hoover gave an interesting little article on the history of the algebraic notation. David S. Hart wrote on the history of the theory of numbers, including the indeterminate and Diophantine analysis. He also contributed several articles on the subject last mentioned. A somewhat lengthy discussion was carried on, on the usefulness of logarithms, by P. H. Philbrick, professor of engineering at the State University of Iowa, and H. A. Howe, professor of mathematics at the University of Denver. The former attempted to show that the use of logarithms greatly augmented the labor of "numerical computation" and led to very erroneous results. Some of the calculations in the magazine in which numerical answers are carried to twenty or more decimal places have no value, either educational or scientific. The names of the contributors for the magazine were about the same as for the Visitor.

To show the good that elementary journals like this may do, we give, as an example, the career of E. B. Seitz. He passed his boyhood on a farm, and afterward pursued a mathematical course of two years at the Ohio Wesleyan University. In 1872 he began contributing problems proposed in the "Stairway" department of the School-day

*The Buffalo Express, August 29, 1886.

Magazine conducted by Dr. Martin. His energies were stimulated, and he became a leading contributor to our periodicals. He astonished his friends by his skill in solving difficult problems, and their admiration for his talents became almost unbounded. His mathematical mind had received the first stimulus from our elementary periodicals. Had he not died in the prime of life, he might have done good original work, provided he had begun to look higher than merely to the solution of difficult problems in our elementary journals. The solving of problems is very beneficial at first, but it becomes a waste of time if one confines himself to that sort of work. The solution of problems is not a high form of mathematical research, and should serve merely as a ladder to more ambitious efforts.

Another journal devoted mainly to the solution of problems is the *School Messenger*, now called the *Mathematical Messenger*, edited and published bi-monthly by G. H. Harvill, at Ada, La. One of the ablest contributors to it is J. W. Nicholson, professor of mathematics at the Louisiana State University. The Messenger commenced February, 1884.

The *Annals of Mathematics* is the continuation, under a new name and different form, of the Analyst. It is edited and published at the University of Virginia by Prof. Ormond Stone and Prof. William M. Thornton. It is of somewhat higher grade than the Analyst, though more elementary than the American Journal of Mathematics. It contains more articles on mathematical astronomy and other subjects of applied mathematics than the American Journal. Our distinguished mathematical astronomer, G. W. Hill, contributes several articles in his specialty. Profs. Asaph Hall, R. S. Woodward, H. A. Howe, and William M. Thornton contribute various articles on applied mathematics. Professor Oliver, of Cornell, has several papers, one on "A Projective Relation among Infinitesimal Elements," and another on the "General Linear Differential Equation." Prof. W. W. Johnson writes on "Glaisher's Factor Tables," the "Distribution of Primes," and other subjects. Professor Halsted gives his demonstration of Descarte's theorem and Euler's theorems. The name of Bohannan, now professor at Ohio University, appears often. Prof. O. H. Mitchell, of Marietta College, discusses the equation of the second degree in two variables. Prof. R. H. Graves has geometrical articles; William E. Heal writes on repetends; S. T. Moreland, on the momental ellipsoid; J. F. McCullogh, on Rolle's theorem. A little space in each number is devoted to the proposing and solving of problems. The list of contributors is too large to be given here in full.

When Professor Sylvester became actively connected with the Johns Hopkins University, in 1877, the university established the *American Journal of Mathematics*, for the publication of original research in pure and applied mathematics. It was the design that this should not be a journal devoted to the publication of solutions to problems, but that it

should be of so high a grade as to command a place by the side of the best European journals of mathematics. It is a source of pride to us that this great aim has been reached. The *American Journal of Mathematics* is to-day as well known and as highly respected in Europe as in America. Among its contributors are found not only the leading scientists of America, but also such foreign investigators as Cayley, Clifford, Crofton, Faà de Bruno, Frankland, De Gasparis, Glashan, Hammond, Hermite, Kempe, Lipschitz, Loudon, Lucas, MacMahon, Muir, Petersen, Poincaré, Roberts, Weichold, and G. P. Young.

The subject which has received most attention in the *American Journal of Mathematics* has been Modern Higher Algebra. The contributions of Sylvester on this subject loom large. In Volume I is found "a somewhat speculative paper" entitled, "An Application of the New Atomic Theory to the Graphical Representation of the Invariants and Covariants of Binary Quantics," followed by appendices and notes relating to various special points of the theory.* Sylvester contributed various memoirs on binary and ternary quantics, including papers by himself, with the aid of Dr. Franklin, containing tables of the numerical generating functions for binary quantics of the first ten orders, and for simultaneous binary quantics of the first four orders, etc. The list of his articles is too extensive to be mentioned here. Since his return to England he has contributed to the Journal a series of "Lectures on the Theory of Reciprocants," reported by J. Hammond.

The larger number of American contributions are from persons who were, or still are, connected with the Johns Hopkins University, either as teachers or students. Dr. W. E. Story, of the Johns Hopkins University, has written on "Non-Euclidean Trigonometry," "Absolute Classification of Quadratic Loci, etc.," and other, chiefly geometrical, subjects. Dr. T. Craig has contributed numerous papers, mainly on the theory of functions and differential equations. Dr. F. Franklin has aided Professor Sylvester in the preparation of papers, and has also made various independent contributions. After the return to England of Professor Sylvester, Professor Newcomb became editor-in-chief. His valuable articles have been noticed elsewhere. Among the contributors who were once students at the Johns Hopkins University, but are now not connected with it, are E. W. Davis, W. P. Durfee, G. S. Ely, G. B. Halsted, A. S. Hathaway, O. H. Mitchell, W. I. Stringham, C. A. Van Velzer, A. L. Daniels, C. Veneziani, D. Barcroft, and J. C. Fields. The Journal has two lady contributors, Mrs. C. Ladd Franklin, of Baltimore, and Miss C. A. Scott, of Bryn Mawr College. The great memoir on "Linear Associative Algebra," by Benjamin Peirce, was published in the American Journal of Mathematics; also articles by his son, C. S. Peirce, on the "Algebra of Logic" and the "Ghosts in Diffraction Spectra." Papers on applied mathematics have been written by Professor Rowland, of the Johns Hopkins University, and George

* *Vide Professor Cayley's* article on Professor Sylvester in Nature, January 3, 1889.

William Hill, of the Nautical Almanac Office. Mr. Hill has done admirable work in mathematical astronomy. For his researches on the lunar theory, published in the American Journal, and for other astronomical papers, published elsewhere, he was awarded the gold medal of the Royal Astronomical Society, in 1887.* Among the writers for the American Journal is Prof. W. W. Johnson, of the U. S. Naval Academy at Annapolis. He is also a frequent contributor to leading European journals, and commands a place among the very foremost of American mathematicians. In the list of American writers to the Journal are H. T. Eddy, J. W. Gibbs, E. McClintock, A. W. Phillips, J. Hagen, E. W. Hyde, H. B. Fine, and others of no less power and originality.

THE U. S. COAST AND GEODETIC SURVEY.

In giving the origin of the U. S. Coast Survey it is desirable to begin with a sketch of the preliminary training of its first superintendent, Ferdinand R. Hassler. He was born in the town of Aarau, Switzerland, in 1770. He was sent to the University of Bern to study law, but he soon drifted into mathematics and became a favorite pupil of Prof. John G. Tralles.† Tralles and Hassler undertook the topographical survey of the Canton of Bern. In 1791 a base-line was measured, and a net of triangles established. The instruments on hand being found insufficient for long distances new ones were ordered from Ramsden, in London. On the receipt of these, in 1797, the survey was resumed, but soon discontinued. The conquering armies of the French came marching into Switzerland. The feeble republic was forced to submit to the dictatorial orders of the war minister of France, which required, among other things, that the places then occupied by the Swiss engineers should be vacated and filled by French. A swarm of sixty French engineers appeared, but soon disappeared without accomplishing anything. The revolutionary tendencies of the times and the unsettled state of the country induced Hassler to quit Switzerland. His fatherland seemed to bear no roses for him. Having landed in Philadelphia, in October, 1805, he formed the acquaintance of Prof. Robert Patterson and Mr. Garnet, of New Brunswick, to whom he showed his mathematical books and instruments.

About this time Congress was considering the feasibility of a survey of the coasts and harbors. Professor Patterson sent to President Jefferson a sketch of Hassler's scientific career in Switzerland, and Mr. Clay, the Representative from Philadelphia, in 1806, asked Hassler whether he would be willing to undertake the survey, in case that the

* *Vide* Monthly Notices on the Royal Astronomical Society, Vol. XLVII, No. 4, February, 1887.

† Translation from the German of Memoirs of Ferdinand Rudolph Hassler, by Emil Zschokke, published in Aarau, Switzerland, 1877, with Supplementary Documents, *published* in Nice, 1882.

Government should decide upon one. Mr. Hassler was, of course, willing. The law authorizing the survey was passed in February, 1807. Hassler received one of the twelve circulars which were sent to scientific men for plans of the contemplated survey. By the direction of President Jefferson, a commission (formed, it appears, of the very gentlemen who had proposed plans, excepting Mr. Hassler) examined the various plans at Professor Patterson's, in Philadelphia. They rejected their own projects and recommended to the President the one suggested by Mr. Hassler. The survey proposed by him was of a kind that had never been previously attempted in this country; it was to be a *triangulation*, and the sides of the triangles were to be from ten to sixty miles in length, such as had, at that time, just been executed in France and was in progress in England. The project was quite in advance of the science of our country. It was fortunate for us that a man of Mr. Hassler's learning, ability, and mechanical ingenuity was available to the Government. He had, meanwhile, been appointed by Jefferson professor at West Point. This position he resigned after three years, and accepted the professorship of mathematics at Union College, Schenectady, N. Y. Politics delayed the work of the survey. The first thing to be done was to procure the necessary instruments. In 1811 Hassler was sent to England by our Government to direct the manufacture of suitable instruments. Shortly after his arrival in Great Britain the War of 1812 broke out, and he was four years in London, in the disagreeable position of an alien enemy, and half the time left by our Government without compensation. He returned to this country in 1814, with a splendid collection of instruments, which had cost nearly forty thousand dollars.

In August, 1816, a formal agreement between the Government and Mr. Hassler was reached, to the effect that he should undertake the execution of the survey. He immediately entered upon the preliminary steps of reconnoitering and the numerous collateral experiments necessary for such a survey. Two preliminary base-lines were measured: One in New Jersey, in the rear of the Highlands, on North River, and nearly six miles in length; the other on Long Island, and of about five miles. Down to the year 1818 eleven stations were occupied, forming the elements of 124 triangles.

To a scientific man, familiar with the many preliminary details which are indispensable to accurate scientific work, but which do not always appear in the ultimate results, the progress which Hassler was making would have seemed highly satisfactory. Congress, however, was displeased. In April, 1818, Mr. Hassler received official notice that he was suspended, accompanied with the remark that the little progress hitherto made in the work had caused general dissatisfaction in Congress. Possibly the feeling on the part of American engineers against this foreigner because he had been preferred to one of them had something to do with this suspension. To Hassler this was a very severe blow; his

brightest hopes seemed dashed into fragments. A year or two later he prepared a defense of himself. He wrote an account of his plans and methods and published it in the Philosophical Transactions of Philadelphia (Vol. III, New Series, 1825). By this article he hoped to vindicate his schemes. It attracted the attention of scientific men everywhere. It was reviewed by leading astronomers in Europe—Bessel, Struve, Schumacher, Férussac, Francœur, Krusenstern, and others—all agreeing that Mr. Hassler's plans were good, and testifying to his inventive genius for solving the difficulties of the Coast Survey, as well as to the certainty that his plans, if carried out, would lead to success. Bessel was certainly a competent judge, for, in addition to his theoretical knowledge, he had had experience in geodetic work in Germany. He had words of only the highest praise for Hassler's scheme.*

After his suspension from the survey, Hassler engaged in various occupations. For a while he was a farmer in northern New York. He afterward went to Jamaica, Long Island, and then to Richmond, Va., giving lessons in mathematics to sons of prominent men. While in Richmond he published his Elements of the Geometry of Planes and Solids, 1828. His Elements of Analytic Trigonometry appeared in 1826. Subsequently he published an Arithmetic, Astronomy, and Logarithms and Trigonometric Tables, with introductions in five languages.

After twelve years in rural retreat, Hassler was recalled to official activity. He became United States gauger, and then was intrusted, from 1830 to 1832, with the difficult mission of regulating the standards of weights and measures throughout the United States, which at that time were very various.

In 1828 the question of the Coast Survey was again agitated. The Secretary of the Navy reported to Congress favorably on Hassler's work, which had been suspended so suddenly ten years previously. The Secretary said that "he [Hassler] had accomplished all that was possible in so short a time." In 1832 Mr. Hassler was re-instated, with the title of "Superintendent of the United States Coast Survey."

In the interval from 1818 to 1832 nothing of permanent value had been accomplished. Attempts had been made to survey portions of the coast, under the direction of the Navy Department, but there had been no general or connected survey. The charts prepared had been expensive and unsafe, and not very creditable to the country.

In 1832 began the most successful and most famous period in Mr. Hassler's life. Though sixty-two years old, there still glowed in him the fire of youth. The survey was begun with vigor. He had a traveling carriage prepared for him, which conveyed him rapidly to all parts of the survey. In this carriage he could seat himself at a writing table or dispose himself for sleep. The work was prosecuted according to the plans first laid out by him. He labored under the great disadvantage of having no skilled assistants. His corps of workmen had all to be

* *Vide* Silliman's Journal, Vol. IX, p. 225.

trained and educated to the refined methods which he was introducing. The work of the survey had to be *systematized*. It continued under his direction until the time of his death, in 1843. He left the work well advanced between Narragansett Bay and Cape Henlopen, and the survey sufficiently organized in all its varied details. His course was, however, frequently criticised in Congress, and it was not always easy to get the necessary appropriations.

Mr. Hassler was very self-confident and independent. This was one cause of the occasional opposition to him. Though not conceited, he was conscious of his superiority over the great mass of men with whom he came in contact in Washington. The following anecdote is characteristic of him: At one time the cry of "retrenchment and reform" had become popular, and a newly appointed Secretary of the Treasury thought he could not signalize his administration more aptly than by reducing the large salary of the superintendent. He sent for Mr. Hassler and said, "My dear sir, your salary is enormous; you receive $6,000 per annum—an income, do you know, quite as large as that of the Secretary of State." "True," replied Hassler, "precisely as much as the Secretary of State and quite as much as the Chief of the Treasury; but do you know, Mr. Secretary, that the President can make a minister of State out of anybody; he can make one even out of you, sir; but if he can make a Hassler, I will resign my place."

Hassler's successor was Alexander Dallas Bache, a great-grandson of Franklin and a graduate of West Point. He exercised a very marked influence over the progress of science among us. He graduated at the head of his class, and the great expectations that were then entertained of him have been fully realized. For eight years he devoted himself to physical science, while professor at the University of Pennsylvania, and gained a wide reputation. The Coast Survey made rapid progress under his management. Congress began to show better appreciation of this sort of work, and made more liberal appropriations. This enabled him to adopt a more comprehensive scheme. Instead of working only at one locality, as had been done previously, he was able to begin independent surveys at several places at once, each section employing its own base. He proposed eight sections, which number was increased on the annexation of Texas, and again on the annexation of California.

Two of the most important improvements of modern geodesy were perfected and brought into use at the beginning of Bache's superintendency, namely, Mr. Talbott's method of determining latitudes and the telegraphic method of determining longitude. Various other refinements in every branch of work were introduced. Systematic observations of the tides, a magnetic survey of the coast, and the extension of the hydrographic explorations into the Gulf Stream were also instituted by Bache.

Having extended the scope of the Survey, Bache needed a greater number of assistants, but the supply was not wanting. Says Prof. T.

H. Safford,* "he found available for its higher geodetic works a number of West Point officers, of whom T. J. Lee was one, and Humphreys, afterward chief engineer of the Army, another. One of the leaders in practical astronomy of the topographical engineers was J. D. Graham; and the work which had been done by that corps upon the national and State boundaries had familiarized a good many Army officers with field astronomy and geodesy.

"Bache, who had been out of the Army nearly twenty years employed his great organizing and scientific capacity in training the Coast Survey corps (including detailed Army officers) into practical methods for its various problems; and the connection between the West Point officers and the able young civilians, who are now the veterans of the Survey, was extremely wholesome.

"Lee prepared a work (Tables and Formulæ) which has served an excellent purpose in bridging the gap between theory and practice; especially for the last generation of West Point officers."

Graduates of West Point are now more closely employed in military and other public duty, and are no longer employed in the Coast Survey.

The work of the Survey was interrupted by the Civil War. Soon after its close Bache died (1867). Benjamin Peirce, his successor in the superintendency, said of him: "What the Coast Survey now is, he made it. It is his true and lasting monument. It will never cease to be the admiration of the scientific world. * * * It is only necessary conscientiously and faithfully to follow in his foot-steps, imitate his example, and develop his plans in the administration of the Survey."

Under Peirce, the survey of the coasts was pushed with vigor, and it rapidly approached completion. He proposed the plan of connecting the survey on the Atlantic Coast with that on the Pacific by two chains of triangles, a northern and a southern one. This project received the sanction of Congress, and thus the plan of a general geodetic survey of the whole country was happily inaugurated.

Benjamin Peirce's successor on the Coast Survey was Carlile Pollock Patterson. He was a graduate of Georgetown College, Kentucky, and had for many years previous to this appointment, in 1874, been connected with the Survey as hydrographic inspector. Under him the extension of the Survey into the interior of our country, as inaugurated by Peirce, was continued. By the completion of this work this country will contribute its fair share to the knowledge of the figure of the earth, which has hitherto been derived entirely from the researches of other nations. On account of this extension, the name, "U. S. Coast Survey," was changed, in 1879, to "U. S. Coast and Geodetic Survey."

Patterson died in 1881, and Julius Erasmus Hilgard became his successor. Hilgard was born in Zweibrücken, Bavaria, came to this country at the age of ten, and at the age of twenty was invited by Bache to become one of his assistants on the Survey. Hilgard soon came to be

*Mathematical Teachings, p. 6.

recognized for great ability and skill, and rose to the position of assistant in charge of the Office in Washington. He held the superintendency from 1881 to 1885, when he resigned. His work consisted chiefly of researches and discussions of results in geodesy and terrestrial physics, and in the perfecting of the methods and instruments employed. The superintendency was next intrusted to Frank M. Thorn, who was succeeded in July, 1889, by T. C. Mendenhall, who now fills the office.

The work of the U. S. Coast Survey has been carried on with great efficiency from its very beginning, and reflects great credit upon America. In making the computations for the Survey, the method of least squares for the adjustment of observations has found extended application. Valuable papers on this subject by Bache and Schott have been printed in the reports of the U. S. Coast Survey.* Charles A. Schott graduated at the Polytechnic School in Carlsruhe, came to this country in 1848, and has since that time been an efficient worker on the U. S. Coast Survey. He is now chief of the computing division.

It will be remembered that interesting researches on least squares had been made quite early in this country by Robert Adrain. Benjamin Peirce invented a criterion for the rejection of doubtful observations.† It proposes a method for determining, by successive approximations, whether or not a suspected observation may be rejected. Tables are needed for its application. Objections have been made to its use, because it "involves a contradiction of reasoning."‡ The criterion is given by Chauvenet in his Method of Least Squares (1864), and has been used to some extent on the U. S. Coast Survey, but it has found no acceptance in Europe. Chauvenet gives an approximate criterion of his own for the rejection of *one* doubtful observation, which is derived, he says, "directly from the fundamental formula upon which the whole theory of the method of least squares is based." But this criterion, too, has been criticised as being "troublesome to use, and as based on an erroneous principle." Stone, in England, offered still another criterion for the rejection of discordant observations, but Glaisher pronounces it untrustworthy and wrong. No criterion has yet been given which has met with general acceptance. Indeed, Professor Newcomb considers it impossible that such a one should ever be invented. Says he (in his second paper mentioned below): "We here meet the difficulty that no positive criterion for determining whether an observation should or should not be treated as abnormal is possible. Several attempts have indeed been made to formulate such a criterion, the best known of which is that of Peirce."

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* See reports for the years 1850, '55, '56, '58, '61, '64, '66, '67, '75.

† Gould's Astronomical Journal, Vol. II, pp. 161–3.

‡ See Prof. Mansfield Merriman's article in the Transactions of the Connecticut Academy, containing a list of writings relating to the method of least squares and the theory of the accidental errors of observation, which comprises 408 titles by 193 authors.

Valuable papers on least squares have been contributed in this country by G. P. Bond,* of Harvard; Simon Newcomb, † C. S. Pierce, ‡ and Truman H. Safford.§ The text-books on this subject generally used in our schools are those of Chauvenet, Merriman, and T. W. Wright.

*"On the use of Equivalent Factors in the Method of Least Squares," Memoirs American Academy, Vol. VI, pp. 179–212.

† "A Mechanical Representation of a Familiar Problem," Monthly Notices of the Astronomical Society, London, Vol. XXXIII, pp. 573–'4; "A Generalized Theory of the Combination of Observations so as to Obtain the Best Results," American Journal of Mathematics, Vol. VIII.

‡ "On the Theory of Errors of Observations," Report U. S. Coast Survey, 1870, pp. 200–224.

§ "On the Method of Least Squares," Proceedings American Academy, Vol. XI.

IV.

THE MATHEMATICAL TEACHING AT THE PRESENT TIME.

The mathematical teaching of the last ten years indicates a "rupture" with antiquated traditional methods, and an "alignment with the march of modern thought." As yet the alignment is by no means rectified. Indeed it has but barely begun. The "rupture" is evident from the publication of such works as Newcomb's series of mathematical textbooks, recent publications on the calculus, the appearance of such algebras as those of Oliver, Wait, and Jones, Phillips and Beebe, and Van Velzer and Slichter; of such geometries as Halsted's "Elements" and "Mensuration;" of such trigonometries as Oliver, Wait, and Jones's; of Carll's Calculus of Variations; Hardy's Quaternions; Peck's and Hanus's Determinants; W. B. Smith's Co-ordinate Geometry (employing determinants); Craig's Linear Differential Equations.

Determinants and quaternions have thus far generally been offered as elective studies, and have formed a crowning pinnacle of the mathematical courses in colleges. It is certainly very doubtful whether this is their proper place in the course. It seems quite plain that the elements of determinants should form a part of algebra, and should be taught early in the course, in order that they may be used in the study of co-ordinate geometry. What place should be assigned to quaternions is not quite so plain. Prof. De Volson Wood introduces their elements in his work on co-ordinate geometry. The professors of Cornell have not taught quaternions directly for some years, but are convinced that most students derive more benefit by a mixed course in matrices, vector addition and subtraction, imaginaries, and theory of functions. The early introduction of determinants seems more urgent than that of quaternions. We think, however, that great caution should be exercised in incorporating either of these subjects in the early part of mathematical courses. Those universities and colleges which are, as yet, not strong enough to maintain a high and rigid standard of admission, and whose students enter the Freshman class with only a very meagre and superficial knowledge of the *elements* of ordinary algebra, would find the introduction of determinants and imaginaries as Freshman studies a hazardous innovation. One of the very first considerations in mathematical teaching is *thoroughness*. In the past the *lack* of thoroughness has poisoned the minds of the American youth with an utter dislike and bitter hatred of mathematics. Whenever a subject is not well understood, it is not liked; whenever it *is* well understood, it is generally liked.

There is almost always some one author whose text-books reach very extended popularity among the great mass of schools. Such authors were Webber, Day, Davies, and Loomis. If we were called upon to name the writer whose books have met with more wide-spread circulation during the last decennium than those of any other author we should answer, Wentworth. Mr. Wentworth was born in Wakefield, N. H., fitted for college at Phillips Exeter Academy, graduated at Harvard College in 1858, and then returned to Phillips Exeter Academy, where he has been ever since. He had for instructors in mathematics, at the academy, Prof. Joseph G. Hoyt, afterward chancellor of the Washington University in St. Louis; and, in college, Prof. James Mills Peirce. "The characteristics of my books," says Mr. Wentworth, "are due to what I have found from a long experience is absolutely necessary in order that a pupil of ordinary ability might master the subject of his reading. To learn by doing, and to learn one step thoroughly before the next is attempted, constitute pretty much the whole story." In point of scientific rigor Wentworth's books are superior to the popular works of preceding decades. It seems to us that the book most liable to criticism is his Elementary Geometry (old edition). He has been greatly assisted in the writing of his books by leading teachers from different parts of our country. Some of the books bearing his name are almost entirely the work of other men.

It is to be hoped that the near future will bring reforms in the mathematical teaching in this country. We are in sad need of them. From nearly all our colleges and universities comes the loud complaint of inefficient preparation on the part of students applying for admission; from the high schools comes the same doleful cry. Errors in mathematical instruction are committed at the very beginning, in the study of arithmetic. Educators who have studied the work of Prussian schools declare that our results in elementary instruction are far inferior. Says President C. K. Adams, of Cornell University:* "In the lowest grades of schools our inferiority seems to me to be very marked. The results of the earliest years of the European course, I mean those devoted to teaching the boy, say from the time he is nine years of age until he is fourteen, when compared with the fruits of the courses pursued during the corresponding years in the average American school, are immeasurably superior." President Adams institutes a comparison between Brooklyn and Berlin schools. Speaking of a Brooklyn boy of fifteen, he remarks: "In the first place it must be said that he has had forced upon him six hours a week in arithmetic, during the whole of the seven primary grades. Then on emerging from the primary school, and coming into the grammar school, he is required to take an average of four hours a week in the same study, during all the eight grades. That is to say, during the whole of the boy's career in school, from the

*New England Association of Colleges and Preparatory Schools; Addresses and *Proceedings* at the Annual Meeting, 1888, p. 24.

time he is seven until he is fifteen, he has devoted no less than five hours a week of recitations to the study of arithmetic alone. If we deduct the hours devoted to reading, penmanship, and music, we find that five-elevenths of what remains is devoted to arithmetic. Making no deductions, and including the hours devoted to the elementary work requiring no preparation whatever, we find that arithmetic occupies in the class-room considerably more than one-fourth of all the student's time, during the whole of seven or eight years."

This statement is applicable with equal force to probably all our schools. The fact is that the study of arithmetic has been, in one sense, greatly overdone in this country. The most melancholy thought in this connection is that, after all, our boys and girls acquire only a deficient knowledge of this subject. Persons who had opportunity for comparison assure us that the American boy does not "figure" as rapidly and accurately as the German boy.

If the above assertions be true, then it behooves the American teacher to inquire wherein the foreign methods of teaching excel his own. In some circles the study of pedagogy has not been popular. This apathy is, we think, partly due to the influence of *some* of our normal schools. Many of our normal schools have been conducted very efficiently, but others have had teachers in their faculties who lacked breadth and depth of scholarship, and who brought the study of pedagogy into disrepute by their narrowness and their lack of elasticity in the application of methods. This aversion to the study of theories of teaching is now happily disappearing. Our universities and colleges are beginning to establish chairs of pedagogy.

Improvements in the teaching of arithmetic might probably be effected by the general introduction of some such method as that of Grube. The first complete exposition of this method was, we believe, published in this country by F. L. Soldan, formerly principal of the St. Louis Normal School. It seems to gain ground here every year.

A most valuable and suggestive monograph on mathematical teaching has been written by Prof. T. H. Safford, of Williams College. Professor Safford is an advocate of the *heuristic method* of teaching. Grube is the representative of this in arithmetic. The method employed by Spencer in his little book on Inventional Geometry is similar to the heuristic, if not identical with it. The heuristic is, in general, the method in which the pupil's mind does the work. It is a slow method. Thus, Grube considers the numbers from 1 to 10 sufficient to engage the attention of a child (of six or seven years) during the first year of school. "In regard to extent, the scholar has not, apparently, gained very much—he knows only the numbers from 1 to 10. But he knows them."* The Germans "make haste slowly," but in elementary education they beat us in the race. Geometry, like arithmetic, should be taught sparingly at a time, but for many years in succession. Profes-

* Grube's method, by F. Louis Soldan, p. 21.

sor Safford strongly recommends the parallelism of the two main mathematical subjects—arithmetic including algebra, and geometry including trigonometry and conic sections. Thereby the study of algebra and geometry can be extended over a longer period of time. According to his ideal programme of study, primary arithmetic is accompanied by notions of form and drawing; arithmetic through rule of three, by rudiments of geometry; universal arithmetic and simple equations, by one or two books in plane geometry; algebra through quadratics, by plane geometry; advanced algebra, by solid geometry, conic sections, plane trigonometry, etc.

"Of course this programme is somewhat variable, but the main principle, that a course of arithmetic must run parallel with one of geometry from the beginning of a school course to the end, is one which is laid down by the best educators since Pestalozzi's time."*

In order to enable the writer to give a view of the present condition of mathematical teaching, the Bureau of Education sent to various universities, colleges, normal schools, academies, institutes, and high schools, a printed letter with a series of questions to be answered. We give a list of the institutions which sent in replies, and state the results as fully as our space will permit.†

STATISTICS ILLUSTRATING THE PRESENT CONDITION OF MATHEMATICAL INSTRUCTION IN THE UNITED STATES.

(*a*) UNIVERSITIES AND COLLEGES.

	Name of institution.	Location.	Name of person reporting.	Title or position of person reporting.
1	University of Alabama...	Tuscaloosa, Ala.....	T. W. Palmer.....	Professor of mathematics.
2	Spring Hill College.....	Mobile, Ala.....	A. S. Wagner.....	Do.
3	Central Female College...	Tuscaloosa, Ala.....	S. B. Foster.....	President.
4	Alabama Polytechnic Institute, Agricultural and Mechanical College.	Auburn, Ala.....	O. D. Smith.....	Professor of mathematics.
5	Huntsville Female College	Huntsville, Ala.....	A. B. Jones.....	President.
6	Talladega College.....	Talladega, Ala.....	Jesse Bailey.....	Principal.
7	Alabama Conference Female College.	Feskegie, Ala.....	John Massey.....	President.
8	Pierce Christian College..	College City, Cal...	D. E. Hughes.....	Professor of mathematics, astronomy, and civil engineering.
9	St. Ignatius College.....	San Francisco, Cal..	T. C. Leonard.....	Teacher of higher mathematics.
10	State Agricultural College	Fort Collins, Colo...	V. E. Stolbrand...	Professor of mathematics and master of literary science.
11	University of Denver....	Denver, Colo.....	H. A. Howe.....	Professor of mathematics and astronomy.

* Monograph on Mathematical Teaching by T. H. Safford, p. 44.

† From this list are omitted some few reports which were sent in too late for insertion, or which did not give the name of the institution or the person reporting, or which were illegible.

(a) UNIVERSITIES AND COLLEGES—Continued.

	Name of institution.	Location.	Name of person reporting.	Title or position of person reporting.
12	University of Colorado	Boulder, Colo	I. M. De Long	Professor of mathematics.
13	State School of Mines	Golden, Colo	Paul Meyer	Do.
14	Storrs Agricultural School	Storrs, Conn	W. P. Washburn	Professor of chemistry and mathematics.
15	Trinity College	Hartford, Conn	F. S. Luther	Professor of mathematics and astronomy.
16	University of Dakota	Vermillion, Dak	L. S. Hulburt	Professor of mathematics.
17	Dakota School of Mines	Rapid City, Dak	L. L. Conant	Do.
18	Georgetown College	Washington, D. C	J. F. Dawson	Professor of physics and mechanics.
19	National Deaf-Mute College.	....do	Joseph C. Gordon A. G. Draper	Professor of mathematics. Assistant professor of mathematics.
20	Howard University	....do	G. W. Cook	Tutor in mathematics.
21	De Land University	De Land, Fla	R. Gentry	Teacher of mathematics.
22	Seminary West of the Suwannee River.	Tallahassee, Fla	G. M. Edgar	President and professor of mathematics and natural science.
23	Florida State Agricultural College.	Lake City, Fla	L. H. Orleman	Professor of mathematics.
24	Bowdon College	Bowdon, Ga	C. O. Stubbs	Do.
25	North Georgia Agricultural College.	Dahlonega, Ga	W. S. Wilson	Do.
26	Hearn Institute	Cave Spring, Ga	E. T. Whatley	President.
27	University of Georgia	Athens, Ga	W. Rutherford	Professor of mathematics.
28	Eureka College	Eureka, Ill	G. A. Miller	Do.
29	Illinois State Normal University.	Normal, Ill	J. W. Cook	Instructor in mathematics.
30	Lombard University	Galesburg, Ill	J. V. N. Standish	Professor of mathematics and astronomy.
31	M'Kendree College	Lebanon, Ill	A. G. Jepson	Professor of mathematics.
32	German-English College	Galena, Ill	Fr. Schaub	President.
33	Lincoln University	Lincoln, Ill	J. W. P. Buchanan	Professor of mathematics.
34	Lake Forest University	Lake Forest, Ill	M. McNeill	Professor of mathematics and astronomy.
35	University of Illinois.	Urbana, Ill	S. W. Shattuck S. H. Peabody	Professor of mathematics. Regent (president).
36	Illinois College	Jacksonville, Ill	J. H. Pratt	Ph. D.
37	North-Western College	Napersville, Ill	H. F. Kletzing	Professor of mathematics.
38	Indiana University	Bloomington, Ind	J. Swain	Do.
39	Wabash College	Crawfordsville, Ind	J. Norris	Do.
40	Earlham College	Richmond, Ind	W. B. Morgan	Do.
41	Rose Polytechnic Institute.	Terre Haute, Ind	C. A. Waldo	Do.
42	De Pauw University	Greencastle, Ind	A. Martin	President.
43	Franklin College	Franklin, Ind	R. J. Thompson	Professor of mathematics.
44	Indiana Normal College	Covington, Ind	O. E. Coffin	
45	Hanover College	Hanover, Ind	F. L. Morse	Professor of mathematics.
46	State University of Iowa.	Iowa City, Iowa	L. G. Weld	Acting professor of mathematics.
47	University of Des Moines.	Des Moines, Iowa	T. M. Blakslee	Ph. D., Yale, 1880.
48	Oskaloosa College	Oskaloosa, Iowa	J. A. Beattie	President.
49	Upper Iowa University	Fayette, Iowa	J. W. Bresell	Do.

(a) UNIVERSITIES AND COLLEGES—Continued.

	Name of institution.	Location.	Name of person reporting.	Title or position of person reporting.
50	Oswego College for Young Ladies.	Oswego, Kan.....	S. H. Johnson.....	Principal.
51	University of Kansas.....	Lawrence, Kan	E. Miller.....	Professor of mathematics.
52	Ottawa University.....	Ottawa, Kan.....	M. L. Ward	Professor of mathematics and political science.
53	Bethany College and Normal Institute.	Lindsborg, Kan.....	W. A. Granville ..	Professor of mathematics.
54	Washburn College.....	Topeka, Kan.....	P. McVicar	President.
55	Kansas State Agricultural College.	Manhattan, Kan	D. E. Lants	Professor of mathematics.
56	West Kentucky Classical and Normal College.	South Carrollton, Ky	W. C. Gaynor.....	President.
57	Millersburg Female College.	Millersburg, Ky	C. Pope.....	Do.
58	Berea College.....	Berea, Ky.....	P. D. Dodge.....	Acting professor of mathematics.
59	Eminence College and Normal School.	Eminence, Ky	H. Boring.....	Teacher of mathematics, Latin, and Greek.
60	Ogden College.....	Bowling Green, Ky .	W. A. Ohmeliain..	President.
61	Kentucky Classical and Business College.	North Middletown, Ky.	S. W. Pearcy.....	Do.
62	Hamilton Female College	Lexington, Ky.....	J. W. Porter.....	Professor of mathematics and Latin.
63	Keachie Male and Female College.	Keachie, La.....	G. W. Thigpen....	Professor of mathematics.
64	Mount St. Mary's College.	Emmitsburg, Md ...	J. A. Mitchell.....	Professor.
65	Western Maryland College	Westminster, Md...	W. R. McDaniel ..	Professor of mathematics.
66	Baltimore City College....	Baltimore, Md	W. Elliott	Principal.
67	Johns Hopkins University	....do	S. Newcomb	Professor of mathematics and astronomy.
68	U. S. Naval Academy	Annapolis, Md.....	W. W. Hendrickson, J. M. Rice.	Professors of mathematics.
69	Maryland Agricultural College.	Agricultural College P. O., Md.	H. E. Alvord.....	President.
70	St. John's College.....	Annapolis, Md.....	J. W. Cain.....	Professor of mathematics.
71	Maine State College of Agriculture and Mechanic Arts.	Orono, Me	J. N. Hart.....	Instructor in mathematics.
72	Colby University.....	Waterville, Me	L. E. Warren	Professor of mathematics.
73	East Maine Conference Seminary.	Bucksport, Me.....	A. F. Chase.....	Principal.
74	Bowdoin College.....	Brunswick, Me	W. A. Moody	Professor of mathematics.
75	Bates College.....	Lewiston, Me.....	J. M. Rand.....	Do.
76	Agricultural College.....	Amherst, Mass	C. D. Warner	Professor of mathematics and physics.
77	Wesleyan Academy	Wilbraham, Mass...	B. S. Annis.....	Instructor in mathematics.
78	The Society for the Collegiate Instruction of Women.	Cambridge, Mass ...	A. Gilman	Secretary.
79	Smith College	Northampton, Mass.	E. P. Cushing	Teacher of mathematics.
80	Lasell Seminary	Auburndale, Mass ..	L. M. Packard ...	Instructor in mathematics.
81	Swain Free School.....	New Bedford, Mass .	A. Ingraham.....	Master.
82	Thayer Academy	Braintree, Mass.....	C. A. Pitkin.....	Master of mathematics and physics.

(a) UNIVERSITIES AND COLLEGES—Continued.

	Name of institution.	Location.	Name of person reporting.	Title or position of person reporting.
83	Amherst College	Amherst, Mass	W. C. Esty	Professor of mathematics.
84	Massachusetts Institute of Technology.	Boston, Mass	J. D. Runkle	Professor of mathematics.
85	Williston Seminary	Easthampton, Mass.	W. C. Boyden	Instructor in mathematics.
86	Williams College	Williamstown, Mass	T. H. Safford	Professor of astronomy.
87	Polytechnic Institute	Worcester, Mass.	J. E. Sinclair	Professor of higher mathematics.
88	Mount Holyoke Female Seminary.	South Hadley, Mass.	E. W. Bardwell	Director of observatory.
89	Michigan Mining School	Houghton, Mich	R. M. Edwards	Professor of mining and engineering.
90	Battle Creek College	Battle Creek, Mich.	J. H. Haughey	Mathematical department.
91	Adrian College	Adrian, Mich	G. B. McElroy	Chairman of the faculty.
92	Hillsdale College	Hillsdale, Mich	A. E. Haynes	Professor of mathematics.
93	Minnesota State University.	Minneapolis, Minn.	J. F. Downey	Professor of mathematics and astronomy.
94	Hamline University of Minnesota.	Hamline, Minn	E. F. Mearkle	Professor of mathematics.
95	Washington University	St. Louis, Mo	C. M. Woodward	Do.
96	Kansas City Ladies' College.	Independence, Mo	J. M. Chaney	President.
97	Missouri State University.	Columbia, Mo	W. B. Smith	Professor of mathematics and astronomy.
98	School of Mines, University of Missouri.	Rolla, Mo	W. H. Echols	Professor of applied mathematics.
99	College of the Christian Brothers.	St. Louis, Mo	Brother Paulian	President.
100	William Jewell College	Liberty, Mo	J. E. Clark	Professor of mathematics.
101	Drury College	Springfield, Mo	A. F. Amadon	Professor of mathematics and physics.
102	Cooper Normal College	Daleville, Miss	T. F. McBeath	President.
103	Agricultural and Mechanical College.	Starkville, Miss	H. C. Davis	Acting professor of mathematics.
104	University of Mississippi.	University P. O., Miss.	C. M. Sears	Professor of mathematics.
105	Doane College	Crete, Nebr	A. B. Show	Librarian.
106	University of Nebraska	Lincoln, Nebr	H. E. Hitchcock	Professor of mathematics.
107	New Hampshire College of Agriculture and Mechanic Arts.	Hanover, N. H	C. H. Pettee	Do.
108	Wake Forest College	Wake Forest, N. C.	L. R. Mills	Professor of pure mathematics.
109	University of North Carolina.	Chapel Hill, N. C	J. L. Love	Associate professor of mathematics.
110	Trinity College	Randolph County, N. C.	J. M. Bandy	Professor of mathematics.
111	Rutherford College	Burke County, N. C.	R. L. Abernethy	President.
112	College of the Sacred Heart.	Vineland, N. J	P. O'Connor	Professor of mathematics.
113	Niagara University	Niagara, N. Y	E. A. Antill	Do.
114	Union College	Schenectady, N. Y	B. H. Ripton	Do.
115	Columbia College	New York, N. Y	W. G. Peck T. S. Fiske	Do. Tutor in mathematics.

(a) UNIVERSITIES AND COLLEGES—Continued.

	Name of institution.	Location.	Name of person reporting.	Title or position of person reporting.
116	University of Rochester	Rochester, N. Y.	G. D. Olds	Professor of mathematics.
117	St. Lawrence University	Canton, N. Y.	H. Priest	Professor of mathematics and physics.
118	The College of the City of New York.	New York, N. Y.	A. S. Webb	President.
119	Syracuse University	Syracuse, N. Y.	J. R. French	Professor of mathematics.
120	U. S. Military Academy	West Point, N. Y.	E. W. Bass	Do.
121	Packer Collegiate Institute.	Brooklyn, N. Y.	W. L. C. Stevens	Professor of mathematics and physics.
122	Brooklyn Collegiate and Polytechnic Institute.	....do.	R. Sheldon	Professor of pure mathematics.
123	Ohio University	Athens, Ohio	William Hoover	Professor of mathematics.
124	Ohio State University	Columbus, Ohio	R. D. Bohannan	Professor of mathematics and astronomy.
125	Miami University	Oxford, Ohio	J. V. Collins	Do.
126	Case School of Applied Science.	Cleveland, Ohio	C. Staley	President.
127	Hiram College	Hiram, Ohio	C. Bancroft	Professor of mathematics and astronomy.
128	Oberlin College	Oberlin, Ohio	H. C. King	Associate professor of mathematics.
129	Denison University	Granville, Ohio	J. L. Gilpatrick	Professor of mathematics.
130	Marietta College	Marietta, Ohio	O. H. Mitchell	Professor of mathematics and astronomy.
131	Buchtel College	Akron, Ohio	C. S. Howe	Do.
132	University of Cincinnati	Cincinnati, Ohio	H. T. Eddy	Professor of mathematics, civil engineering, and astronomy.
133	Pacific University	Forest Grove, Oregon.	W. N. Ferrill	Professor of mathematics.
134	The State Agricultural College of Oregon.	Corvallis, Oregon	J. D. Letcher	Professor of mathematics and engineering.
135	Dickinson College	Carlisle, Pa	F. Durell	Professor of mathematics.
136	Bryn Mawr College	Bryn Mawr, Pa	Charlotte A. Scott.	Associate professor of mathematics.
137	Pardee Scientific Department of Lafayette College.	Easton, Pa	J. G. Fox	Professor of civil and topographical engineering.
138	Lehigh University	South Bethlehem, Pa.	C. L. Doolittle	Professor of mathematics.
139	Swarthmore College	Swarthmore, Pa	S. J. Cunningham	Do.
140	Haverford College	Haverford, Pa	Isaac Sharpless Frank Morley	Professor of mathematics. Instructor in mathematics.
141	Muhlenberg College	Allentown, Pa	D. Garber	Professor of astronomy.
142	Central Pennsylvania College.	New Berlin, Pa	H. R. Kelly	Professor of mathematics.
143	Brown University	Providence, R. I	N. F. Davis	Assistant professor of mathematics.
144	Furman University	Greenville, S. C	C. H. Judson	Professor of mathematics and mechanical philosophy.
145	University of South Carolina.	Columbia, S. C	E. W. Davis	Professor of mathematics and astronomy.
146	Columbia Female College.	....do	J. G. Clinkscales	Professor of mathematics.

(*a*) UNIVERSITIES AND COLLEGES—Continued.

	Name of institution.	Location.	Name of person reporting.	Title or position of person reporting.
147	Fisk University	Nashville, Tenn	H. H. Wright	Professor of mathematics and instructor in vocal music.
148	University of Tennessee	Knoxville, Tenn	Wm. W. Carson	Professor of mathematics and civil engineering.
149	Grant Memorial University.	Athens, Tenn	D. A. Bolton	Professor of mathematics.
150	Bethel College	McKenzie, Tenn	W. W. Hamilton	Do.
151	Chattanooga University	Chattanooga, Tenn	E. A. Robertson	Do.
152	Vanderbildt University	Nashville, Tenn	Wm. J. Vaughn	Do.
153	University of Texas	Austin, Tex	G. B. Halsted	Professor of pure and applied mathematics.
154	Agricultural and Mechanical College of Texas.	College Station, Tex.	L. L. M'Innis	Professor of mathematics.
155	Randolph-Macon College	Ashland, Va	R. B. Smithey	Do.
156	Emory and Henry College.	Emory, Va	S. M. Barton	Do.
157	Hampden-Sidney College	Hampden Sidney, Va	J. R. Thornton	Do.
158	University of Virginia	Charlottesville, Va	C. S. Venable	Do.
159	Bethel Military Academy.	Bethel Academy P. O., Va.	E. S. Smith	Instructor in higher mathematics and modern languages.
160	Virginia Agricultural and Mechanical College.	Blacksburg, Va	J. E. Christian	Professor of mathematics and civil engineering.
161	Polytechnic Institute	New Market, Va	W. H. Smith	President.
162	Vermont Methodist Seminary.	Montpelier, Vt	E. A. Bishop	Principal.
163	Norwich University	Northfield, Vt	J. B. Johnson	Professor of mathematics.
164	University of Washington.	Seattle, Wash	J. M. Taylor	Do.
165	Wheeling Female College.	Wheeling, W. Va	H. R. Blaisdell	President.
166	West Virginia College	Flemington, W. Va	T. E. Peden	Do.
167	Ripon College	Ripon, Wis	C. H. Chandler	Professor of mathematics and physics.
168	Beloit College	Beloit, Wis	T. A. Smith	Do.

Are students entering your institution thorough in the mathematics required for admission?

"No," "not generally," "by no means," "seldom:" 3, 5, 6, 7, 8, 9, 12, 13, 14, 16, 17, 19, 20, 21, 22, 23, 25, 27 (but growing better), 29, 31, 33, 36, 39, 40, 42, 43, 44, 47, 49, 51, 52, 53, 56, 57, 58, 59, 60, 61, 63, 66, 67, 68, 69, 70, 72, 73, 76, 80, 82, 83, 86, 88, 91, 92, 93 (but marked improvement every year), 94, 96, 97, 99, 102, 104, 105, 109, 110, 111, 113, 117, 160, 161, 162, 163, 164, 165, 166, 167, 168 (and this is one of the evils of our times).

"Fairly so," "reasonably so:" 10, 11, 15, 30, 32, 34, 37, 46, 55, 84, 95.

"Not as thorough as we desire:" 35, 38, 41, 71, 74, 87, 107, 108, 114, 116.

"Yes:" 1 (most of them), 28, 45 (generally), 54 (or fail to enter), 64, 65 (usually), 78 (a fair proportion), 79 (generally), 89, 90 (or fail to enter), 100 (or they are placed in preparatory department), 101, 112 (generally), 115.

Is the mathematical teaching by text-book or by lecture?

This question was answered by all who sent in reports. The following forty-six answers were "by text-book," without indicating that any lectures whatever were given: 6, 7, 11, 12, 14, 17, 18, 20, 21, 24, 25, 26, 29, 30 (it is impossible to teach mathematics by lecture), 32, 33, 37, 44, 50, 51, 53, 63, 65, 68, 73, 74, 85, 88, 101, 102, 105, 107, 109, 113, 114, 125, 133, 141, 142, 147, 149, 150, 151, 161, 163, 164.

The following sixty-five answers were "mainly by text-book," "text-book principally," "text-book as a basis," "text-book and informal lectures," or some similar phrase, indicating that the text-book predominated: 1, 3, 8, 9, 15, 19, 22, 28, 31, 34, 38, 40, 41, 43, 45, 47, 48, 52, 55, 57, 58, 59, 60, 62, 66, 71, 72, 75, 77, 80, 86, 87, 89, 90, 91, 92, 93, 95, 97, 100, 103, 106, 110, 116, 118, 119, 120, 122, 123, 128, 133, 134, 135, 136, 137, 143, 144, 148, 155, 156, 159, 162, 166, 167, 168.

The following fifty-five answered "by both," or "by text-book and lecture," without saying which predominated: 2, 4, 5, 10, 16, 23, 27, 35, 36, 39, 42, 46, 49, 54, 56, 61, 64, 67, 69, 70, 76, 78, 79, 82, 83, 84, 94, 96, 98, 99, 104, 108, 111, 112, 115, 117, 121, 124, 126, 127, 129, 130, 131, 132, 139, 140, 145, 146, 152, 153, 154, 157, 158, 160, 165. The answer of number 13 is "by lectures, except elementary geometry;" and of 81, "by lecture."

What mathematical journals are taken?

The following answered "none:" 1, 3, 5, 6, 7, 9, 14, 15, 17, 20, 21, 22, 24, 25, 29, 30, 32, 33, 36, 39, 40, 43, 44, 45, 48, 49, 50, 53, 56, 57, 58, 59, 60, 63, 65, 69, 70, 71, 73, 74, 76, 77, 78, 79, 80, 81, 82, 85, 87, 88, 89, 91, 94, 96, 99, 101, 102, 103, 104, 105, 107, 108, 112, 114, 116, 117, 121, 122, 128, 129, 133, 134, 141, 147, 149, 150, 151, 159, 160, 162, 163, 166, 167, 168. Some of these answers were "none by the college," "none that are purely mathematical," "several scientific and engineering journals," but most of them were simply "none." In addition to this list, numbering eighty-four, we may safely add thirty-three that did not answer this question in their report, making one hundred and seventeen institutions out of one hundred and sixty eight that take no mathematical journal of any sort devoted to pure mathematics.

The following reported as taking simply the American Journal of Mathematics: 10, 27, 28, 54, 55, 72, 75, 84, 95, 131, 145.

The following, as taking simply the Annals of Mathematics: 4, 8, 13, 16, 23, 42, 64, 90, 100, 110, 148, 157.

University numbered 11 is taking *d, k, l*, marked below;* 12, *b, d, n;* 19, *k;* 35, *b, d, i, s;* 37, *k;* 38, *b, d, q, n;* 41, *d, m, l;* 46, *b, k*, etc.; 47, *b, d;* 51, *b*, nearly all the foreign journals are expected after this year; 67, all the leading ones; 68, nearly all mathematical journals; 83, *b, d, j;* 86, *b, h, u;* 92, *k, l, o*, etc.; 93, *b, d;* 97, *b, j*, etc.; 98, *b, d, n;* 106, *a, b*, Jahrbuch d. Vortsch. d. Math.; 109, *d, j, p;* 111, any we can get; 115, *a, b, c, d, e, f, g, h, i, j, m, n, p, s, t, u;* 152, *b, d, j, m, p, s, t;* 153, *b, d, t;* 154, *d, s;* 155, *d, p;* 158, *d, p*, and others; 161, *q;* 164, *d, k, l.*

Are there any mathematical seminaries or clubs, and how are they conducted?

All answered in the negative, except the following:

15. No clubs, unless special classes for voluntary and outside reading be so designated. Such classes are conducted like all other classes.

38. A club. The meetings of the club occur on alternate Tuesdays. Membership about 25; topics are assigned to or chosen by the student at his option; assistance is given him as he may need. The work is pedagogical, rather than original.

41. One. Reading and exposition of the more difficult parts of Williamson's Calculus.

51. In connection with the Science Club; by lectures.

67. There is a mathematical society, in which there is free choice of subjects for communication, and there are two or three seminaries for post-graduate students, conducted by the teacher on special lines.

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*(*a*) Acta Mathematica. (*b*) American Journal of Mathematics. (*c*) Annali di Matematica. (*d*) Annals of Mathematics. (*e*) Archiv d. Math. u. Physik. (*f*) Bulletin des Sciences Math. et Astron. (*g*) Bulletin de Société Math. (*h*) Comptes Rendus. (*i*) Journal de Math. (Lionville). (*j*) Journal f. reine u. angew. Math. (Crelle). (*k*) Mathematical Magazine. (*l*) Mathematical Visitor. (*m*) Mathematische Annalen. (*n*) Messenger of Mathematics. (*o*) Proceedings London Math. Society. (*p*) Nouvelles Annales de Math. (*q*) School Visitor. (*r*) School Messenger. (*s*) Quarterly Journal of Math. (*t*) Zeitschrift f. Math. u. Phys. (*u*) Zeitschrift f. Vermessungskunde.

Are there any mathematical seminaries or clubs, and how are they conducted?—Continued.

115. Yes, one. It proposes to discuss the literature of mathematics, to solve problems given by members, and to make original investigations.

136. No clubs, but seminaries, through part of regular course, but not very formal; they are intended to afford students opportunity of working problems under guidance.

143. There are men in each class studying for honors. The principal part of their work is the solution of original problems. I meet them frequently for discussions and suggestions.

158. No clubs; the lectures and recitations regularly extend through an *hour and a half*, and at each original solutions of problems are given, and next time are called for. Each meeting of each class is a seminarium.

Are there any scholarships or fellowships for graduate students in mathematics?

All who answered this said "no," except the following:

67. Yes.

84. None for mathematics exclusively.

86. One is occasionally given to a man of high promise.

93. Fellows are allowed to choose mathematics.

97. One fellowship; two scholarships await instantly State appropriation for support.

115. Yes, an annual fellowship in science.

136. One fellowship awarded yearly to a *properly qualified* graduate of any college.

145. Yes, there is one, retainable for two years.

159. Occasionally conferred on deserving students wishing to prosecute their studies at other institutions.

Is the percentage of students electing higher mathematics increasing or decreasing?

The following reported "increasing:" 1, 5, 6, 8, 9, 12, 15 (among scientific students), 16, 21, 24, 25, 26, 27, 28, 31, 32, 33 (?), 35, 37, 38, 39, 40, 42 (?), 46, 50, 51, 53, 54, 55, 56 (with gentlemen), 57, 59, 63, 73, 75, 76, 78, 79, 82, 88, 90, 93, 97, 98, 99, 101, 102, 106, 108, 111, 112, 113, 116, 117 (?), 119, 123, 124, 125, 126, 128, 129, 132, 137, 140, 142, 144, 145, 149, 150, 151, 153, 154, 156, 157, 158, 159, 160, 161, 164, 165, 166.

The following reported "about constant," "neither increasing nor decreasing:" 3, 4, 23, 30, 48, 67, 70, 74, 80, 84, 86, 91, 92, 100, 104, 107, 114, 115, 118, 122, 127, 130, 131, 141, 147, 148, 162, 163.

The following reports indicated a "decrease:" 15 (among classic students), 44, 49, 56 (with ladies), 83.

The following reported that none of the mathematical studies were elective: 10, 11, 17, 18, 20, 22, 36, 41, 45, 47, 52, 65, 71, 85, 87, 94, 96, 103, 105, 108, 120, 133, 135, 139, 142, 146, 155.

Does the interest in mathematics increase as students advance to higher subjects?

"Yes:" 3, 5, 6, 8, 10, 11, 13, 16, 17, 18, 21, 22, 23, 26, 28, 29, 30, 32, 35, 36, 37, 38, 40, 41, 42, 46, 48, 50, 51, 52, 53 (very much), 54 (generally), 55, 57, 59, 60, 61, 63, 65, 66 (?), 73, 77, 79, 82, 84, 87 (generally), 88, 90, 92, 93, 94, 95, 98, 101, 102, 104 (?), 105 (?), 108, 111, 112, 113, 116, 118, 119, 122, 123, 126, 131, 132, 133, 134, 139, 140, 145 (generally), 146, 147, 150, 151, 153, 154, 156, 157, 158 (emphatically so), 159, 160, 161, 163, 164, 165, 166, 167.

"With the best students only," "with those students whose mental tendencies are along mathematical lines," or some similar remark: 1, 4, 9, 12, 24, 25, 31, 33, 34, 43, 44, 45, 47, 56, 58, 68, 70, 71, 72, 74, 75, 76, 80, 81, 85, 107, 114, 120, 121, 124, 125, 127, 128, 129, 130, 136, 137, 141, 142, 143, 148, 149, 152, 155.

"The interest increases so long as the student sees the bearing of his work upon practical scientific investigation or can be assured that it has such a bearing." "It increases as application to practical matters is shown:" 15, 19, 69, 103.

"All who understand the principles show a growing interest," "where proper preparation on the part of the student has been attended to and the teacher is a live man, it does" (110), "The interest is according to the clearness of apprehension of mathematical truths. Hence, the more evolved or abstruse the matter, the greater the interest to those who succeed" (144): 27, 91, 97, 110, 115, 144.

"No:" 2, 7, 20, 34 (for poor students), 39, 43 (for the majority), 49, 86, 100, 106, 109, 135, 162, 168.

Are prizes awarded for excellence of daily class-room work, or for success in orginal research?

"No prizes awarded:" 1, 4, 6, 7, 8, 10, 13, 14, 16, 17, 19, 20, 21, 23, 28, 29, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 43, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 58, 59, 60, 61, 63, 64, 65, 68, 69, 70, 71, 72, 73, 76, 78, 79, 80, 81, 84, 85, 87, 88, 89, 90, 91, 93, 94, 95, 98, 100, 101, 103, 104, 105, 106, 107, 108, 111, 112, 114, 115 (except class honors), 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 135, 136, 138, 139, 140, 141, 142, 144, 145, 147, 148, 149, 150, 151, 152, 154, 156, 157, 158, 160, 161, 164, 166, 167, 168.

"For both:" 2, 9, 36, 67 (bestowal of scholarships and fellowships is based upon both the considerations), 82, 99, 134, 155 (but I do not believe in prizes).

"For work in the class-room:" 18, 22, 25, 26, 62, 66, 77, 86, 96, 118, 137, 143, 159, 162.

"For outside work, not generally original:" 15, 116.

"For original investigation only:" 97, 102, 117.

"Yes, prizes are awarded:" 3, 5, 11, 12, 24, 27, 54, 57, 74 (scholarship of $300 to best Sophomore in mathematics), 92, 97, 109, 110, 113, 165.

What mathematical subjects are preferred by students?

The answers given point to no definite conclusion. For want of space, they are here omitted, except the following: "*Their preferences are generally for the particular subject which they have had the best elementary training in*" (148).

Are topics assigned to students for special investigation?

1. Yes.
2. Problems are proposed.
3. Sometimes prize problems are given to students.
5. Yes.
6. Yes.
7. For the higher classes.
8. Yes; often.
9. Yes.
10. Not in general.
11. To a small extent.
12. Occasionally.
14. No.
17. Yes.
18. Sometimes.
19. Occasionally.
20. No.
21. Yes.
22. No.
23. Yes.
24. Yes.
25. No.
26. There are.
27. Independent problems given in all the classes for solution, reported on paper.
28. Once each term.
29. No.

Are topics assigned to students for special investigation?—Continued.

30. Not much. The man who pursues original investigation with the average student will make a failure.
31. We have not been accustomed to do so.
32. Not to any extent that deserves mention.
33. Practically, no.
34. Not to under-graduates. There are no graduate students in mathematics at present.
35. Yes.
36. Yes; but necessarily elementary.
37. Yes, sir.
38. Yes, in connection with the club and for graduation theses.
39. Yes.
40. Sometimes; but our classes are generally so closely occupied by their various studies, there is but little time for extra work.
41. No.
42. Yes.
43. Yes.
44. Frequently.
45. They are, and form a very essential part of the work.
46. Students in the higher classes are assigned such topics.
47. None advanced enough.
48. Yes.
49. Sometimes.
51. Yes.
52. Occasionally.
53. Yes.
54. Yes.
55. No.
56. Yes.
58. Yes.
59. Yes; with good success.
60. Yes.
63. Yes.
64. No.
65. No.
67. They are in the seminaries.
68. No.
69. A few.
70. To a limited extent.
71. No.
72. Not to any great extent.
73. Occasionally.
74. In elective divisions.
76. To some extent.
77. Rarely.
78. Yes.
79. No.
81. Yes.
82. Yes.
84. No.
85. No.
86. Yes, to post-graduates.
87. Yes.
88. Not often.

Are topics assigned to students for special investigation?—Continued.

89. In applied mechanics, yes.
90. Not for original investigation, but otherwise.
91. Occasionally.
92. Yes, for thesis in general geometry and calculus.
93. Yes, especially in elementary geometry, analytical geometry, and calculus.
94. Yes.
95. Rare.
96. Only to a limited extent.
97. Such assignment has hitherto been only exceptional, hereafter to be made regular.
98. In applied mathematics theses are required on special subjects, and original investigation encouraged.
99. Yes.
100. No.
101. Yes.
102. Yes, in all the different branches, especially in applied mathematics.
103. The graduating and other theses are on subjects divided among the departments.
104. No.
105. No.
106. They are.
107. Yes.
108. Yes.
109. No.
110. Yes, this is encouraged in all the classes, but is secured best in the higher classes.
111. Yes.
112. Occasionally original theorems and problems are given.
114. Occasionally, results submitted in original theses.
115. No, except the work done in the seminary.
116. Yes, in all classes of all departments.
117. No.
118. Yes, to a large extent in geometry.
119. In pure mathematics very seldom; not in applied mathematics.
120. Yes.
121. No.
122. No.
123. No.
124. We have this in view for next term.
125. None as yet.
126. Yes.
127. Yes.
128. Yes.
129. To some extent.
130. Yes.
131. Occasionally.
132. No.
133. To some extent.
134. Occasionally.
135. No.
136. I should consider this exercise profitable only to *very* advanced students; and have not had occasion to employ it yet.
137. Yes, to some extent.
138. No.

Are topics assigned to students for special investigation?—Continued

139. No.
140. Occasionally.
141. Not to any extent.
142. Yes, in applied geometry, surveying, and physics.
143. Occasionally to advanced students.
144. Only exercises in theorems and problems.
145. Yes.
146. Yes.
147. No.
148. The solution of problems related to the recitations is required. Nothing else.
149. No.
150. No.
152. No.
153. Yes.
154. Yes.
155. In the higher classes topics are occasionally assigned.
156. No.
157. Original exercises are given at intervals.
158. To graduate students, candidates for the degree of Ph. D.
159. But few outside of text-book.
160. Yes.
161. No.
162. No.
163. No.
164. Yes.
165. Yes.
166. Yes.
167. To some extent.
168. Occasionally.*

(a) Is any attention given to the history of mathematics? (b) Does it make the subject more interesting?

(a) "Yes:" 1, 5, 9, 18, 34, 35, 37, 39, 42, 44, 46, 48, 52, 53, 61, 63, 64, 65, 72, 80, 81, 90, 92, 97, 98, 99, 101, 102, 108, 112, 114, 116, 123, 126, 129, 131, 135, 136, 138, 145, 152, 153 (great), 154, 156, 157, 158, 160, 164.

(a) "Very little," "only incidentally," "not much", etc.: 4, 6, 8, 11, 12, 13, 16, 17, 19, 21, 23, 24, 25, 27, 28, 30, 38, 40, 41, 43, 45, 47, 51, 54, 55, 56, 58, 59, 60, 66, 67, 68, 73, 74, 75, 76, 78, 79, 82, 86, 88, 91, 93, 94, 100, 104, 106, 107, 111, 115, 118, 119, 120, 121, 124, 125, 127, 128, 130, 133, 137, 143, 144, 147, 151, 159, 163, 168.

(a) "No:" 3, 7, 10, 14, 20, 22, 29, 32, 33, 36, 49, 50, 57, 69, 70, 71, 77, 83, 85, 87, 89, 95, 96, 103, 105, 109, 110, 117, 122, 132, 134, 139, 140, 142, 146, 148, 149, 150, 161, 162, 166, 167.

(b) "Yes," "it does," "most decidedly," was the experience of all who had given any attention, in the class-room, to mathematical history, except the following, who were in doubt: 11, 15, 16, 47, 56, 76, 104, 120. Even these were inclined to say "yes." *No one* answered that it did *not* make the subject more interesting—*a clear case.*

How does analytical mathematics compare in disciplinary value with synthetical?

1. I regard both methods equally important.

4. I think synthetical has much the greater disciplinary value; analytical has much the greater value for practical application. Analysis is the principal tool for investigation and work.

* Widely different views seem to be implied in the above answers as to what constitutes a "topic for special investigation."

How does analytical mathematics compare in disciplinary value with synthetical?—Cont'd.

5. Analytical superior.

6. The former used more largely in the Grammar Department.

8. Analytical mathematics gives the better mental discipline.

9. I think both necessary to full mental development, but if I were obliged to choose I should prefer analytical.

10. I can not say fairly, for my teaching has been wholly in analytical mathematics. In my studies I prefer that method.

11. I use combination of both and so can not well answer.

12. The development of reason is more regular, rapid, and substantial in geometry than in any other branch of the mathematical course. For advanced students I would count algebraic analysis a superior discipline.

13. It seems to be a question of individuality.

15. I regard analytical mathematics as the more valuable and the more important.

16. The former is superior.

17. It is superior.

18. Analytical seems to be better.

19. Common geometry, considered as an application of logic, especially in the demonstration of *easy* "riders" and in very simple exercises in constructions, is of pre-eminent value to quite young and undisciplined minds. At different stages each has its peculiar and really unmeasurable value.

20. They are of equal value.

21. I have not data enough for an opinion.

22. Superior, if the two are divorced; but the synthetical should be united with it.

23. Favorably.

24. Analytical greater.

25. With the majority of students more satisfactory results are obtained through the synthetical method of reasoning.

26. Analytical preferred.

27. As a rule, I have found that students stand better in geometry than in algebra. When *analytical* geometry is clearly comprehended, it affords the best discipline for the mind.

28. Synthesis seems to give better discipline.

30. Analytical preferable.

31. The former is a better test for form and figure, the latter seems to tax the memory.

32. We have no classes sufficiently advanced to test the relative value extensively.

33. If the work be the same in both, the synthetical.

34. My own preference is analytical.

35. Could not get along with either method left out (Professor Shattuck). Each has its special function; as well ask whether braces or tie-rods are of most service in a bridge-truss (Regent Peabody).

36. Synthetical more valuable.

37. Disciplinary value of former is greater.

38. We give the analytical the first consideration after the student is led up to it.

40. Superior.

41. I think the former the more valuable as an instrument of research, the latter as a means of discipline.

42. The analytical is more valuable simply as a means of discipline.

43. The synthetical is better for younger students; the analytical for those more mature.

How does analytical mathematics compare in disciplinary value with synthetical?—Cont'd.

45. It is evidently far superior.

46. Each affords excellent discipline.

47. It is superior.

48. For college grade we think the analytical produces the best results.

49. Better.

50. Better for discipline, but we have not used it as yet.

51. Somewhat superior in value.

52. Superior.

53. Analytical training is more beneficial.

54. Favorably.

55. They are superior.

56. Prefer the analytical.

57. We use both methods, but give preference to former.

58. Can we do without either? I should say both are necessary, but analytical is less taught.

59. Analyzing the whole into its elements is valuable, but building the whole from elements is *very* valuable.

60. Superior for advanced students.

63. The analytical the more valuable.

65. Analytic mathematics is far superior in its disciplinary value.

67. The latter is probably the more valuable discipline in early stages of a mathematical education; but after the elements of geometry are mastered, probably the reverse is true.

68. In general we prefer analytical methods.

69. Latter preferred.

70. Doubtful; students prefer synthetical.

73. In my judgment the analytical method is to be preferred.

75. For the average student the synthetical gives better results.

76. I think analytical mathematics better for mental discipline.

79. Its disciplinary value is less than that of the synthetical.

80. The synthetical is more valuable.

81. They interact; but the latter is an indispensable prerequisite for the former.

82. I should place analytical as greater in disciplinary value.

84. Analytical is inferior to synthetic.

86. Both methods are essential and I am not aware of any difference. Perhaps I do not understand the question.

87. Superior; yet this depends, perhaps, on the mind of the individual student.

90. Very favorably, so far as our experience has gone.

91. I prefer the former for advanced students—the latter for beginners, or students of a low grade.

92. Synthetical seems best for the less advanced students.

93. I do not believe that the two can be separated and compared. I believe with Sir William Hamilton, "Analysis and synthesis are only the two necessary parts of the same method. Each is the relative and correlative of the other." Neither without the other would be of much value.

94. The synthetical is absolutely necessary as a foundation of good work; after the foundation, the former is desirable.

95. Do you mean graphical (or geometrical) by synthetical? I think descriptive geometry has the finest disciplinary value.

97. As commonly taught, unfavorably; as taught here, with special stress on Morphology and by aid of determinants, very favorably.

98. In favor of the former.

How does analytical mathematics compare in disciplinary value with synthetical?—Cont'd.

99. Are in favor of the analytical.

101. Superior.

102. That depends upon the peculiar natural bent of the pupil's mind. For some, analysis, and for others, synthesis is more valuable.

104. Can not institute comparison, we use both in combination.

106. I would answer this by saying, that I consider special geometry better for mental discipline than analytical geometry, and geometry better than algebra.

109. Analytical superior.

110. In my judgment the analytical is so far superior to the synthetical that there is left little room for comparison. Permit me to say that reason wants *light*, not *darkness*.

111. They are about equal. We use Peck's and Davies' methods.

112. The comparison is in favor of analytical mathematics.

113. The analytical method, in my opinion, produces better results than the synthetical.

114. Superior.

115. Each has its special value; both are desirable (Professor Peck). Analytical gives the more rigorous training. Each plays its own part (Tutor Fiske).

116. Synthetical better for training in formal logic; in other respects analytical is unquestionably superior.

117. Synthetical seems to give better results.

118. For older students the analytical methods are superior; for those below the Sophomore class, this is doubtful.

119. I think the analytical is better.

120. Analytical mathematics predominates here, and therefore has the greater disciplinary value. If comparing equal times in the two, I should say synthetical.

122. Synthetic best for discipline; analytic best for use.

123. Former is better.

125. It is hard for me to answer this. Perhaps the latter is superior for dull or average students, while the former is preferable for the more able.

126. Both necessary for proper discipline.

128. Well.

129. Analytical mathematics is the better.

130. Sometimes seems to me superior; sometimes seems to me inferior, depending upon the mental character of the student.

132. We teach no synthetical mathematics in the university, except a book of elementary mechanics, which is good in its place, but analytical mathematics alone develops real mathematical power.

133. I regard the analytic method as much superior in way of developing habit and power of investigation.

134. I use both and would not willingly part with either. Deem them of about equal value.

135. Equally valuable though in different way.

136. I should be inclined to give preference to analytical; but where there is a strong natural mathematical bent, possibly more discipline is derived from synthetical mathematics.

137. Rather unfavorably with the average student.

139. Superior.

140. Better.

143. Both valuable; both necessary.

144. Analytical is favorable for advanced students; synthetical, for younger students.

How does analytical mathematics compare in disciplinary value with synthetical?—Cont'd.

145. The value of the discipline depends upon the closeness of the student's application rather than upon the methods employed.

146. Superior to it.

147. I consider the analytical far superior to the synthetical.

148. In my opinion, the analytical is far superior to the synthetical.

149. I think the analytic is better.

150. The former is of more disciplinary value than the latter.

151. The analytical mathematics in most cases most satisfactorily fulfils the end or object of mathematical study.

152. The former, in my opinion, is preferable in almost every respect.

153. Analytical mathematics is very far inferior to synthetic in disciplinary value.

155. Analytical mathematics has, I think, a higher disciplinary value than synthetical.

156. The synthetical is more valuable, I think, but by no means should either be adopted to the exclusion of the other.

158. Impossible to make a comparison in so short space.

159. I regard analytical mathematics as possessing higher disciplinary value, when properly taught.

160. Analytical mathematics is superi or to synthetical in disciplinary value.

161. Favorably, both should be used.

163. I favor analytical mathematics.

164. Analysis is superior in disciplinary value.

165. Superior.

166. They are about equal.

167. Any true method of study seems to me to use them both, with so frequent changes that comparison is difficult. Moreover, their relative value differs with different pupils.*

(*a*) *What method of treating the calculus do you favor, that of limits, the infinitesimal, or some other?* (*b*) *Does the infinitesimal seem rigorous, and to satisfy the mind of the student?*

1. (*a*) Limits. (*b*) Does not satisfy the student.

3. Calculus is not taught in this college.

4. (*a*) I favor the method of rates, though I use the method of limits and infinitesimals—the latter in mechanics. (*b*) It does not in my experience.

5. We do not teach it.

8. (*a*) That of limits. (*b*) At first it does not seem rigorous to the student, nor to satisfy his mind.

9. (*a*) We think, with many others, the subject needs both. (*b*) Not sufficiently so, and hence the advantage of calling "limits" to its aid.

10. (*a*) Calculus is not taught here. Personally I prefer infinitesimals. (*b*) I think so, more than that of limits, which is better for the mathematician than the student.

11. (*a*) I favor none exclusively. I teach "rates", "infinitesimals", and "limits." (*b*) It does not seem rigorous.

12. (*a*) The method of limits (now made familiar in geometry) seems most satisfactory. (*b*) Not to beginners. Later this method should be studied also.

13. (*a*) Defining $f'(a)$ as co-efficient in development: $f(x)=f(a)+f'(a)(x-a)+\ldots\ldots$. (*b*) It does not seem rigorous as usually represented, but could be made so, but I doubt whether for beginners.

15. (*a*) I favor the method known as that of "rates." (*b*) I think not.

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*For collateral reading on this question see President C. W. Eliot's article, "What is a Liberal Education?" in the Century Magazine, June, 1884; Report of the (English) Commission in 1852; Report of the French Commissioners in 1870.

(*a*) *What method of treating the calculus do you favor, that of limits, the infinitesimal, or some other?* (*b*) *Does the infinitesimal seem rigorous, and to satisfy the mind of the student?*—Continued.

16. (*a*) A combination of limits and infinitesimals. (*b*) Combined with the method of limits, it does; alone, no.

17. (*a*) Infinitesimal; a little about limits. (*b*) I have never yet had a student to whom I could not make it perfectly satisfactory.

18. (*a*) Limits. (*b*) No.

19. (*a*) In theory, Buckingham's "direct method of rates;" practically, the infinitesimal as set forth by Olney and others, on account of its practical advantages. (*b*) The philosophy at the base of this method seems to involve one in a maze of absurdities, but I have had too little experience with pupils in the calculus to speak positively upon this point.

20. We do not teach the calculus.

21. (*a*) Doctrine of limits. (*b*) It does not.

22. (*a*) That of limits. (*b*) Not in every case.

23. (*a*) Limits.

24. (*a*) Limits.

25. (*a*) The method of limits. (*b*) It does not.

26. (*a*) Limits.

27. (*a*) I explain and illustrate both limits and infinitesimal analysis. (*b*) When properly explained and illustrated, I think it does.

28. (*a*) The infinitesimal. (*b*) It does, *i. e.*, generally.

30. Method of limits, not the Newtonian of passing to zero.

31. (*a*) Have been accustomed to take the limits.

32. We have no classes in calculus.

33. (*a*) Had experience only with infinitesimals. (*b*) No; certainly Olney's presentation can be improved upon.

34. (*a*) The infinitesimal, if properly presented. (*b*) Yes, when the student can appreciate mathematical reasoning.

35. (*a*) Teach both methods, do not favor either. (*b*) Yes; Lagrange's method of derived functions is considered the best in theory (Professor Shattuck).

36. (*a*) Method of limits. (*b*) No.

37. (*a*) Limits.

38. (*a*) Method of limits. (*b*) No.

39. (*a*) Limits. (*b*) No.

40. (*a*) I teach the infinitesimal, prefer it in general. (*b*) Occasionally a student will not accept its theories; I then try him on limits and show him their relation.

41. (*a*) The latter, with a sprinkling of the former. (*b*) The infinitesimal method is just as rigorous, when understood, as the method of limits, but it is my experience that the latter more quickly removes the logical difficulties in the way of the *beginner*.

42. (*a*) The method of limits. (*b*) It does, provided its relation to the method of limits be shown; otherwise not.

43. (*a*) The infinitesimal. (*b*) Students have generally preferred it to the method of limits.

45. (*a*) The infinitesimal. (*b*) It does when it is known that results do not vary.

46. (*a*) I use the method of limits; the method of infinitesimals is also presented. (*b*) One method seems to do as well as the other if properly presented.

47. (*a*) Method of rates (see Taylor, Rice and Johnson, Buckingham).

48. (*a*) Generally by limits. (*b*) To the first part, yes; to the second part, generally not very satisfactory to those going over the subject for the first time.

49. (*a*) The infinitesimal. (*b*) Not always.

(*a*) *What method of treating the calculus do you favor, that of limits, the infinitesimal, or some other?* (*b*) *Does the infinitesimal seem rigorous, and to satisfy the mind of the student?*—Continued.

51. (*a*) Method of limits; use both. (*b*) Not so rigorous as that of limits.

52. (*a*) The infinitesimal. (*b*) So far as I know it does.

53. We have no classes in calculus.

55. (*a*) The calculus is not a part of our course of study; personally, I prefer the method of limits.

56. (*a*) The infinitesimal. (*b*) Yes.

57. (*a*) This is elective—no students yet.

58. (*a*) Have used the integral.

59. (*a*) Infinitesimal. (*b*) Yes, when the student is well drilled in what should precede.

60. (*a*) So far, the method of limits. (*b*) Have not found it so, generally.

62. We do not teach anything higher than trigonometry.

63. (*a*) That of limits. (*b*) No.

64. (*a*) Infinitesimal. (*b*) Yes.

65. (*a*) The infinitesimal. (*b*) Not entirely; but the ideas of the calculus are obtained better through this than any other method.

66. (*a*) That of limits. (*b*) Not altogether so.

67. (*a*) The two methods named are not essentially distinct; I regard the method of infinitesimals, based upon the doctrine of limits, as the best mode of treating the subject. (*b*) Not unless it is based upon the doctrine of limits.

68. (*a*) The method of rates, passing later to the method of limits.

69. No calculus.

70. (*a*) Infinitesimal. (*b*) Not entirely.

71. (*a*) The method of rates, combined with the method of limits. (*b*) It is very little used, and only after the others have been taken up.

72. (*a*) Limits. (*b*) Students have seemed satisfied when that method has been used.

73. (*a*) The infinitesimal. (*b*) Healthful.

74. (*a*) Infinitesimal method for students taking brief course. (*b*) Generally, yes.

75. (*a*) The infinitesimal. (*b*) By sufficient explanations.

76. (*a*) Limiting ratios preliminary to the more direct method of infinitesimals. (*b*) Somewhat, but often fundamental investigations are made more intelligible to the beginner by this method.

78. (*a*) Limits. (*b*) It does when properly taught.

79. (*a*) Limits. (*b*) No.

80. (*a*) The method of limits. (*b*) Yes, if the student persists until it is conquered.

81. (*a*) The first at the outset. All should be introduced (see Wundt, *Logik*).

82. (*a*) That of limits. (*b*) Hardly.

83. (*a*) Limits. (*b*) Yes.

84. (*a*) Both limits and infinitesimals. (*b*) Not when the two methods are presented together.

85. Calculus is not studied with us.

86. (*a*) Limits, decidedly. (*b*) Not until the student has mastered the method of limits.

87. (*a*) Infinitesimal. (*b*) Our students seem to understand this best.

88. (*a*) That of limits.

89. (*a*) Limits. (*b*) No.

90. (*a*) The infinitesimal on account of its simplicity, but the new method by General C. P. Buckingham is excellent. (*b*) Not always.

(*a*) *What method of treating the calculus do you favor, that of limits, the infinitesimal, or some other?* (*b*) *Does the infinitesimal seem rigorous, and to satisfy the mind of the student?*—Continued.

91. (*a*) The method of limits. (*b*) It does not.

92. (*a*) Infinitesimal. (*b*) It does.

93. (*a*) The fluxionary method. (*b*) Not entirely; it is taught from text-book—fluxionary by lectures.

94. (*a*) Infinitesimal. (*b*) Yes.

95. (*a*) Infinitesimal. (*b*) It does.

97. (*a*) The German method of limits, not the popular English and French.

98. (*a*) The rigorous method of limits. (*b*) *No;* there is an evident loss of faith at this point for students on first reading.

99. (*a*) We use both. (*b*) Yes.

100. (*a*) Limits. (*b*) Do not teach it.

101. (*a*) Newtonian fluxions. (*b*) No.

102. (*a*) Limits. (*b*) No; I find *few pupils* satisfied with it.

103. (*a*) Do not teach calculus; favor limits.

104. (*a*) We use both; sometimes we prefer one to the other.

106. (*a*) The infinitesimal for *practical* use, but that of *rates* as a logical basis. (*b*) Not as satisfactory as the theory of rates as given by Buckingham.

107. (*a*) Limits for general proof and infinitesimals for doing examples. (*b*) No, not alone.

108. (*a*) Limits.

109. (*a*) Limits.

110. (*a*) The method of limits is the only logical, or rational, way of treating it; though the infinitesimal has an advantage in application. (*b*) No; how a quantity can have another quantity taken from it and not decrease the quantity so diminished, is the skeleton that will not down.

112. (*a*) The infinitesimal. (*b*) Yes; both rigorous and satisfactory.

113. (*a*) The method of limits. (*b*) It does not.

114. (*a*) Limits. (*b*) Not perfectly.

115. (*a*) The method of infinitesimals, if properly taught. (*b*) Perfectly so, when properly taught (Professor Peck). (*a*) Limits. (*b*) Only when explained in connection with that of limits (Tutor Fiske).

116. (*a*) Limits. (*b*) Not unless based on the theory of limits.

117. (*a*) Limits. (*b*) I do not find it to.

118. (*a*) We use infinitesimals. (*b*) Yes; in general.

119. (*a*) I favor the method of *fluxions*, but use the infinitesimal, mainly because I could not get a suitable text-book in fluxional method until recently. (*b*) Yes; better than the method of limits; I have no trouble after a little explanation.

120. (*a*) The method of rates (the Newtonian), using also the principles of limits in connection therewith. (*b*) No, decidedly no; if not established by the principles of limits.

121. (*a*) The infinitesimal for practical use; limits as a means to an end. (*b*) Yes; sufficiently so for all practical purposes.

122. (*a*) Infinitesimals for use; for demonstration, limits. (*b*) Yes.

123. (*a*) Infinitesimal. (*b*) Yes.

124. (*a*) We teach both methods simultaneously. Having understood thoroughly the rigor of the method of limits, the student has no trouble in handling infinitesimals, practically, in mechanical problems. (*b*) Yes, after he has once thoroughly understood Taylor's theorem, not as a formula for development in series merely, but as the means of determining the value of a function at one point from its value at another.

125. (*a*) The first, but the infinitesimal should also be given careful statement. (*b*) *Yes, if* properly presented.

(*a*) *What method of treating the calculus do you favor, that of limits, the infinitesimal, or some other?* (*b*) *Does the infinitesimal seem rigorous, and to satisfy the mind of the student?*—Continued.

126. (*a*) Infinitesimal. (*b*) Yes.

127. (*a*) The conception of calculus as the "science of rates. (*b*) To some minds it is not satisfactory.

128. (*a*) Limits, supplemented by the conception of rates. (*b*) As usually stated, no.

129. (*a*) The infinitesimal. (*b*) Yes.

130. (*a*) About as given in the calculus of J. M. Taylor. (*b*) Not in the earlier part of his course, but later. Yes.

131. (*a*) Limits. (*b*) *The students find it easier, and most college students are not very critical.*

132. (*a*) We use Todhunter's treatises, who employs limits in the differential calculus, and infinitesimals in the integral calculus, and we find it to work well. (*b*) Not at first, but later, when calculus is used in analytical mechanics and mathematical physics, it carries conviction and satisfaction.

133. (*a*) The infinitesimal. (*b*) It may seem rigorous at first, but I think ultimately he is better satisfied of its advantages as mental drill.

134. (*a*) Our course does not include calculus.

135. (*a*) Infinitesimal in the main, though with many references to the theory of limits. (*b*) As so taught, it does.

136. (*a*) I prefer the infinitesimal method, but I do not hesitate to use such assistance as can be derived from the method of limits. (*b*) It appears to me that it is only familiarity with the proofs that makes either method seem rigorous; but the difficulties seem no greater in regard to one than the other.

137. (*a*) The infinitesimal for beginners, limits for the advanced. (*b*) I think it does if properly presented.

138. (*a*) Infinitesimals at first, afterward the method of limits may be introduced. (*b*) With 99-100, yes.

139. (*a*) Limits is most mathematical, infinitesimals most easily comprehended. (*b*) It is satisfactory to ordinary students.

140. (*a*) Limits for the theory, infinitesimals for the practice. (*b*) No.

141. (*a*) Limits.

142. (*a*) Infinitesimal.

143. (*a*) I hardly know. Each has its advantages and disadvantages. I now use the method of rates as given in Rice and Johnson's treatises. (*b*) It does not, as usually presented.

144. (*a*) Limits for theory, inclusive of rates and of infinitesimals. (*b*) It sometimes seems to satisfy the students, but never the professor.

145. (*a*) The infinitesimal, but in connection with the other methods. (*b*) It does, if any method does.

146. We do not teach calculus.

147. (*a*) The infinitesimal at first; I use both to some extent. (*b*) Not altogether.

148. (*a*) That of fluxions, with demonstrations by limits. (*b*) No; its equations (with one exception) have never been proved true, and may easily be shown false. Its results, however, are absolutely true, as experience proves, and as may be shown by theory.

149. (*a*) That of limits.

150. (*a*) Infinitesimal. (*b*) It does.

151. (*a*) Limits. (*b*) It does not.

152. (*a*) The method of limits. (*b*) I don't use the method in teaching.

153. (*a*) Infinitesimals founded upon limits. (*b*) The infinitesimal method is never rigorous unless founded upon limits.

(*a*) *What method of treating the calculus do you favor, that of limits, the infinitesimal, or some other?* (*b*) *Does the infinitesimal seem rigorous, and to satisfy the mind of the student?*—Continued.

154. (*a*) Limits. (*b*) It is the practical method, but is not satisfactory, at first, to the student.

155. (*a*) I am decidedly in favor of the method of limits. (*b*) It does not seem rigorous, and does not satisfy the mind of the student.

156. (*a*) The infinitesimal. (*b*) Perhaps not at first.

157. (*a*) That of limits.

158. (*a*) Limits. (*b*) Yes; can be presented in a perfectly satisfactory manner.

159. (*a*) The theory of limits, as presented by Todhunter. (*b*) My experience with classes has been to the contrary. It does not.

160. (*a*) Limits. (*b*) It has not done so with my classes.

161. (*a*) By limits. (*b*) It does not.

163. (*a*) By limits. (*b*) No.

164. (*a*) The method of rates and fluxions, as developed by Rice and Johnson. (*b*) No.

165. (*a*) No specialty.

166. (*a*) Limits. (*b*) Yes.

167. (*a*) The method of rates, combined with that of limits. (*b*) I have not been able to make it as real to my students as I desire.

168. (*a*) The infinitesimal with some use of that of limits. (*b*) Yes; except with a few students.

(*a*) *Do scientific or classical students show greater aptitude for mathematics?* (*b*) *Which sex?*

1. (*a*) No special difference is noticed.

2. Classical.

3. Scientific.

5. Classical.

6. (*b*) Male.

8. (*a*) No difference; our Civil Engineering students have shown most aptitude. (*b*) Male sex.

9. (*a*) I think it depends on talent.

10. (*b*) Male, generally.

11. (*b*) Girls for rote work, boys for original work.

12. (*a*) Classical. (*b*) Male.

15. (*a*) I observe little or no difference.

16. (*a*) Classical. (*b*) Male.

17. (*b*) Male.

20. (*a*) Scientific. (*b*) Male.

21. (*a*) Classical. (*b*) With us, females.

23. (*a*) Scientific.

24. (*a*) Scientific. (*b*) Equal.

25. (*a*) Scientific. (*b*) Male.

26. (*a*) Scientific. (*b*) Boys.

27. (*a*) Scientific, as a rule.

28. (*a*) Scientific. (*b*) Male.

29. (*b*) I note no material difference in our work.

30. (*a*) I am inclined to think scientific. (*b*) Male, but sometimes female.

31. (*a*) Have not been able to note great difference. (*b*) In my classes the young ladies have, as a rule, excelled.

32. (*a*) Scientific. (*b*) The male.

(*a*) *Do scientific or classical students show greater aptitude for mathematics?* (*b*) *Which sex?*—Continued.

33. (*a*) The greater the drill, the greater the aptitude for anything. Therefore classical. (*b*) The young men for persistent, the young ladies for instantaneous grasp.

35. (*a*) Scientific. (*b*) Male.

36. (*a*) Good students in either course. (*b*) Males as a rule.

37. (*a*) Classical. (*b*) Males.

38. (*a*) I would say classical. (*b*) There are more males than females.

40. (*a*) Difficult to answer. (*b*) The ladies are not inferior.

42. (*a*) Generally the scientific. (*b*) I see no difference.

43. (*a*) No difference noted. (*b*) Males average higher, but a female often stands first.

44. (*b*) A greater number of males succeed, but a few females excel.

45. (*a*) Cannot see any difference.

46. (*a*) Classical students not required to study analytical mathematics. No comparison is possible, as we have no preparatory school. (*b*) Young men form the larger part of our higher classes. As far as comparison is possible, the two sexes show about equal aptitude for the study.

47 (*a*) Our best have been classical here, but the reverse has been my experience elsewhere. (*b*) Girls in the text-book; boys outside.

49. (*a*) No difference. (*b*) Male.

51. (*a*) There is no difference. (*b*) Some of our best students in mathematics have been young ladies.

52. (*a*) No difference. (*b*) Interest about equal.

53. (*a*) The scientific. (*b*) Male.

54. (*a*) Generally scientific. (*b*) About equally.

55. (*a*) The former. (*b*) No perceptible difference as to sex.

56. (*a*) Classical. (*b*) Gentlemen.

57. (*a*) Generally the former.

59. (*b*) Equal in *applied*, but more males in *pure*.

60. (*a*) Scientific.

61. (*a*) Classical.

63. (*b*) Male.

64. (*a*) Scientific.

65. (*a*) Scientific. (*b*) Male

66. (*a*) Classical students.

73. (*a*) Scientific (*b*) No difference.

75. (*a*) Good students in other departments are equal in mathematics as a rule. (*b*) It is rather difficult to answer directly. The ladies average fully as high as the gentlemen.

76. (*a*) Scientific. (*b*) Male.

77. (*a*) Classical. (*b*) Male.

79. (*a*) Classical, thus far.

80. (*a*) Classical.

82. (*a*) Scientific, usually. (*b*) I find little difference.

83. (*a*) Classical.

84. (*b*) The male sex.

85. (*a*) The classical, as a rule; our very best mathematical students have been scientific.

86. (*a*) No difference.

87. (*a*) No classical. (*b*) Young men.

88. (*a*) Classical.

90. (*a*) Scientific. (*b*) Males.

91. (*a*) Scientific. (*b*) Male for analysis, female for book work.

(a) Do scientific or classical students show greater aptitude for mathematics? (b) Which sex?—Continued.

92. (*a*) Scientific or philosophical. (*b*) Nearly alike on the text-book work. The gentlemen seem more successful in *original* investigation. May not the reason for this be found in the fact that it has been assumed for an indefinite period that woman is not capable of doing such work, and so she has not been *required* to do it, thus leading to a dwarfing of this part of her mind?

93. (*a*) Seem not to divide on this line. (*b*) See no difference. Few girls elect calculus.

94. (*a*) No appreciable difference. (*b*) The females do closer work on lessons and tasks assigned.

95. (*a*) Scientific.

97. (*a*) Classical, and students of the exact (not merely scientific) sciences. (*b*) Male.

98. (*a*) Scientific. (*b*) Male.

99. (*a*) Scientific.

100. (*b*) Male.

101. (*a*) Scientific. (*b*) Male.

102 (*a*) The scientific. (*b*) Males in quantity, females in quality.

104. (*a*) No difference, high classics generally carry high mathematics.

105. (*a*) No difference. (*b*) Hard to say. Boys a little better reasoners.

106. (*a*) Classical. (*b*) On the whole, young men.

108. (*a*) Can't say. (*b*) Males.

113. (*a*) Scientific.

114. (*a*) Classical students frequently show a greater aptitude, but scientific students, after having a practical end in view, more frequently become accomplished mathematicians.

115. (*a*) Scientific (Professor Peck). (*a*) Mixed. (*b*) Male (Tutor Fiske).

116. (*a*) Classical generally.

117. (*a*) Scientific. (*b*) Male.

118. (*a*) We see little difference.

119. (*a*) Classical. (*b*) About equal.

121. (*a*) Scientific.

122. (*a*) Scientific.

123. (*a*) Rather the classical. (*b*) Young men.

124. (*a*) Scientific. The mathematics in the classical course ends with trigonometry. (*b*) Only a few girls take mathematics. Can't answer satisfactorily. Some girls do excellent work. I doubt whether sex has much to do with natural mathematical ability.

126. (*a*) Other conditions being equal, no difference. (*b*) No difference, following similar preliminary training.

127. (*a*) Difficult to decide. (*b*) Not much difference.

128. (*a*) Our experience not a fair test—students have been so largely classical. (*b*) Male, on the whole.

129. (*a*) Classical.

130. (*a*) Scientific.

131. (*a*) About the same. (*b*) Male.

132. (*a*) No difference in aptitude, but classical students find no time for extended courses. (*b*) Comparatively few females excel, though some are as good as any of the males.

133. (*a*) Very little difference. (*b*) Males.

135. (*a*) Scientific students. (*b*) Male.

136. (*a*) Classical students. (*b*) No experience.

137. (*a*) Scientific or technical.

138. (*a*) So far as we can test it, classical.

(*a*) *Do scientific or classical students show greater aptitude for mathematics?* (*b*) *Which sex?*—Continued.

139. (*a*) No difference in aptitude. (*b*) No difference in sex.

140. (*a*) Classical are more frequently the best.

142. (*a*) Classical. (*b*) The male by a large percentage.

144. (*a*) Classical.

145. (*a*) Scientific.

146. (*a*) I notice no difference.

147. (*a*) The classical are superior. (*b*) No difference.

148. (*a*) The scientific, but, for the reason (I think) that those who dislike it elect a literary course.

149. (*a*) Classical, I think. (*b*) Male and female equally is my observation here.

150. (*a*) Classical. (*b*) Males.

151. (*a*) Classical. (*b*) Male sex.

153. (*a*) Scientific. (*b*) Male.

154. (*a*) We have no classical students.

155. (*a*) Scientific.

156. (*a*) Scientific.

157. (*a*) Scientific.

159. (*a*) Scientific.

160. (*a*) Scientific.

161. (*a*) The classical. (*b*) Male.

162. (*a*) I think classical. (*b*) Male.

164. (*a*) Scientific. (*b*) Male.

166. (*a*) Classical. (*b*) I do not see any difference.

167. (*a*) Generally the scientific. (*b*) Males as a rule, at least in higher branches than elementary trigonometry.

168. (*a*) The classical.

(*a*) *In what other subjects are good mathematical students most successful?* (*b*) *In what least successful?*

1. (*a*) Good students in mathematics generally stand well in all other studies.

4. (*a*) As a rule none but the better students pursue mathematics more than two years. Good mathematical students are successful in the scientific branches taught here. (*b*) In English.

5. (*a*) Geography, logic, history, chemistry, and natural philosophy. (*b*) Grammar, rhetoric.

8. (*a*) In logic and in the physical sciences. (*b*) Ancient languages and English literature.

9. (*a*) In philosophy, chemistry, analytical mechanics, geodesy, etc. (*b*) I have not observed.

10. (*a*) I can not say. (*b*) Do not know; hard to determine.

11. (*a*) Am not certain; should say physics and ancient languages. (*b*) Can't say.

12. (*a*) I do not know certainly, but I have never noticed any inverse relation between linguistic and mathematical endowments. Chemistry and mathematics are less friendly.

15. (*a*) Scientific subjects, especially physics. But I observe that good mathematicians usually do well in almost any subject which interests them. (*b*) Subjects which involve much "committing to memory."

16. (*a*) A good student is successful everywhere. I have found that my best students in mathematics were, as a rule, "best students" in other departments.

17. (*a*) In any subject in which continued reasoning is necessary. (*b*) I am unable to specify.

(*a*) *In what other subjects are good mathematical students most successful?* (*b*) *In what least successful?*—Continued.

18. (*a*) In natural philosophy, in metaphysics, and generally in Greek. (*b*) Literature.

19. (*a*) They do equally well in Latin, as a rule. (*b*) English.

20. (*a*) Mechanics.

21. (*a*) A student good in mathematics is apt to be successful in all branches.

22. (*a*) My experience goes to show that a student who is good in mathematics is capable of coming off with good standard in almost any other study. I have known a few apparent exceptions. (*b*) In languages.

24. (*a*) Sciences. (*b*) Classical studies.

25. (*a*) Physics.

26. (*a*) Physics, logic, chemistry. (*b*) Rhetoric.

27. (*a*) In engineering, physics, astronomy. Mathematical training seems to make lawyers more successful in the clear statement of their cases. (*b*) Literary pursuits.

28. (*a*) In mental, moral, and natural philosophy. (*b*) In belles-lettres.

30. (*a*) In almost every other. (*b*) Perhaps, literature.

32. We can hardly give an intelligent answer to this question with our grade of work.

33. (*a*) Mathematics, as we are compelled to teach it, is largely mechanical; therefore, in subjects not requiring great originality. (*b*) Answered.

35. (*a*) In engineering. (*b*) In languages.

37. (*a*) Varies with the student.

38. As a rule, our mathematical students are excellent in all their studies. Languages are not unfrequently hard for good mathematical students.

39. (*a*) Greek, Latin—often—mainly—various forms of graphics. (*b*) Scientific research, *i. e.*, natural sciences.

40. (*a*) The various branches of natural science, metaphysical studies.

41. (*a*) Usually in all other subjects of our course. (*b*) Occasionally in languages.

43. (*a*) We often have fine work upon topics related to general geometry.

44. (*a*) Chemistry, physics, languages. (*b*) History, literature.

45. (*a*) Generally in whatever is undertaken, I believe success in any branch is in proportion to application.

46. (*a*) Whatever they undertake. (*b*) Whatever they give the least attention to.

47. (*a*) Logical. (*b*) Linguistic.

48. (*a*) As a rule in all subjects requiring judgment, reason, discrimination. (*b*) In subjects requiring the memory as the chief element of the mind.

49. (*a*) Sciences. (*b*) Languages.

50. (*a*) Languages. (*b*) History.

51. (*a*) All others, that is, according to circumstances.

52. (*a*) Our good mathematical students are good in languages and sciences.

53. (*a*) Chemistry and physics. (*b*) Have not noticed.

54. (*a*) Some in one subject and some in another, according to native aptitude and application.

55. (*a*) In chemistry, physics, and logic. Good mathematical students rarely show weakness in any study. (*b*) Literature (and modern languages?).

56. (*a*) Natural science. (*b*) Language.

57. (*a*) Natural science.

59. (*a*) The *majority* of good mathematical students are good in everything else, but sometimes a mathematical mind fails in letters, and *vice versa*, as some appreciate only *demonstrative* reasoning, and some *moral*.

60. (*a*) Physics, astronomy, and natural science. (*b*) Languages.

(a) *In what other subjects are good mathematical students most successful?* (b) *In what least successful?*—Continued.

63. (a) I find good mathematical students successful, generally, in all other subjects.

66. (a) In most subjects.

68. (a) Our best students are about equally successful in all mathematical branches.

69. (a) Natural science. (b) Can't say.

70. (a) Languages. (b) History and literature.

73. (a) Physics and astronomy. (b) My best mathematicians are best in other lines.

74. (a) Since I have observed here (four years), the best mathematical students are usually also among the best in all studies; otherwise in natural sciences, English and Greek, history and political science. (b) Languages.

75. (a) The good mathematicians are those whose general standing is high, but of course there are exceptions to this; I should say that they are more likely to excel in the sciences, logic, and metaphysics.

76. (a) Chemistry, logic, mental science. (b) Language, history, rhetoric, oratory.

79. (a) Languages, I should say, in general.

80. (a) Latin and science. (b) History and literature.

82. (a) As a rule, I think, in all subjects, although occasionally I find one who is weak in language and literary studies.

84. (a) Physics and mechanics. (b) The languages.

86. (a) No special difference so far as I know.

87. (a) Chemistry, physics, and applied mechanics. (b) Languages.

88. (a) Our records show that good mathematical students are successful in all other subjects.

89. (a) In all other subjects taught in the school.

90. (a) In historical studies, natural philosophy, and mathematical astronomy. (b) Literary, but not always.

91. (a) In the lateral sciences, *e. g.*, physics, chemistry; also in logic. (b) Languages and history.

92. (a) As a rule, I think, in the sciences, and especially in original investigations in science. (b) So far as my observation goes, in languages, as a rule.

93. (a) With rare exceptions they are good in all the subjects. The converse is not so general, *i. e.*, students often excel in one or two departments without excelling in mathematics.

94. (a) They average well all around. (b) No uniformity.

95. (a) Draughting, physics, chemistry, logic.

96. (a) As a rule, those good in mathematics are good in all others, but especially in natural sciences, psychology, and logic.

97. (a) In all the more introspective, and such as require prolonged and strenuous thought, not mainly observation (like stone—or bug—lore). (b) In these latter so-called experimental sciences.

98. (a) Applied arts, engineering, physics, etc. (b) Languages, metallurgy, analytical chemistry.

99. (a) Philosophy. (b) Composition.

101. (a) Philosophical. (b) Linguistic.

102. (a) In natural sciences, history, geography, logic. (b) Languages.

104. (a) All scientific pursuits, drawing, arts, generally.

106. (a) In such as require concentration of mind, and close reasoning. (b) If in any, in such as depend upon observation and experiment.

107. (a) Generally in all others, if they are interested.

109. (a) Natural philosophy, chemistry, Greek, Latin. They can generally do well, wherever they try. (b) English, political science.

(*a*) *In what other subjects are good mathematical students most successful?* (*b*) *In what least successful?*—Continued.

110. (*a*) My observation has been that where students were good in mathematics, they were good in all their other studies.

111. (*a*) Logic and analytical studies.

112. (*a*) They generally stand high in all subjects.

113. (*a*) As a rule they are successful in all other studies; more so in metaphysics, theology. (*b*) The higher study of literature.

114. (*a*) Very difficult to generalize. Many excellent mathematicians are "all-around" men. Others excel in science, and are least successful in languages and speculative subjects.

115. (*a*) A good student in mathematics is generally a good student in all other branches (Professor Peck).

116. (*a*) Generally in logic and psychology.

117. (*a*) Mechanics, physics, chemistry, logic. (*b*) Classics.

118. (*a*) They are generally good in all subjects. (*b*) Subjects requiring memory only.

120. (*a*) Chemistry (including heat, physiology, electricity, and magnetism), mineralogy and geology, engineering, ordnance and gunnery, and law. (*b*) Drawing, Spanish, and French, relatively. Good mathematical students are generally good in all other branches (Professor Bass, professor of mathematics).

Charles W. Larned, the professor of drawing at West Point, answers as follows: I differ somewhat from the inferences to be drawn from the answer to this question by the professor of mathematics.

In so far as any influence is to be implied by mathematical proficiency upon other studies, an examination of the standing of the last five graduating classes tends to show very positively that law belongs to the category of those studies in which there exists the greatest discrepancy, and this, notwithstanding that law is studied two years after mathematics is completed and when habits of study and ability to master a wider range of subjects is more highly developed by the study of intermediate synthetic studies.

There is a much greater range of discrepancies also between the group of studies comprised under the head of chemistry (which includes electrics, mineralogy and geology, and heat) and mathematics than would naturally be inferred from the grouping made. Even in natural philosophy the aggregate discrepancies were greater than I had supposed probable.

The standings of the graduating classes of 1888, 1887, 1886, 1885, and 1884 in law, chemistry, drawing, English, and French were reduced to the same standard, and the differences between these and mathematics in each case were obtained, and the aggregate in each subject for these classes is as follows:

Discrepancies as compared with mathematics.

	Law.	Chemistry.	Drawing.	French.	English.
1888	356	342	481	418	413
1887	925	714	1,021	936	996
1886	1,102	1,096	1,694	1,166	1,376
1885	336	250	332	295	301
1884	195	144	366	254	241

In regard to drawing it is proper to observe that a marked distinction exists between technical graphics and free-hand drawing. The standing given includes *free-hand* drawing, occupying one-fourth of the course. In this the possession

(*a*) *In what other subjects are good mathematical students most successful?* (*b*) *In what least successful?*—Continued.

of natural graphical talent exercises a much greater influence in producing discrepancies in standing. In technical graphics, however, throwing out a few men, perhaps one-half dozen in each class, with pronounced natural ability, standing in plane and descriptive geometry has a decidedly beneficial influence on standing in drawing. In other respects, intelligence, whether mathematical or liberal, will tell in the work. Leaving out four or five exceptional men in each class the discrepancies in drawing, even with free-hand included, fall below those of French, English, and law.

121. (*a*) Physics and astronomy. (*b*) Latin, French, etc.

122. (*a*) First-rate mathematical students generally do well in all other studies.

123. (*a*) In mathematics of physics. (*b*) Possibly the biological sciences and languages.

124. (*a*) My experience is that a man who is good in mathematics has mental ability sufficient to make any subject of an ordinary college course comparatively easy. Good in mathematics—good everywhere.

125. (*a*) Applied sciences, of course—astronomy, physics, logic, and metaphysics. (*b*) Languages and literature, sometimes. Still hardly think that is true, as a rule.

126. (*a*) Advanced work in physics and engineering. (*b*) Our best students in mathematics are best everywhere.

129. (*a*) Good mathematical students are good in all their work. (*b*) Rarely unsuccessful in any line of study.

130. (*a*) Physics, astronomy, logic.

131. (*a*) Classics, sciences, but there are many exceptions. (*b*) English, probably.

133. (*a*) Think that, on the whole, our best mathematical students are best, generally, in other studies.

134. (*a*) Sciences.

136. (*a*) Greek is often combined with mathematics.

137. (*a*) They are generally good all around. (*b*) In languages, but only in exceptional cases.

138. (*a*) In all other subjects. (*b*) None.

140. (*a*) Physics and astronomy. The good ones are also usually good in classics and everything.

141. (*a*) Generally also in the classical studies.

142. (*a*) Physics, chemistry, logic. (*b*) Moral and mental philosophy.

143. (*a*) All subjects requiring accurate thought. In our college this is especially noticeable in mental and moral philosophy. (*b*) Those requiring mere memory.

144. (*a*) Usually in all others.

145. (*a*) Some in one, some in another. No general rule. (*b*) No general rule.

146. (*a*) In this they differ, though they are possibly better in scientific studies. (*b*) Languages.

148. (*a*) I have not observed that a successful student in mathematics is more apt to succeed in one subject than he is in another, except where the subject rests on mathematics.

150. (*a*) Generally in everything else.

151. (*b*) Belles-lettres.

152. (*a*) Naturally in subjects depending upon a knowledge of mathematics, and generally in whatever else they may study.

153. (*a*) In logic, physics, engineering, medicine. (*b*) Languages.

(*a*) *In what other subjects are good mathematical students most successful? In what least successful?*—Continued.

154. (*a*) In all branches that require accurate observation and close reasoning. (*b*) In languages.

155. (*a*) In branches of natural science. (*b*) In the languages, I think.

156. (*a*) In logic and, as a rule, the natural sciences.

157. (*a*) They are apt to be more successful in all subjects connected with the sciences than in the study of languages.

158. (*a*) As a rule, all students standing very well in mathematics will achieve success in any other subject. I have seen but few exceptions.

159. (*a*) In moral and mental philosophy, logic and civil law. (*b*) Synthetic languages.

160. (*a*) In the mathematical sciences. A good mathematical student is good at everything he undertakes.

161. (*a*) Physics and chemistry. (*b*) Languages and history.

162. (*a*) Oftener classics.

163. (*a*) Engineering and physics. (*b*) Languages.

164. (*a*) In any subjects requiring reflection. (*b*) Those requiring perception and memory only.

166. (*a*) Logic, chemistry, philosophy, political economy, and astronomy. (*b*) Language.

167. (*a*) I believe that a really first-class mathematical student is generally successful in nearly all subjects, but those a grade lower are most likely to excel in the physical sciences than in other lines. (*b*) Perhaps in belles-lettres.

168. (*a*) I don't think I can tell, for there is such diversity; yet I think that those who are good in mathematics are good all-around students, as a rule.

What is the relative prominence of mathematics in your course of study as shown by hours per week and per year?

1. In classical and scientific courses the same number of hours is given to mathematics as to any other study. In the engineering course about twenty-five per cent. more.

2. About one-fourth part of the class-time is devoted to the study of mathematics.

3. No study has more attention, and some have less.

4. Five hours per week out of sixteen hours for recitation and lectures are devoted to mathematics for three years; the last year three hours per week during the year.

5. More prominence given to mathematics than to any other study.

6. Large.

7. Occupies more time than any other subject.

8. In classical course one-fourth of student's time is devoted to mathematics; relatively more in scientific and civil engineering course.

9. The principle studies in our college receive equal attention; mathematics one hour and a quarter daily.

10. More stress on mathematics as a whole than upon any other subject, I think.

11. Mathematics takes one-fourth of the time in the scientific course, one-seventh in the literary, and over one-fifth in the classical.

12. It leads Greek, and is on a par with Latin and physics.

13. Considered of fundamental importance and continued throughout the four years of study; twenty hours per week for the four classes (pure mathematics only).

14. During the year, four hours per week out of a total of fifteen hours.

15. In the classical course about ten per cent. of the work is in mathematics, and in the scientific course about fifteen per cent. I have counted only the *prescribed* work and the pure mathematics, so-called.

What is the relative prominence of mathematics in your course of study as shown by hours per week and per year?—Continued.

16. It has the same prominence as do the classics.

17. Preparatory, two-thirds of the entire *work done* is mathematics; first year, one-half; second, about two-fifths; third, one-fifth; fourth, only applied mathematics.

18. One hour of mathematics each day, *i. e.*, six hours per week; about three and three-fourths of other studies.

19. Our regular course of study practically covers five years, divided into three terms each; mathematics occupying one-third of each term for the ten terms ending with first term of the Junior year (Professor Gordon). In the early years of the course, equal to any other subject except English (Professor Draper).

20. Mathematics and classics each occupy five times as many hours as science.

21. Twenty per cent.

22. Daily recitation required of every student.

23. It is of the first prominence.

24. No special prominence observed.

25. Six hours per week; about one hundred and eighty per year.

26. One-fifth of time.

27. Mathematics has seventeen hours per week; the ancient languages, fifteen hours per week; English, twelve hours per week.

28. One-fourth of all the time during Freshman and Sophomore years is devoted to pure mathematics; and one-tenth of all the time in the Junior and Senior years.

29. Mathematics extends through two-thirds of the course. Takes about one-fourth of time during that period.

30. The time spent is about the same as in the average college.

31. More time is given than to any other one topic.

32. It ranks with the natural sciences and the ancient and the foreign languages.

33. As prominent as any other chair, if not more so.

34. Freshman year, three-fifteenths; Sophomore year, three-fifteenths; Junior year, two-fifteenths (elective); Senior year, two-fifteenths (elective).

35. In some courses one-third the time for one year; in others one-third the time for seven terms out of twelve, with applied mathematics for eleven terms more. In both, eighteen units out of thirty-six.

36. Nearly one-third in Freshman and Sophomore years.

37. The same as other studies; three and three-fourths hours per week.

38. One year's work is required of all students. Four years are required of mathematical students. Many elect mathematics for one or more years.

39. The time is about equally distributed between Latin, Greek, modern languages, and mathematics.

40. Different in the various courses. Mathematics is more prominent in the scientific course, claiming about one-fourth the student's time (perhaps one-third).

41. Freshman year twenty-one hours out of sixty-one, 798 per year; Sophomore eighteen out of fifty-four, 684 hours per year; Junior thirteen out of sixty, 494 per year. In this estimate two hours of preparation are usually reckoned with each hour of recitation.

42. Sixteen hours per week.

43. Upon an equality with Latin and Greek.

45. It is taught five hours per week until the end of the Sophomore year.

47. Co-ordinate with Latin and Greek.

What is the relative prominence of mathematics in your course of study as shown by hours per week and per year?—Continued.

48. About one hour out of every four.

49. Five hours a week for two years and one year additional, which is elective.

50. About the same.

51. As great as that of any other subject.

52. Equal with language and science, until Junior year.

53. Has more time than any other study.

55. Five out of fifteen per week for three years of the course. No mathematics in the last year of the course.

56. It receives more time than any other subject taught.

57. First.

59. About two-fifths of the time in the various schools.

60. As about five to four in comparison with language and natural science.

61. First.

62. It ranks with any other study in prominence.

63. It is desired to make it equally prominent with other subjects.

64. One hour daily in class-room.

65. Four hours per week are devoted to mathematics throughout the course. It receives about equal attention with any other subject.

66. Quite as prominent as classics.

67. Under our "group" system, under-graduate students who include in their "group" of studies a *minor* course in mathematics devote one-third of their time for one year (as measured by hours per week) to mathematics; those who take a *major* course in mathematics devote to it one-third of their time for two years. The whole time of an under-graduate course is three years. A student need not include any mathematics in his group. Our entrance requirements include trigonometry and some analytical geometry.

68. About one-fourth of required time is devoted to pure mathematics.

69. Stands near bottom of the list.

70. Classics, science, and mathematics have equal prominence.

71. During the first year for all students, thirty-three and one-third per cent. of recitation periods is for mathematics. The time for preparation would be larger. During the second and third years the engineers gave about twenty-five per cent. to pure and twenty-five per cent. to applied mathematics. During the Senior year about twenty-five per cent. to applied mathematics. Other students give but little time to mathematics after the first year.

72. Four hours out of fifteen per week in Freshman year.

73. Thirty-three and one-third per cent.

74. Freshman year four hours per week, *i. e.*, twenty-five per cent. is required throughout the year. Four hours per week elective is offered in Sophomore and Junior years.

75. Five hours per week for the first two years of the course.

76. It stands third in the course.

77. Less prominent than the classics, except in the academy and in the inductive science courses.

78. It stands on the same level with Latin and Greek—our courses being (like those of Harvard College) elective.

79. It is on an equality with Greek.

80. Algebra, plane geometry, plane trigonometry are required, five recitations per week during Freshman and Sophomore years. Mathematics is elective three hours per week during rest of course.

82. Five hours per week for thirty-eight weeks per year, or nearly one-third of the whole work.

84. In first year, one-fourth the time; second and third, one-fifth; none in the *last.*

What is the relative prominence of mathematics in your course of study as shown by hours per week and per year?—Continued.

85. One-third of the time.

86. Of required work, mathematics has about fifteen per cent. out of the fifty-two hours weekly.

87. Our students average in three years six and one-third hours per week of mathematics, to four and one-half of language, to four of physics and chemistry, mineralogy and geology altogether.

88. It is different in the different years. For the four years it is about one to ten.

89. One to six.

90. About par.

91. The hours are about equal, taken as a whole. For the degree of B. S. they far exceed.

92. A high importance; fifteen terms (including the preparatory course); five hours per week, forty weeks per year, for five years.

93. Sub-Freshman year, two-sevenths of entire work; Freshman year two-ninths; Junior year (elective) one-fourth; Senior year (elective) one-sixth. This does not include mechanics, surveying, and other applied mathematics.

95. Four hours per week; most other studies (non-professional), three.

96. Daily recitations, one-half hour each.

97. In the scientific courses it is first; in the classical and literary, second (Latin and Greek, respectively, English first).

98. Mathematics is the ground work of the institution, preparatory and coincident with the courses in engineering.

99. Two hours daily devoted to mathematics during the session of ten months.

100. Fourteen-sixtieths of the four years' course.

101. Twenty-five per cent.

102. About one-third of the whole time; ten hours per week, 480 hours per year.

103. Each class averages five hours per week, per year.

104. Same, no prominence.

105. In the preparatory course, total hours per week fifteen; mathematical average, three and one-third. In college classes, total hours per week, fifteen; mathematical average, two and eleven-twelfths.

106. The mathematical course for the majority of our courses is completed the Freshman year, having five-sixteenths of the time.

107. Leading study.

108. Five twenty-fourths of the whole.

109. Mathematics ranks with Latin and Greek throughout, each getting four-fifteenths of the time in first two years; elective in third.

110. Until this year, more than half the time was given to mathematics; now, perhaps, one-third.

111. We require two and one-half hours per week.

112. Considering the hours devoted to mathematics, it ranks with any other subject.

113. Mathematics and Latin have each three hours a day.

114. Freshman year, one-third; Sophomore year, four-fifteenths; Junior and Senior years, optional.

115. The courses in my department are elective. Question cannot well be answered.

116. Five hours per week out of fifteen in Freshman year; three hours per week out of fifteen, Sophomore year for classical; four out of fifteen for scientific.

What is the relative prominence of mathematics in your course of study as shown by hours per week and per year?—Continued.

117. Freshman year, five hours per week out of sixteen; Sophomore, five out of seventeen in first term, and three out of eighteen, second term.

118. In scientific course sixteen one-hundredths of whole time, or twenty-eight one-hundredths, including descriptive geometry and mechanics and astronomy. In classical course thirteen one-hundredths and sixteen one-hundredths.

119. Four hours per week required during the entire Freshman and Sophomore years, and the second term of the Junior year. From two to six hours per week may be elective during the other terms.

120. During the first year, time devoted to mathematics is to time devoted to modern languages as four and one-half is to three. During the second year, time devoted to mathematics is about the same as the time devoted to languages and drawing.

123. It is on an equality with the subjects taught in the other departments.

124. Mathematics is one of our most important subjects. Three professors give jointly forty-five hours per week to mathematical instruction, for twenty-six weeks, and thirty-nine hours per week the remaining eleven weeks of the session.

125. Freshmen five-fifteenths, Sophomores three-fifteenths, Juniors (elective) two-fifteenths, Seniors (elective) two-fifteenths.

126. It occupies about one-third of the whole course.

127. Full work in all subjects fifteen hours per week. In mathematics five hours per week whenever any mathematical subject is studied. In the preparatory department algebra is required in all the courses three terms (one full year), and in the science courses four terms. Plane geometry is required through the last term of the Senior preparatory year. Then, in the college course, we have solid geometry, trigonometry, surveying, analytical geometry, and calculus, one term each. A second term of analytical geometry and calculus is required of the scientific students; and in the philosophical course, students elect between a second term of calculus and practical chemistry.

128. All courses are four or five hours a week. In classical course, mathematics have 378 hours, required and elective, out of a total of 4,077 hours.

129. Two hundred and seventy hours distributed through two years with opportunity of election in addition.

130. Four hours out of fifteen per week during two years for all the class; then four out of fifteen during another year for electives.

131. One-fourth up to second term of Sophomore year; from that point all subjects in the course are elective.

132. We have too many courses to make any general statement.

133. About the same as ancient languages.

134. Greater than others, except English.

135. Freshman year, five hours out of eighteen per week.

136. Mathematics is on an equality with all other courses.

137. A little more than one-fourth of the student's time is given to mathematics in Freshman and Sophomore years; a little less than one-fourth during the remainder of the course.

139. Full course, four hours per week.

140. About twenty-five per cent. of total.

141. On a par with the classical.

142. First in the course.

143. Pure mathematics thirteen and six-tenths per cent. of required work. It may be thirty-four per cent. of elective work. It may be one-fifth of the whole course.

What is the relative prominence of mathematics in your course of study as shown by hours per week and per year?—Continued.

144. First year about thirty-three and one-third per cent. of time to mathematics; second year, fifteen per cent.; third year, twenty-five per cent.; fourth year, fifteen per cent.

145. Out of the required eighteen hours per week, literary students get in the first year five, second year three, third year nought, fourth year one and one-half; total, nine and one-half out of seventy-two; scientific students get in the first year five, second year four and one-half, third year four and one-half, fourth year one and one-half; total, fifteen and one-half out of seventy-two.

146. We regard it of greatest importance.

147. In the preparatory course one-third of the study is mathematical, *i. e.* 185 hours a year out of 555. The same in the Freshman year. In the Sophomore year 135 hours out of 555. After that, none.

148. Twenty-five per cent. of the student's time is devoted to mathematics until he completes the Sophomore year. Besides, the students in engineering devote twelve and one-half per cent. of their time in Junior year to mathematics.

149. About one-fourth the time is devoted to mathematics.

150. Mathematics to science about equal; mathematics to language about four to one.

151. About the same time is given to mathematics as to other branches, viz, five recitations per week, except in last year, three times.

152. Mathematics, Latin, and Greek have each four hours per week for Freshman and Sophomore years; no other subjects have as much time. After Sophomore year mathematics is elective.

153. More prominent than any other subject except English and equal to that.

154. It stands first.

155. It is as prominent as any other branch of study. The Junior class, with which the college work properly begins, has five recitations per week, each one hour long. The intermediate class has four per week, the Senior has three per week, and in applied mathematics there are three per week.

156. About twenty hours per week, or eight hundred hours per session.

157. It is given as much time as other subjects, five hours per week in the Freshman year, five hours per week in the Sophomore year, three hours per week in the Junior year, and two hours per week in the Senior year.

158. Our system of independent schools and free elective courses enables us to give a positive statement that, as a rule (with a few exceptional years), the school of mathematics is the most largely attended school in the academic department. The number of lecture hours per week for under-graduates is thirteen.

159. Mathematics occupies a more prominent position in our schedule than any other branch.

160. First.

161. It occupies about one-third of the time devoted to the course of instruction.

162. Five sections per week or nearly one-third of time for first two years.

163. It is probably on about the same footing as the other chief branches of study.

164. Our course in mathematics is very prominent, requiring one-third the student's time through the Sophomore year.

165. Three to two.

166. One-half.

167. Classical course: Freshman year, one-third of time required; Sophomore year, one-third elective; Junior year, one-ninth elective. Scientific course: Freshman year, one-third required; Sophomore year, two-ninths required,

What is the relative prominence of mathematics in your course of study as shown by hours per week and per year?—Continued.

one-ninth elective; Junior year, one-ninth required; Senior year, one-ninth elective.

168. Almost the least prominent thing in the course, as ours is classical, with a leaning to natural sciences.

(*a*) *Do you favor memorizing rules in algebra?* (*b*) *What reforms are needed in teaching the same?*

1. (*a*) No.

2. (*a*) No. (*b*) It ought not to be taught to such young boys, who contract the incurable habit of learning it by rote.

3. Principles, but not rules.

4. (*a*) No. (*b*) Rules and principles should be deduced from examples; a more thorough drill in algebraic language, especially in the meaning and use of signs, exponents, etc.

5. Do not use text-book too closely.

6. (*a*) Yes. (*b*) More practical application should be given.

7. No.

8. No.

9. (*a*) I prefer formulas. (*b*) More thoroughness and better understanding of elementary principles, with reviews and drilling.

10. (*a*) No. (*b*) Algebra should be taught just as arithmetic, wholly by the analytic method.

11. (*a*) Some of the rules. (*b*) I do not know.

12. (*a*) To a limited extent. (*b*) More of the spirit and reason and less mere mechanical solution.

13. (*a*) No. (*b*) The modern methods, as determinants, etc., should be introduced as soon as possible.

14. (*a*) Yes.

15. (*a*) No. (*b*) A larger number of *simple* problems; a less number of difficult demonstrations, such as those in logarithms, the binomial formula, etc.; an earlier introduction to the methods and notation of the calculus.

16. (*a*) No.

17. (*a*) Very little. (*b*) Anything which will make it less a collection of dry bones, and more a living and beautiful science.

18. (*a*) Yes.

19. (*b*) By proper classification the number of propositions could be materially reduced and the number of important theorems and constructions for original work could be materially increased. (Professor Gordon). (*a*) No. (*b*) More attention should be paid to explaining and illustrating the principles involved in operations, and the embodying of questions to test the understanding of those principles; *e. g.*, *why* $x - b = 10$ is equal to $x = 10 + b$; *why* does $+ \times - = -$? etc. (Professor Draper.)

20. (*a*) Yes. (*b*) That the sense of the rules shall be known when the memorizing is complete.

21. (*a*) No. (*b*) Teaching needs to be less mechanical. The reasons for processes need to be taught more.

22. No; the inductive method should be used first.

23. (*a*) Yes.

24. No; thorough drill in substituting numerical quantities for literal.

25. (*a*) I do not. (*b*) In teaching the elements as few formal demonstrations as possible should be used—first a working knowledge, and then philosophize.

26. (*a*) Do not. (*b*) Practical examples.

27. No; more attention to fundamental principles, clear teaching *why* signs are changed in transposition, etc.

(a) *Do you favor memorizing rules in algebra?* (*b*) *What reforms are needed in teaching the same?*—Continued.

28. No; more prominence to principles and less of method. Students should do more private work.

29. No; it should be freed from its mechanical character. Algebra should be *seen*.

30. Not much; less memorizing, more analysis, more thoroughness.

31. We require the principles involved, rather than the exact words of a rule.

32. Yes; only the most important rules and theorems should be memorized, but those *thoroughly*.

33. Emphatically *no;* more principle and why; less toughing, disgusting gymnastics.

34. No.

35. No; more familiarity with technique; less mechanism.

36. No.

37. No, sir.

38. (*a*) Not in general. (*b*) In general, I should say a more thorough teaching of the principle and reasoning of algebra.

39. (*a*) Yes. (*b*) More ought to be taught.

40. (*a*) Students are urged to state operations and principles clearly and briefly without much regard to the text. (*b*) Many problems (original and otherwise) should be solved mentally.

41. (*a*) No. (*b*) More careful attention to the interpretation of literal equations.

42. (*a*) To some extent.

43. (*a*) No.

44. (*a*) I do not. (*b*) More drill on simple exercises, and fewer difficult problems.

45. (*a*) I do not. (*b*) Such as will render the mind able to deal with the principles in forming rules.

46. (*a*) Some of them. (*b*) Too little time seems to be given to the study of algebra.

47. (*a*) Not verbatim. (*b*) More rigorous proofs; more noting of analogies; more as a preparation for higher work than the solving of problems as mere puzzles.

48. (*a*) Yes.

49. (*a*) No.

50. (*a*) I do.

51. (*a*) No. (*b*) Methods are learned by practice, and rules evolved therefrom.

52. (*a*) No.

53. (*a*) Very few. (*b*) There should be more practical application of its principles.

54. (*a*) Not mechanically. (*b*) The reform of good common sense, and clear, simple presentation.

55. (*a*) No.

56. (*a*) No. (*b*) Keep students out of it until they have passed the discussion of arithmetic.

57. (*a*) Yes. (*b*) We need a simpler and at the same time fuller elementary book.

58. (*a*) At the beginning. (*b*) More familiarity with principles.

59. (*a*) Not generally. (*b*) The teacher should assist the pupil to make his own rules.

60. (*a*) No. (*b*) A correct reading of algebraic expressions in algebraic language, and a clear analysis of work done.

(*a*) *Do you favor memorizing rules in algebra?* (*b*) *What reforms are needed in teaching the same?*—Continued.

61. (*a*) I do not. (*b*) Require pupils to think and not to be machines or jugs to be filled.

62. (*a*) To a certain extent. (*b*) There is need of impressing the students in some way with the idea of the practical value of the study and of creating an interest in it.

63. (*a*) No. (*b*) Teachers should wait till their pupils are prepared to begin the study. It should be thoroughly taught.

64. (*a*) Yes.

65. (*a*) Yes. (*b*) The subject ought to be presented freer from technicalities than text-books give it. Unnecessary parts ought to be left out.

66. (*a*) Yes.

68. (*a*) No.

69. (*a*) No. (*b*) Drill on the principles and *raison d'être* for formulæ.

70. (*a*) Yes; when once thoroughly understood.

71. (*a*) No. (*b*) The use and meaning of exponents and of the negative sign are not made as clear as they should be. More accuracy.

72. (*a*) No.

73. (*a*) No. (*b*) Less rules and more thinking. The less memorizing in mathematics, the better the results.

74. (*a*) No.

75. (*a*) Yes.

76. (*a*) No. (*b*) The teacher should lead with the general demonstration of each subject in form of lectures.

77. (*a*) No. (*b*) Pupils should be required more generally to demonstrate principles and work from them rather than from rules and formulæ.

78. (*a*) We do not teach elementary geometry.

79. (*a*) Yes. (*b*) The rules should be proved as strictly as any proposition in geometry.

80. (*a*) No. (*b*) More classification of subjects.

82. (*a*) No. (*b*) The chief cause of failure in many cases is not doing enough miscellaneous examples for practice.

83. (*a*) Yes.

84. (*a*) Some. (*b*) None.

85. (*a*) A more logical arrangement of the different sections of the subject; more examples, and so given as to form a constant review of the ground already gone over; application of business methods to the revision of many rules and methods.

86. (*a*) No. (*b*) More thoroughness, practicality, and solidity of teaching—*the German system.*

87. (*a*) To some extent. (*b*) For a course of study like ours I think more emphasis should be put on thoroughness than extent of ground covered.

88. (*a*) Yes.

89. (*a*) No.

90. (*a*) No. (*b*) More attention to reasoning processes.

91. (*a*) I do not. (*b*) More independence of books and greater original investigation.

92. (*a*) No. (*b*) Less memorizing and more thinking, both on the part of teacher and student.

93. (*a*) Yes. (*b*) Pupils should be taught to state a proposition and follow it with a general demonstration, as in geometry.

94. (*a*) Yes—No! Teach the pupil to develop the principle, and to formulate his own rule for it and for his process.

95. (*a*) Not much. (*b*) Omit attempts to exhaust each subject as it comes up.

96. (*a*) No.

(a) *Do you favor memorizing rules in algebra?* (b) *What reforms are needed in teaching the same?*—Continued.

97. (*a*) No. (*b*) Blind, unreasoning, mechanical solution of equations needs abatement; the doctrines of forms and series, advancement.

98. (*a*) *No!! Rattle the bones of the algebraic skeleton, as exhibited generally in this country, and show it in its living, breathing continuity and beauty of* FORM. *Give a conception of the magnificent power of analysis.*

99. (*a*) No. (*b*) Much desired in text-books, at least many of them.

100. (*a*) Not verbatim.

101. (*a*) No.

102. (*a*) No. (*b*) More attention to principles and less to problems.

103. (*a*) For immature students, yes. (*b*) The method of teaching must, I think, vary under different circumstances. The principle idea should be to prevent the student *thinking* it *difficult*.

104. (*a*) No. (*b*) Knowledge, generally.

105. (*a*) No.

106. (*b*) I favor thorough mastering of the *reasoning* used in deducing formulæ, also memorizing for *ready* use.

107. (*a*) Not word for word. (*b*) Digesting subject as a whole, especially on review.

108. (*a*) No.

109. (*a*) No, with few exceptions. (*b*) A more thorough treatment of a smaller number of subjects; use of determinants, less fractions.

110. (*a*) No. (*b*) More stress should be laid on factoring, less on the theory—more of the solid work with a broader view of its application.

111. (*a*) We do not. (*b*) More mental exercise and less blackboard work.

112. (*a*) I do not. (*b*) More attention should be paid to generalization than it usually receives.

113. (*a*) No.

114. (*a*) No.

115. (*a*) That depends. (*b*) We need no reforms (Professor Peck). (*a*) Yes. (*b*) In the preparatory schools more work should be done independently of the text-book, and a more elaborate elucidation of fundamental principles should be given (Tutor Fiske).

116. (*a*) No. (*b*) In general, greater attention to accuracy; in particular, more attention to theory of exponents and radicals.

117. (*a*) Yes; either those of the text-book or carefully prepared ones. More "why" needed.

118. (*a*) Yes; so far as to secure accuracy of expression and as a mode of fixing methods clearly in the mind.

119. (*a*) Not to a large extent. (*b*) I think the student should be taught to rely upon his logical powers, rather than his memory.

120. (*a*) No. (*b*) Methods that develop a clear understanding of each process and ability to explain clearly, in place of a knowledge of rules without understanding.

121. (*a*) No. (*b*) More of the *inductive* method; and the abolition of much that may be interesting theoretically, but of little practical use.

122. (*a*) In very few cases.

123. (*a*) Yes. (*b*) An improvement in the speed with which the mechanical processes are done.

124. (*a*) Yes; saves time. (*b*) Get teachers who know more.

125. (*a*) Hardly. (*b*) 1. Opposite numbers ought to be given a full treatment, including all the rules for signs, with illustrations and a considerable number of examples and problems in their use, *before the literal notation is begun*. 2. In the former the reason for the use of + and − to mark the series ought to be

(*a*) *Do you favor memorizing rules in algebra?* (*b*) *What reforms are needed in teaching the same?*—Continued.

brought out simply and plainly, and justified. 3. The fact that in elementary algebra the letters always stand for *numbers* ought to be reiterated to avoid obscurity of ideas in the learner's mind. 4. The treatment of the equation should be analogous to that employed in geometry. The method of writing references to axioms, etc., at the right of the page, familiar to those who have used Wentworth's Geometry, can be employed in algebra to even greater advantage.

126. (*a*) No.

127. (*a*) Some of them. (*b*) Examinations of students from many places convince me that algebra should be taught more thoroughly than it is in most of the schools.

128. (*a*) No. (*b*) Explanation to be really so, and work done at time of explanation as far as possible. *Many* comparatively simple problems, not puzzles. *New* work in hour. Students to be ranked according to actual work done in problems. Much board work by entire class.

129. (*a*) Yes and no. (*b*) Better elementary text-books, better preparation on part of teacher; more rigid demonstrations of the principles of a science.

130. (*a*) Not in general. (*b*) More attention should be given to the axioms and the fundamental laws and their connection with the subject, and more attention to the theory of simultaneous equations.

131. (*a*) Yes, most important, but not necessarily in the words of text. (*b*) In the larger colleges algebra is mostly taught by tutors, who hold temporary appointments, and do not expect to make teaching their life work. Algebra as well as calculus should be taught by a permanent professor.

133. (*a*) Yes, but not in rigorous form. (*b*) Greater facility in their use with a more intelligent understanding of them.

134. (*a*) Yes, but they must also be thoroughly understood.

135. (*a*) Yes, for average student. (*b*) Examples given should be made more modern and practical. *The theory of functions should be incorporated, beginning with simple elements. This will make the whole subject of series, etc., easy for the student.*

136. (*a*) Very few.

137. (*a*) Yes. (*b*) Let us have live, enthusiastic, and competent teachers—such as will teach the subject rather than the text-book.

138. (*a*) The more important, yes. (*b*) For the preparatory work, greater thoroughness is much needed.

139. (*a*) No. (*b*) Explain by common sense and not by rule.

140. (*a*) It does no harm. (*b*) The current text-books are too arithmetical.

141. (*a*) No.

142. (*a*) No.

143. (*a*) No. (*b*) Less mechanical work; more thought. Students should be taught to think! think!! think!!!

144. (*a*) No. (*b*) With such text-books as Hall and Knight's Elementary Algebra and same as C. A. Smith's or Todhunter's Higher Algebra; *no reform needed.*

145. (*a*) No. (*b*) *Greater attention to mental and inventional algebra, and to numerical and geometrical applications and illustrations.*

146. (*a*) No.

147. (*a*) I recommend the memorizing of the rules, unless the pupils furnish a good working rule of their own (a rare case).

148. (*a*) No. (*b*) A more thorough drill in factoring and in fractions, and in putting into words the ideas conveyed by its symbols, equations, and operations. Also greater precision of expression.

(a) *Do you favor memorizing rules in algebra?* (b) *What reforms are needed in teaching the same?*—Continued.

149. (a) I do not.

150. (a) No.

151. (a) I do not. (b) The pupils learn to do by doing. Hence, instead of having pupils waste their time on abstract demonstrations, let them solve numerous problems of every variety. It is only practice that makes perfect.

152. (a) No.

153. (a) No. (b) The founding of all algebra upon the laws of operation.

154. (a) No.

155. (a) Yes.

156. (a) Not at all. (b) The pupil should be taught to *think* rather than *to work by rule.* More thoroughness needed.

157. (a) I do not. (b) I think that the student should be required to construct his own rules as far as possible.

159. (a) But few. (b) Principles are apt to be lost sight of in the strict and close adherence to rules.

160. (a) No. (b) More thorough drill is needed, especially in the elementary principles.

161. (a) Yes. (b) Broader views of algebraic operations; more generalizing and greater exactness of language.

162. (a) Yes.

163. (a) Yes. (b) The rules should be demonstrated oftener than they are.

164. (a) No. (b) To develop the subject by original investigation.

166. (a) Yes.

167. (a) To but very slight extent. (b) Less formality and more "realism;" introduction of principles often held back until higher branches are reached, *e. g.*, factors of direction, differentials, etc.

168. (a) Only very few. (b) More attention to problems involving principles and less to puzzles.

(a) *To what extent are models used in geometry?* (b) *To what extent and with what success original exercises?* (c) *Do you favor memorizing verbatim the theorems (not the demonstrations) in geometry? What reforms are needed in teaching the same?*

1. (a) Class-room very poorly supplied, but we use the few we do possess as much as possible. (b) Such exercises are given every day and are found to be very beneficial. (c) No.

2. (a) Moderately, to explain effects of perspective on the black-board. (b) To a very moderate extent with the great majority of students, to a great extent with the best.

3. (a) When models have been used it has facilitated the work. (c) Yes.

4. (a) They are used to a limited extent. I question very much the advantages of using models, except with beginners, or rather with those who are studying works introductory to regular demonstration. (b) To a limited extent. (c) Yes; more original work; more attention to logical processes, clearness and accuracy of statement. I change the figures, *i. e.*, their relative position, so that the demonstration shall be reasoning and not memory.

5. (a) Use them to a great extent. (b) Original exercises with fine success. (c) No; some.

6. (a) The text-book quite closely followed. (b) Some daily and with good success. (c) Yes.

7. (a) Not used. (b) Used to some extent. (c) No.

8. (a) To a large extent. (b) One-fifth of work in geometry is in original exercise; the success is good. (c) Yes.

(*a*) *To what extent are models used in geometry?* (*b*) *To what extent and with what success original exercises?* (*c*) *Do you favor memorizing verbatim the theorems (not the demonstrations) in geometry? What reforms are needed in teaching the same?*—Continued.

9. (*a*) To a considerable extent in the lower grades. (*b*) Very extensively and with very satisfactory results. (*c*) As a general thing, I am of the opinion that too little time is given to the subject *to secure the best results.*

10. (*a*) Largely, both in class-room and out-doors. (*b*) To no great extent and with no marked success, as yet.

11. (*a*) Very little. (*b*) The exercises in Welsh's Geometry are used. *Some* of them seem hard to the students, but on the whole they do fairly with them. (*c*) Almost verbatim.

12. (*a*) Only moderately. (*b*) They take one-third of the whole time and make the life of the work. (*c*) The rigorous requirement of original well-graded work from the very first.

13. (*a*) In descriptive geometry only. (*c*) It ought to be taught more from a comparative point of view.

14. (*a*) None. (*b*) To a very limited extent and *not* with marked success. (*c*) No.

15. (*a*) Models are largely used in geometry in three dimensions. (*b*) To a small extent and without marked success. (*c*) No; better drawing in the text-books, especially in geometry in three dimensions; more attention to the drawing of the students; less geometry, altogether; I think the importance of Euclidian geometry as mental discipline is greatly overestimated.

16. (*b*) To a very large extent and with excellent success. (*c*) It is left optional with the student.

17. (*a*) Very little. (*b*) Subordinate to a marked degree. I am trying to change this state of affairs. (*c*) Yes; more original work, also more comparative, not purely descriptive work.

18. (*a*) Always used in teaching solid geometry and in teaching conic sections.

19. (*a*) Forms are used in solid geometry, etc., freely, to aid the mental conception of the *perfect ideals* of mathematics (Professor Gordon). But little used in plane geometry (Professor Draper). (*b*) Very simple exercises, arithmetical application of geometrical principles, constructions, and problems are freely used. Very simple "catch" theorems or "corollaries" involving some absurdity are occasionally introduced *to be proved!* Students of ordinary intelligence generally succeed with exercises graduated to their state of advancement (Professor Gordon). About one-fifth of the time is given them. Those who do well in the text and stand questioning upon it are fairly successful with originals. (Professor Draper). (*c*) Yes; except in a few cases where I think the theorem itself can be improved. Would begin it in childhood of pupil; would spend more time on its elementary principles (Professor Draper).

20. (*a*) Models are used in solid geometry and spherical trigonometry. (*b*) To a limited extent and with good results. (*c*) Yes. That the sense of the theorem be known when the memorizing is complete.

21. (*a*) Very little in plain geometry; more, but not very extensively, in solid geometry. (*b*) Used largely and with unqualified success. (*c*) Yes. Using figures just as given in book, using only propositions already proven, and many other things of a similar kind need reformation.

22. (*a*) To a limited extent. (*b*) To limited extent and with good success. (*c*) Yes, at first; it promotes accuracy of expression. Greater latitude may be allowed with advanced students.

23. (*a*) Models have just been obtained.

24. (*a*) None, except to illustrate solid bodies. (*b*) Extensively and successfully.

(*a*) *To what extent are models used in geometry?* (*b*) *To what extent and with what success original exercises?* (*c*) *Do you favor memorizing verbatim the theorems (not the demonstrations) in geometry? What reforms are needed in teaching the same?*—Continued.

25. (*a*) From want of funds the supply is limited to such rude models as teacher and student can make. (*b*) Original exercises in connection with every study are used freely and with good results. (*c*) More original exercises.

26. (*c*) No. Variation of letters, etc., to represent angles.

27. (*a*) I have relied upon models to a great extent. I require all studying solid geometry to construct the five regular polyhedrons with pasteboard, giving reason why only five can be formed. (*b*) All classes work at original propositions. The results have shown the practice to be very important.

28. (*a*) No models used in plane geometry. The sphere, the cone, and a few others are used in solid geometry. (*b*) One-half of all the time for geometry is devoted to original exercises. Success very satisfactory. (*c*) Some. More original work. Demonstrations varying from those of the author should be encouraged.

29. (*a*) Not to a great extent. (*b*) They are freely used with the best results. (*c*) Yes, substantially; the gravest error is the memorizing of demonstrations—an evil that seems unavoidable, if text-books are employed. The ideal method is oral instruction, in which the mental movements of the pupils are under the eye of the instructor. It is a *pity* that a subject that has such possibilities for pupils should be so taught as to become a mere "memory grind."

30. (*a*) To no great extent, but figures extensively. (*b*) No great extent. Human nature is not original. Originality is the exception. The average student who spends his time on original exercises will fail of that discipline in *method* which he needs. (*c*) Yes, the student will become benefited in learning of a formula of words expressing truths. Stick to the Euclidian method; there is no "royal road" to geometry.

31. (*a*) Such as we are able to improvise.

32. (*a*) They are not used. (*b*) We intersperse them throughout the entire course of geometry. (*c*) While they should be memorized, the student should learn to state them also in good language of his own.

33. (*a*) Practically to no extent. (*b*) Great success when used. (*c*) More, *much more* original work and simplification of demonstrations.

34. (*a*) Very little. (*b*) One-third to one-half of work assigned. Great success with the better students. (*c*) No.

35. (*a*) Used in the study of the geometry of space, of surfaces of the second order. (*b*) Geometry is crowded into short time for necessary reasons; some original work done, less than would be useful. (*c*) Yes. Greater familiarity with definitions and axioms. The constructive method of carrying on demonstrations (*i. e.*, omit drawing figure in full, beforehand).

36. (*a*) We approve of their extensive use. (*b*) Throughout the course; success indifferent. (*c*) Yes. More attention to the form of demonstration and accuracy of statement.

37. (*a*) Not used to any extent. (*b*) Original exercies are extensively used, and a greater interest in the study. (*c*) More attention paid to original exercises well graded.

38. (*a*) Models are used. (*b*) All the examples in Wentworth's Geometry are solved, together with selections outside. We are more successful each succeeding year. (*c*) No. In general, less text-book routine and more problems, not so difficult, but well graded.

39. (*a*) Very little. (*b*) Much and with great success. (*c*) Yes. More demonstrations should be written out, both in the elements and among original exercises.

(a) To what extent are models used in geometry? (b) To what extent and with what success original exercises? (c) Do you favor memorizing verbatim the theorems (not the demonstrations) in geometry? What reforms are needed in teaching the same?—Continued.

40. (*a*) Not very much. I prefer that students should learn as soon as possible to form mental pictures of the figures and reproduce them on the board. (*b*) Frequent original problems are given and are very valuable. (*c*) Original problems and propositions should be given in connection with the lessons from the beginning.

41. (*a*) They are used in teaching the higher surfaces, especially the warped surfaces in descriptive geometry. (*b*) A few original exercises are given with the text-book work, and with marked success. (*c*) No. More reliance upon the imagination for the figures and less upon the blackboard.

42. (*a*) Limited. (*b*) Very largely and with great success. (*c*) Not necessarily.

43. (*a*) A limited extent. (*b*) Original demonstrations are required on one day of each week of second term. (*c*) Yes.

44. (*b*) From first to last with good success. (*c*) I do not. More original work.

45. (*a*) They are all represented by the blackboard. (*b*) As much as possible; usually daily. (*c*) I do. Less demonstrations in the book; more propositions for the student.

46. (*a*) Geometry is with us a preparatory study. (*b*) Constantly, with success. (*c*) No. The geometrical teaching in our public schools seems to be excellent.

47. (*a*) To a large extent. (*b*) With good success when in printed form; otherwise not so. (*c*) Unless the student can hold himself to an equally clear form. A union of the old rigor with modern improvements.

48. (*a*) The usual blocks, etc. (*b*) About one exercise out of every ten with fair success. (*c*) If the text is given in definite form and is well worded, yes.

49. (*a*) None, excepting the elementary models. (*c*) In part.

50. (*a*) As far as needed in all cases. (*b*) Not much success as yet, but hopeful. (*c*) I do. More use of exercises and original work.

51. (*a*) We have about one hundred dollars' worth of models for pure mathematics. (*c*) No.

52. (*a*) For solid. (*b*) From the first and with gratifying success. (*c*) Yes. Practical application of principles in concrete problems.

53. (*a*) Only ordinary models, or those commonly used. (*b*) Occasionally with good success. (*c*) Yes. Less speed and more thoroughness.

54. (*a*) To a limited extent. (*b*) Made prominent and with good effect. (*c*) To discourage mere effort to demonstrate by memory, rather than by intuition and train of reasoning.

55. (*a*) As far as possible, especially in solid geometry. (*b*) A great many original exercises. They are the best measures of the student's ability. (*c*) No. Less memorizing of demonstrations and more original work.

56. (*b*) The representative theorems are all demonstrated by original work as far as possible. (*c*) Yes. Any plan that will prevent students from memorizing the demonstrations.

57. (*a*) Not at all. (*b*) Have not tried this plan yet. (*c*) No. Not prepared to suggest.

58. (*a*) Not much, and mostly in spherical geometry. (*b*) Have had some original work with profit. (*c*) Yes. More familiarity with relations of parts to each other, and less dependence on the wording of the demonstrations as given in the book.

59. (*a*) We usually use diagrams. (*b*) To considerable extent and with eminent success. (*c*) Yes. All hail! to the man who will devise means to prevent the pupil from committing to memory the demonstrations.

(a) To what extent are models used in geometry? (b) To what extent and with what success original exercises? (c) Do you favor memorizing verbatim the theorems (not the demonstrations) in geometry? What reforms are needed in teaching the same?—Continued.

60. (*a*) To a moderate extent. (*b*) Largely, and with great success. (*c*) Yes. Guarding against use of memory too much by students in demonstrations of propositions.

61. (*a*) In lecturing only. (*c*) Yes. Thorough understanding of relatively important principles.

62. (*a*) To a very limited extent. (*b*) We use a great many original exercises with much success. (*c*) Yes. Too many allow students to memorize the demonstrations and thus miss the great advantage in geometry, a development of the reasoning faculties.

63. (*a*) So far as to illustrate triangles, parallelograms, circles, pyramids, prisms, cones, cylinders, and spheres. (*b*) Limited. (*c*) Yes. Demonstrations ought not to be memorized. Pupils ought to be shown that the truth of each proposition is established by a course of logical reasoning.

64. (*a*) For illustrating solid geometry, mensuration, conic sections. (*c*) Yes.

65. (*a*) Not at all. This is due to the school not being provided with models, and not to the teacher's preference. (*b*) They are used only occasionally, but with considerable success when used. (*c*) Yes.

66. (*a*) Whenever necessary. (*c*) Yes.

68. (*a*) Not at all in geometry, to slight extent in descriptive geometry. (*b*) Cadets are frequently required to submit exercises. (*c*) No.

69. (*a*) Largely. (*b*) Few, but satisfactory. (*c*) No. *Latitude*—so long as object is clearly stated, and demonstration is concise and complete.

70. (*a*) Very limited. (*b*) To a limited extent, but with good success. (*c*) Yes. More extended use of models.

71. (*a*) The spherical blackboard and models are used considerably. (*b*) They are being introduced with good success. (*c*) No; except for those students who must in order to understand them. Students should learn to depend less on the printed denonstrations.

72. (*a*) Definitions are taught by means of models. (*b*) The extent varies with different classes. The success is good with about one-third of the class. (*c*) No.

73. (*a*) Little. (*b*) To some trifling extent, always with profit. (*c*) Yes.

74. (*a*) Where models seem to make principles clearer, or their application practical, they are used in teaching solid geometry and spherical trigonometry. (*c*) Yes.

75. (*b*) Nearly one-half the time is spent upon original work and with marked success. (*c*) Not absolutely.

76. (*a*) In metrical geometry models are used altogether for illustration. (*b*) Our time being limited, we spend little on original exercises, but with fine success. (*c*) Yes. The student should be required to carefully write each demonstration upon the board.

77. (*a*) In solid geometry all the principal figures are thus illustrated. (*b*) To a considerable extent in plane geometry and with excellent success. (*c*) Yes. More original work and less memorizing of demonstrations.

78. (*a*) Very little. (*b*) To a very considerable extent and with marked success. (*c*.) No. More attention should be given to original work.

79. (*a*) To no great extent. (*b*) Original exercises are given as optional work and a few students are very successful in them. (*c*) Indifferent, provided they are given clearly and concisely.

80. (*a*) I use them very frequently. (*b*) Original exercises form a part of nearly every lesson. With a few exceptions the results are excellent, or at least satisfactory. (*c*) Yes. More original work.

(*a*) *To what extent are models used in geometry?* (*b*) *To what extent and with what success original exercises?* (*c*) *Do you favor memorizing verbatim the theorems (not the demonstrations) in geometry? What reforms are needed in teaching the same?*—Continued.

82. (*a*) Somewhat in solid geometry. (*b*) To a very large extent in daily work and with very satisfactory results. (*c*) I hold students responsible for a knowledge of the theorem, but not verbatim.

83. (*b*) Original exercises are used and with good success. (*c*) Yes.

84. (*a*) Not at all. (*b*) To a considerable extent and with much success. (*c*) No. More attention to logical form and precision of statement.

85. (*a*) In course on "form." (*b*) As far as the time allotted will allow, and with great success. (*c*) Yes. A greater use of objects. A leaving of parts of the demonstrations to be filled in, thus training for original work.

86. (*c*) No. The adoption of the *heuristic* method.

87. (*a*) Very little. (*b*) Original exercises comprise a very large part of the work, say one-half, in geometry. (*c*) Yes; those to be frequently referred to in subsequent work; others, no.

88. (*b*) Much used in geometry, and very successfully. (*c*) Yes.

89. (*a*) None. (*b*) Numerous practical problems with, I think, good success. (*c*) No.

90. (*a*) To a small extent. (*b*) A very large extent and excellent results. (*c*) No. Cultivation of more originality by means of graded exercises.

91. (*a*) They are not much used. (*b*) They are used whenever there is an opening; success is good. (*c*) No.

92. (*a*) They are used to some extent in solid and general geometry. (*b*) To considerable extent, with very satisfactory results. (*c*) As a rule, yes; for the reason that they are usually stated much more concisely than the student would state them. More original demonstrations.

93. (*a*) Full sets of Schröder's (Darmstadt) models. In solid geometry students make models from pasteboard. (*b*) Such exercises in connection with nearly every lesson, and with gratifying success. (*c*) Yes; number of proposition and book should not be memorized. More problems and practical applications; more theorems for original demonstration by pupils.

94. (*a*) Very sparingly; find them hurtful rather than helpful. "Normal school" methods are a failure in geometry. Have tried both and seen both tried. (*b*) In connection with nearly every theorem and every lesson. Success good. (*c*) Yes—no, depends on the student and the sort of drill he needs. A more rigorous insistence on founding everthing on the axioms.

95. (*b*) Many new exercises with great success. (*c*) Introduce more exercises and require variation in figures.

96. (*a*) None. (*b*) Very considerable extent and good success. (*c*) Not verbatim, but clearly and fully in substance.

97. (*a*) Hitherto but little; henceforth very great (if the appropriation asked of the State be granted). (*b*) If unassisted, or only slightly assisted, demonstration and solution be meant, great and good. (*c*) No! *Rupture with the traditional Euclidian methods, alignment with the march of modern thought.*

98. (*a*) To a small extent in descriptive geometry (warped surfaces, etc.). (*b*) To a great extent and as much as possible, and with marked success. (*c*) No, only to a slight extent for beginners. More original exercises, and *more modern geometry of position.*

99. (*a*) Average. (*b*) Tested daily. (*c*) Yes.

100. (*a*) We use models of the usual geometrical forms for illustration. (*b*) Frequent exercises in geometry; success only moderate. (*c*) No.

101. (*a*) In solid geometry. (*b*) One-third. Good results with fair success. (*c*) Yes. Larger per cent. of original work required.

102. (*a*) None. (*b*) Extensively used, and results very gratifying. (*c*) Yes. More original exercises and a more rigid reference to first principles.

(*a*) *To what extent are models used in geometry?* (*b*) *To what extent and with what success original exercises?* (*c*) *Do you favor memorizing verbatim the theorems (not the demonstrations) in geometry? What reforms are needed in teaching the same?*—Continued.

103. (*a*) None. (*b*) In geometry, with fair success; in practical surveying, leveling, etc. (*c*) Yes. As a rule, not allowing any lettering on board, etc.

104. (*a*) None. (*b*) None. (*c*) No.

106. (*a*) To a very limited extent. (*b*) To the extent which the time will permit, and with increasing degree of success. (*c*) I do not. I think reform needed in regard to grasping the truth, and giving it in good language, of the pupil's selection.

107. (*a*) Very little. (*b*) Slight extent. (*c*) No. More original work.

108. (*a*) Small. (*c*) Yes.

109. (*a*) Very little. (*b*) Considerable, with fair success. (*c*) No. Incorporation of some treatment of modern geometry.

110. (*a*) To a considerable extent in teaching solid geometry and spherical geometry. (*b*) Original exercises constitute half of the work, and with satisfactory success. (*c*) Yes. Require more solutions of practical problems; this tests the ability of the student and teaches him to walk alone.

111. (*a*) None. (*b*) We use few. (*c*) It is not material with us.

112. (*a*) Only to a limited extent in illustrating some of the properties of planes and solids. (*b*) To a great extent, and with satisfactory results. (*c*) I do. It is necessary that the student should know what he is required to demonstrate. Theory and practice should go hand in hand.

113. (*a*) They are used for every demonstration in solid geometry. (*c*) No.

114. (*a*) Very little. (*b*) It has not seemed profitable to spend much time on original work in geometry, which is a study of the Freshman year. (*c*) No.

115. (*b*) A great number of original exercises are given with complete success. (*c*) I do not favor memorizing anything except such principles as are needed in after work. For purposes of illustration we have a full set of models of solid and descriptive geometry. (*b*) Original exercises are given as regular and extra work to all classes. (*c*) Yes. The elementary principles of logic should be explained in connection with elementary geometry. (Tutor Fiske.)

116. (*a*) None. (*b*) Increasing number from year to year. Successful with first third of the class.

117. (*a*) Students make models in solid geometry. (*b*) With excellent results and to a large extent in geometry and trigonometry. (*c*) Yes. Throw students more on their own resources.

118. (*a*) Merely in explaining and illustrating. (*b*) They are required more or less throughout the course, especially in geometry. (*c*) Yes.

119. (*a*) We use the globe and the usual geometrical solids. (*b*) Original exercises are frequently given; success is very fair. (*c*) Yes.

120. (*a*) None are used in plane geometry. We have twenty-six fine models of warped and single-curved surfaces for use in descriptive geometry. (*b*) Original exercises are given out at each recitation, with great success as regards the development of mathematical knowledge. (*c*) No. With each lesson the student should have several original exercises involving the principles, to solve or demonstrate.

121. (*a*) Always in the teaching of geometry of space, when I find it helpful. (*b*) Continually given as *voluntary* text-work, excusing the student from formal examination in proportion to her success in it. (*c*) No. The abolition of *indirect* proof, and the use of symbolic notation, with special attention to *form*.

122. (*a*) In solid geometry. (*b*) To a limited extent. (*c*) Yes. More frequent *direct* application to problems in which dimensions are to be found.

123. (*a*) To a moderate extent. (*b*) To a considerable extent and with as good success as can be expected. (*c*) Yes. More frequent tests on original theorems and problems.

(*a*) *To what extent are models used in geometry?* (*b*) *To what extent and with what success original exercises?* (*c*) *Do you favor memorizing verbatim the theorems (not the demonstrations) in geometry? What reforms are needed in teaching the same?*—Continued.

124. (*a*) The students make models of the regular polyhedrons. (*b*) Large numbers given in each class; this is one of our chief methods of drill and training; *it is the only way in which fundamental principles can be so thoroughly ingrained in the mental make-up of a student that he is no longer conscious of an effort of memory in his knowledge.* (*c*) Yes, to encourage exact expressions and as a tribute to order which is the soul of geometrical reasoning.

125. (*a*) Very little at present; hope to use them extensively. (*b*) To as large an extent as possible. With good success from the majority of students. (*c*) I do not. (1) Better trained teachers; (2) more thinking and less memorizing; (3) use of thoroughly good text-books, like Byerly's Chauvenet; (4) emphasis of logic side; (5) generalization and summing up of truths proved, etc.

126. (*a*) No geometry taught excepting descriptive geometry. Students construct their own models. (*b*) Original exercises in almost daily use. (*c*) Yes. Demonstration of theorems without letters or figures.

127. (*a*) We have a set of "geometrical solids," which we use on occasion. (*b*) Such exercises are often required, and they are valuable—increasing the interest and testing the student's knowledge.

128. (*a*) Not largely, but so far as the students seem to need them. (*b*) Very largely and with good success. (*c*) A clean-cut, accurate statement, whether verbatim or not. Teacher to make sure of actual mastery of principles—no memory work; much use of original exercises.

129. (*a*) So far as is necessary for the pupil to get a clear conception of the geometrical concept. (*b*) In the preparatory course for admission to Freshman class, limited. In Freshman used to large extent. (*c*) Yes.

130. (*a*) Models of solids are used in solid geometry. (*b*) A good deal of use is made of them. Success good with best students. (*c*) No. More time should be given to leading the student to discover theorems for himself.

131. (*a*) Not very much. (*b*) As much as possible and with gratifying results. (*c*) No. More original exercises.

132. (*a*) Geometry is finished before entrance, except descriptive geometry, in which we use no models.

133. (*a*) To a limited extent with sphere and regular polyhedrons. (*b*) The original work in Wentworth's Geometry, with fair success. (*c*) Yes.

134. (*a*) In solid geometry. (*b*) Exercises and constructions on each book, and with good success. (*c*) No. Something like Wentworth's system of demonstrating propositions.

135. (*a*) None except sphere and cone. (*b*) At least one original exercise is given as a part of each lesson. Great success. (*c*) No. Students should be taught to master new processes or methods of proof rather than individual theorems; so come to look on theorems and proofs as illustrations of processes, or methods of investigation.

136. (*a*) Not at all. (*b*) Very little, in obligatory mathematics; and in regular course there is hardly any pure geometry; but when there is any, such exercises are helpful. (*c*) Only in elementary work, and even in that the attendant dangers are great.

137. (*a*) To a slight extent only. (*b*) Limited extent, but with good success. (*c*) Yes.

138. (*a*) To a limited extent. (*b*) They are much used and with good results. (*c*) No.

139. (*a*) Very slight. (*b*) About fifty original exercises are given and are well done. (*c*) Yes.

140. (*a*) Slightly. (*b*) Considerably, with success. (*c*) No.

(*a*) *To what extent are models used in geometry?* (*b*) *To what extent and with what success original exercises?* (*c*) *Do you favor memorizing verbatim the theorems (not the demonstrations) in geometry? What reforms are needed in teaching the same?*—Continued.

141. (*a*) Small. (*b*) Considerable extent and with commendable success. (*c*) Yes.

142. (*b*) With good success, as a rule. (*c*) Yes.

143. (*a*) Our models are such as we make ourselves. We illustrate, so far as possible, in solid and descriptive geometry. (*b*) All that I can have time for and can get the students to solve; great success with the few, little with the mass. (*c*) No; but I require a clear statement in somebody's words. More original work should be given; the student should be taught to depend upon himself more, and less upon book or teacher, to *think*, to originate, not memorize, not absorb.

144. (*a*) Not at all, except for young pupils. (*b*) Daily use and with good success. (*c*) Yes. Young pupils should be drilled in practical exercises, with use of instruments.

145. (*a*) Have been used but little. (*b*) Constantly and successfully. (*c*) No. *An earlier start, with main attention, at first, to training observation. Greater freedom from formalism.*

146. (*a*) Whenever possible. (*b*) Limited. (*c*) Yes.

147. (*a*) I make considerable use of models, especially in solid geometry. (*b*) Original exercises are required at a few places (two or three lessons). Good success. (*c*) Yes; yet I do not insist on keeping every word, provided the sense is kept.

148. (*a*) We do not use models. (*b*) We lay great stress on original exercises. When properly selected they are of the utmost service. (*c*) No. A clearer comprehension of the definitions; a more frequent enumeration of facts already proved; a more explicit enumeration of facts to be established in demonstrating any particular theorem.

149. (*a*) In a very limited degree. (*b*) For the past few years I have used them freely with gratifying success. (*c*) No.

150. (*a*) To a limited extent. (*c*) No.

151. (*b*) The study of geometry would fall far short of its object if original work were not required. I devote one recitation hour each week to it, and I am pleased with the results. I judge of the mental development by the original work done by pupils. (*c*) Yes. The facts of geometry must come first, concrete object lessons; can't reason about that concerning which we know little or nothing.

152. (*a*) But little, except in descriptive geometry. (*b*) Much time is given to solution of problems, both from text-book and from other sources. (*c*) Yes.

153. (*a*) Largely, especially in solid geometry. (*b*) Continually and copiously and with great success. (*c*) No. *The rejection of the words "direction" and "distance" from elementary geometry.*

154. (*a*) Largely, especially in conic sections and descriptive geometry. (*b*) Weekly exercises and with very satisfactory results. (*c*) No.

155. (*a*) To a limited extent. I expect to use models to a greater extent in the future. (*b*) Original exercises are greatly used. I value them very highly, and I am much pleased with the results I have obtained by using them in all my classes. (*c*) I do not.

156. (*a*) None used. (*b*) Special prominence is given to the use of original exercises, with encouraging success. (*c*) Yes. Originality should be encouraged.

157. (*a*) They are used to a very limited extent, simply because the college is not supplied with them. (*b*) Frequent exercises are given with quite good success. (*c*) I do not; I think that a student should be required always to express his thoughts in his own language, if for no other reason than to acquire *facility in expression.*

(*a*) *To what extent are models used in geometry?* (*b*) *To what extent and with what success original exercises?* (*c*) *Do you favor memorizing verbatim the theorems (not the demonstrations) in geometry? What reforms are needed in teaching the same?*—Continued.

158. (*a*) To only a small extent. (*b*) Largely given, and, I think, with great success in promoting intellectual pluck and thoroughness of attainment. (*c*) Yes; and also to learn to state them in one's own words. Subject too large for space. I will say, however, that the schools should give more exercises for solution, and train the boys from the beginning in original solution.

159. (*a*) Models are used in conic sections and for surfaces of revolution in analytic geometry of three dimensions. (*b*) Special attention given to original exercises. A taste for such work is easily developed in every lover of mathematics. (*c*) No; would prefer that the student thoroughly understand the truth to be demonstrated and express same in his own language.

160. (*a*) Very extensively, especially in solid geometry. (*b*) Largely, and with decided success. (*c*) Yes. I would have it made more practical.

161. (*a*) None. (*b*) Original exercises are frequent and attended with encouraging success. (*c*) I do. Demonstrations should be less verbose, and expressed to a greater extent by algebraic symbols.

162. (*a*) Considerably. (*b*) With almost every lesson, and with much success. (*c*) To some extent.

163. (*a*) Used to illustrate definitions. (*b*) We devote some time to them now; shall devote more; excellent success. (*c*) I do. Greater care that the pupil understand the reasons for every statement.

164. (*a*) Models used but little in first presenting the subject. (*b*) Original exercises given throughout the course, with good success. (*c*) No. Geometry should be taught as algebra and arithmetic by original work.

165. (*a*) Limited. (*b*) Largely, with great success. (*c*) Yes.

166. (*a*) We have none. (*b*) On each recitation, when there is time. (*c*) Yes. Less text-book and more originality.

167. (*a*) Very little. The attempt is made to lead the student to form his magnitudes in space, and without even a drawing, if possible. (*b*) To a very considerable extent, especially in test-work; and with excellent success in about one-half the cases. (*c*) No; but I would insist on concise and accurate statement. (1) Less bondage to text-book. (2) Encouraging original demonstrations. (3) Clearer distinction between leading steps of proof and details.

168. (*a*) Only slightly, because we are not able to afford them. (*b*) They are used as much as time will permit, and with good success. (*c*) No. Less time given to theorem-demonstration and more to original exercises, with the proper change in text books.

Is elementary geometry preceded or accompanied by drawing?

"Preceded": 21, 22, 29, 49, 51, 53, 55, 58, 62, 73, 76, 82, 90, 92, 98, 102, 103, 110, 112, 114, 139, 147.

"Accompanied:" 8, 10, 12, 13, 14, 20, 24, 25, 27, 28, 30, 33, 39, 41, 42, 43, 44, 48, 54, 56, 57, 59, 60, 61, 66, 70, 71, 84, 94, 96, 97, 99, 101, 105, 106, 122, 127, 134, 135, 137, 138, 140, 142, 150, 153, 154, 457, 159, 164, 165, 166.

"Both preceded and accompanied": 4, 9, 11, 69, 85, 87, 88, 107, 111, 119, 124, 146, 160, 161.

"Neither preceded nor accompanied": 1 (except engineering students), 15, 16, 17, 19, 31, 32, 34, 35, 37, 38, 45, 63, 77 (except in industrial course), 81, 91, 93, 100, 108, 109, 115, 116, 117, 120, 125, 128, 129 (except in scientific course), 130, 131, 149, 150, 155, 156, 167, 168.

"Yes; either preceded or accompanied": 23, 26, 123, 145, 163.

State time of your special preparation for teaching mathematics, number of hours you teach per week, and what other subjects you teach.

1. Teach twenty hours per week; teach no other subjects.

3. About ten years before entering the University of Alabama, where I spent five years; physics and astronomy.

4. I teach twenty-five hours a week, and during five months in the year give about six hours a week to special work in surveying; I teach no other subjects.

5. Six and a half hours per day in the entire school, with one-half hour recitation for classes at different times.

6. From 1880 to 1886; nine hours; geology, astronomy, elocution.

8. Two years; thirty-five hours per week; book-keeping, six hours per week.

9. I teach only mathematics, and give five lessons of one and one-fourth hours each per week.

10. Four years; seventeen hours per week; no other subjects save military science and tactics.

11. Five years' study at the Cincinnati Observatory after graduating from college. At present I teach thirty recitation hours (forty-five minutes each) a week. Astronomy (popular).

12. It has been my specialty for eight years; twenty hours per week; none.

13. Pure mathematics is taught twenty hours per week.

14. Teach mathematics three and four hours per week; my chief subject is chemistry, while mathematics is a secondary subject here.

15. I have taught mathematics since graduating from college (1870). I teach from eight to twelve hours per week. Astronomy is also in my charge.

16. I teach from twenty to thirty hours per week, and teach no other subject.

17. Five years; twenty hours per week; none.

18. The classes of mathematics, except the first class, are taught by the professors of the regular classical course. Each professor teaches only one class of mathematics.

19. Ten to fifteen hours per week, mental science and chemistry, etc. (Professor Gordon). Preparation has nearly all been made since I began and while teaching; ten hours; Latin (Professor Draper).

20. I teach mathematics twenty hours per week. I teach no other subjects.

21. Mathematics was my specialty in college two years; teach it fifteen hours per week. I teach no other subjects.

22. About three hours per diem devoted to teaching. Teach, besides, chemistry and elocution.

23. I teach twenty hours per week, and also military science and tactics.

25. Three hours per day. I teach nothing but mathematics.

27. I teach only pure mathematics.

28. Thirty; teach no other subject.

29. Fifteen hours a week; physics five hours a week for six months; astronomy five hours a week for four months; psychology five hours a week for three months.

30. Sixteen to twenty hours. I teach philosophy, astronomy, logic, moral philosophy.

32. I give fifteen to twenty hours of instruction, and teach several natural sciences, besides some German.

33. One year and a half; twenty hours; no other subjects.

34. Several graduate courses at Princeton College, and private study. I teach from eleven to fifteen hours per week. Am also engaged in teaching astronomy.

State time of your special preparation for teaching mathematics, number of hours you teach per week, and what other subjects you teach—Continued.

35. Have taught and have been a student in the higher mathematics the past twenty years; fifteen hours per week; not any. (Professor Shattuck.) Algebra, five hours per week for two terms; geometry, same; natural philosophy, physiology, botany, English, rhetoric, Latin, Greek. (Regent Peabody.)

37. One year; twelve hours per week, besides teaching some in preparatory department.

38. I teach mathematics and astronomy about twenty hours per week; no other subjects.

39. Fifteen hours; no other subjects.

40. Teach about twenty hours per week. I teach regularly no other subject. and only the mathematics in the college department.

41. I spend from one to two hours per lesson on mathematical works, directly or remotely connected with the recitation. I teach twenty hours per week and only mathematics.

43. Teach twenty hours per week; teach no other subject except astronomy.

44. Eighteen hours per week; history and vocal music.

45. Everything pertaining to or suggested by the lesson is prepared. All lessons are five hours per week with one exception; geometry has four.

46. I teach mathematics and astronomy fifteen hours per week; assistant teaches mathematics fifteen hours per week also.

47. After graduating at Madison, spent four years in post-graduate study at Yale; fifteen to twenty hours; astronomy and political economy.

50. We have four teachers of mathematics, who spend about fifteen hours a week in their classes.

51. Am at it nearly all my time that can be secured from other work; first term, fifteen hours; second term, twenty hours.

52. Thirty hours per week; political economy.

53. All afternoon for preparation; twenty-five a week; none.

55. Have taught it for eighteen years, six years exclusively. Teach mathematics twenty hours per week; teach nothing else.

56. I took the two years' collegiate course required and took the post-graduate course, spending three months on special work in mathematics; fifteen hours; natural science.

57. Some class almost every hour in the day, averaging, perhaps, twenty-five hours per week.

58. Scarcely any two terms the same.

59. About three-fifths of my time is given to mathematics and about two-fifths to Latin and Greek.

60. Three years; twenty hours; political science, astronomy, and German.

61. Fifteen hours per week; mental philosophy and logic.

62. I teach mathematics fifteen hours per week and have some classes in Latin. My preparation is done each night before the work of the following day.

63. Six to ten hours; eighteen hours; Latin, physiology, physical geography, English literature.

64. One hour daily; twelve hours per week; astronomy, natural philosophy, chemistry, geology, mineralogy, drawing.

65. Four years at college, one in private work, and two at Johns Hopkins University. Twenty-four hours per week. Nothing else.

66. Four hours per week.

70. Mathematical course at Yale College together with four years subsequent study. Political economy and English literature.

71. Since completing the course in this college, three years ago, I have spent in private study a considerable portion of my time; five to ten hours per *week; rhetoric* and drawing.

State time of your special preparation for teaching mathematics, number of hours you teach per week, and what other subjects you teach—Continued.

72. Eight, six, or four hours per week, according to term, whether it be fall, winter, or spring. Lecture on art.

73. The only special preparation I employ is the light reading of new text-books which come to hand. For over twenty years have had no difficulties in mathematical instruction. I also teach, as occasion calls, metaphysics, morals, political economy, history, literature, etc.

74. Teach on an average twelve hours per week.

76. Three years; about fifteen hours per week; physics and meteorology.

77. I teach twenty-five hours per week, five hours of which are devoted to industrial drawing.

80. Three years in which I did the four years' work in college mathematics, required and elective, together with outside special work in same subject. I teach five or six classes per day in mathematics. I have only astronomy, besides mathematics.

82. Harvard College, A. B., with electives in mathematics.

85. A four-years' course in both State Normal school and college; no other subject.

86. I teach mathematics about five to nine hours weekly, astronomy three to fifteen hours weekly; and, *this term*, am teaching algebra. In addition, I have certain duties connected with the observatory, and a requirement of the founder of my professorship, viz, I have to contribute to the advancement of astronomical science.

87. Two gentlemen here are occupied in teaching mathematics, exclusive of analytical mechanics and civil engineering; occupied in class-room eighteen or twenty hours.

89. Five hours per week and five hours for assistant; mining, surveying, mechanics.

90. Two years; twenty-two and one-half hours; no other subjects.

91. Several hours each day; twenty hours per week. (I only teach the higher branches.) My assistant teaches all up to and including analytic geometry, moral science, etc.

92. The mathematics of an ordinary college course and two summer vacations' study with the late Dr. Edward Olney, of Michigan University; thirteen and one-third hours per week; physics and astronomy.

93. Two years (1871–73) of post-graduate work, and fifteen years since as specialist; teach about seventeen hours per week; part of the work done by an assistant; astronomy.

94. Twelve and one-half hours; political economy.

95. I have calculus four times per week, mechanics six times, and thermodynamics (Clausius) three times.

97. Two years' private study, three years' study in Europe; fifteen or twenty; none.

98. Five hours' class-work per week for five years, full course.

99. Ten per week.

101. Eighteen hours per week; physics.

103. Preparation, four years' course at the United States Military Academy; average time, ten hours per week; arts and science of war and tactics.

104. Forty-nine years; fifteen hours per week; no other subject.

105. The professor of mathematics has not had special training, but has special aptitude in this direction. Has usually taught chemistry.

106. My teaching is limited to mathematics; fifteen hours per week.

107. Four years at Dartmouth College, and two years at the Thayer School of Civil Engineering; about ten hours per week; mechanics, astronomy, meteorology, surveying.

State time of your special preparation for teaching mathematics, number of hours you teach per week, and what other subjects you teach—Continued.

108. Twenty hours per week; no other subject.

109. Graduated Ph. B. at the University of North Carolina, and spent one year studying mathematics at the Johns Hopkins University; eight hours per week; English four hours.

110. Time of teaching varies from eighteen to twenty-one hours per week.

112. Teaching hours, sixteen per week; I teach no other subjects.

113. Generally prepare in one-half hour; teach two hours; Latin and Greek one hour each.

114. Five years; fourteen hours per week; no other subjects.

115. Our professors teach but one subject.

116. My college course, supplemented by three years' study in Germany; I teach ten hours per week; no other subject.

117. Have been teaching mathematics and science for fifteen years. At present most is done here by our instructor.

119. From ten to sixteen hours per week; I teach nothing but mathematics.

120. I devote eighteen hours per week to teaching; I do not instruct in any other subject.

121. Mathematics and physics, also lectures on general astronomy and some on physical geography.

123. Average eighteen; none.

124. Graduate of the University of Virginia, with degrees of bachelor of science, civil engineer, mining engineer; two years a student of mathematics in Cambridge University, England, and fourteen months at Göttingen, Germany; seventeen hours per week (mathematics and astronomy). Assistant Prof. C. E. Kilbourne, graduate of United States Military Academy, teaches ten hours per week, and Assistant Prof. G. W. McCoard (Bethany College, W. Va.) teaches eighteen hours per week.

125. One and one-half years in university, besides work done privately. Mathematics, twelve hours per week; two hours in civil engineering and astronomy.

127. I teach mathematics from fifteen to twenty hours per week, and French from five to ten hours.

128. Aside from regular college and private work, a year's partial work at Harvard. Teach fifteen hours a week. No other subject, except Bible, one hour a week.

129. From ten to fifteen hours; astronomy, surveying, and bridge construction.

130. Four years, seventeen to eighteen hours; astronomy and elementary mechanics are included in the seventeen or eighteen hours.

131. Post-graduate student two years at Massachusetts Agricultural College and Johns Hopkins University. Teach twelve to sixteen hours per week. Astronomy also.

133. No special training aside from a regular college course and private study; ten hours; mechanics, physics, astronomy.

134. Teach three hours each day; surveying and mechanics.

135. One year, after graduation from college; eleven hours per week; astronomy.

136. Graduated with honors at Girton College, Cambridge, 1880; four years' subsequent residence and attendance at Professor Cayley's lectures; ten or eleven hours per week; no other subject.

139. Amount of time occupied in teaching is regularly eighteen hours, often increased by extra work. I teach no other subject.

140. Prof. Isaac Sharpless, L. B. (Harvard); seven hours; none. Prof. Frank *Morley, A. M.* and eighth wrangler of Cambridge; fourteen hours; none.

State time of your special preparation for teaching mathematics, number of hours you teach per week, and what other subjects you teach—Continued.

142. Usual college course; twelve hours per week; Latin.

143. A college course and all the mathematics, both regular and extra, I could crowd into it. I teach now thirteen to fifteen hours per week. No other subject.

144. Algebra, five times; geometry, five; trigonometry, three; analytical geometry, three; calculus, three; analytical mechanics, three; trigonometry and surveying by an assistant.

145. Five years; thirteen hours per week; no other subject.

147. I have never taken a *special* course in mathematics, but have studied advanced works to some extent. I teach mathematics four hours a day. I have in charge vocal music, which takes forty minutes a day.

148. I teach (personally) twenty to thirty hours a week. About one-fourth of this time is occupied with mathematics, the other three-fourths with mechanics and civil engineering.

149. I teach no other subjects. I teach twenty hours per week.

150. Twenty-five hours per week; none.

151. No special preparation; twenty hours per week. I teach no other studies.

153. I devoted twelve years to my special preparation for teaching mathematics.

154. I am a student of mathematics; have charge of the department, but only teach the higher classes.

155. I first took a college course in mathematics. After this I spent several years in post-graduate work. I teach eighteen hours per week, and I teach no other subjects.

156. About seventeen hours per week; no other subject.

157. I have a course in civil engineering. I also have charge of a commercial course.

158. Number of hours given above one and a half at thirteen per week. But this does not include office hours of myself and assistant for meeting students and giving explanation. My assistant does not teach, but simply keeps office hours for consultation and solution and explanation of difficulties in lectures or assigned work. I will add that the under-graduate course in pure mathematics is the most extensive and thorough one given in any university in the United States.

159. With assistant, twenty-four hours per week; modern languages (French and German).

160. Twenty hours per week; civil engineering.

161. I teach mathematics four hours each day, and also teach physics and chemistry.

163. Sixteen hours per week; eighteen hours; French, Latin, history.

164. About twenty-five hours per week.

166. We give each class one hour each day. Any subject in the course.

165. Twenty years' experience in the mathematical class-room; about ten hours per week in mathematics, and five hours per week in physics.

168. The number of hours of teaching varies from five to twenty per week. College course and then three years at Yale; physics and astronomy.

(b) NORMAL SCHOOLS.

	Name of institution.	Location.	Name of person reporting.	Title or position of person reporting.
169	State Normal School.....	Jacksonville, Ala...	C. B. Gibson	President.
170	State Normal School.....	Florence, Ala	J. K. Powers.....	Do.
171	Normal School.....	Tuskegee, Ala	Maria A. Benson..	Instructor in mathematics.
172	Branch Normal School of the Arkansas Industrial University.	Pine Bluff, Ark	J. C. Corbin.....	Principal.
173	State Normal School.....	San José, Cal.....	R. S. Holway.....	Teacher in normal school.
174	State Normal School.....	Madison, Dak.....	William F. Gorrie.	President.
175	Washington Normal School.	Washington, D. C...	E. S. Atkinson	Principal.
176	Southern Illinois Normal University.	Carbondale, Ill.....	G. V. Buchanan ...	Teacher of mathematics.
177	Tri-State Normal College.	Angola, Ind.....	L. M. Sniff.....	President.
178	State Normal School.....	Terre Haute, Ind ...	Nathan Newby ...	Professor of mathematics.
179	State Normal School.....	Cedar Falls, Iowa...	D. S. Wright.....	Do.
180	State Normal School.....	Emporia, Kans.....	M. A. Bailey	Do.
181	State Normal School.....	Gorham, Me	W. J. Corthett	Principal.
182	State Normal School.....	Baltimore, Md	George L. Smith ..	Professor of mathematics.
183	State Normal School.....	Westfield, Mass.....	J. C. Greenough ..	Principal.
184	State Normal School.....	Worcester, Mass....	E. H. Russell	Do.
185	Iuka Normal Institute....	Iuka, Miss	E. L. Sherwood ...	Professor of natural science and mathematics.
186	State Normal School.....	Warrensburg, Mo...	George H. Howe ..	Professor of mathematics.
187	North-Western Normal School.	Stanberry, Mo	A. Moore	Principal.
188	State Normal School.....	Kirksville, Mo	J. L Nelson	Professor of mathematics.
189	Fremont Normal School..	Fremont, Nebr.	W. H. Clemmons..	President.
190	Normal College of New York.	New York, N. Y	J. A. Gilbt	Professor of mathematics and physics.
191	State Normal and Training School.	Genesee, N. Y.....	R. A. Waterbury..	Professor of higher mathematics and methods in arithmetic.
192	State Normal and Training School.	Cortland, N. Y	D. E. Smith	Professor of mathematics.
193	State Normal School.....	Oswego, N. Y	W. G. Rappleye ...	Teacher of mathematics.
194	State Normal and Training School.	New Paltz, N. Y	F. S. Capen	Principal.
195	State Normal School.....	Albany, N. Y	E. P. Waterbury ..	Do.
196	State Normal School.....	Buffalo, N. Y.....	J. M. Cassedy.....	Do.
197	State Colored Normal School.	Plymouth, N. C	H. C. Crosby	Do.
198	Normal Training School ..	Cleveland, Ohio.....	Ellen E. Reveley..	Do.
199	North-Western Normal and Collegiate Institute.	Wauseon, Ohio	J. H. Diebel.....	Instructor in mathematics.
200	State Normal School	Ashland, Oregon....	J. S. Sweet.....	President.
201	Drain Academy and State Normal.	Drain, Oregon	W. C. Hawley.....	Do.
202	State Normal School.....	Bloomsburg, Pa.....	G. E. Wilbur.....	Professor of higher mathematics and history.
203	State Normal School.....	Clarion, Pa	J. H. Apple	Professor of mathematics.
204	Philadelphia Normal School.	Philadelphia, Pa	G. W. Fetter.....	Principal.
205	*Cumberland Valley State Normal School.*	Shippensburg, Pa...	E. H. Bughee	Teacher of mathematics.

(*b*) NORMAL SCHOOLS—Continued.

	Name of institution.	Location.	Name of person reporting.	Title or position of person reporting.
206	State Normal School.....	West Chester, Pa...	D. M. Sensenig....	Professor of mathematics.
207	Central State Normal School.	Lock Haven, Pa	O. W. Kitchell	Instructor in mathematics.
208	Richmond Normal School.	Richmond, Va	S. T. Beach	Principal.
209	State Normal School.....	Fairmont, W. Va....	C. A. Sipe.....	Do.
210	State Normal School.....	Farmville, Va.....	Celestia S. Parish.	Teacher of mathematics.
211	State Normal School.....	Johnson, Vt	A. H. Campbell ...	Principal.
212	State Normal School.....	Oshkosh, Wis.....	E. F. Webster.....	Teacher of mathematics.
213	State Normal School.....	River Falls, Wis....	.....	

State time of your special preparation for teaching mathematics, number of hours you teach per week, and what other subjects you teach.

169. Four years; twenty hours; physics, chemistry, and astronomy.

170. Two years; ten hours; no other subject.

171. Twenty hours; reading.

172. I personally teach, at present, one class each in algebra, arithmetic, and geometry, for five days in the week, one in natural philosophy

173. Twenty hours; no other subject.

174. Algebra five hours; geometry, Latin, zoölogy, history of education.

176. Regular course in university; twenty-five hours; no other subject.

177. Twenty hours; no other subject.

179. Twenty hours; no other subject.

180. Twenty years; twenty-four hours; no other subject.

182. Drawing and physics.

183. The teacher of mathematics has ten hours; physiology.

185. Two years; six hours; natural science, history, rhetoric, and book-keeping.

186. Six classes per day, forty minutes each, five days per week; no other subject.

188. Five hours per day; astronomy one hour per day.

189. Fifteen hours.

190. Twelve hours; chemistry, physics.

191. Have taught mathematics almost exclusively for seventeen years, and principally for twenty-five years; thirty hours.

192. Election of all mathematics I could get in college course; eighteen and three-fourths hours; class on school law.

193. Graduate of Cornell; twenty-five hours; none.

195. Two years; twenty hours; none.

196. Mathematical course at Dartmouth; fifteen hours; astronomy.

197. Ten hours; physiology, history, moral science, and English literature.

198. Teachers of mathematics not specialists.

199. Six hours per day; no other subject.

200. Twelve hours; book-keeping, philosophy, psychology, art of school management.

201. Seven hours per week.

202. Four years; twenty-five periods, forty-five minutes each, per week; civil government.

203. Classical college course; twenty-five hours; no other subject.

204. Theory and practice of teaching and school government.

205. Twenty-two hours; no other subject.

State time of your special preparation for teaching mathematics, number of hours you teach per week, and what other subjects you teach—Continued.

206. Graduate of both elementary and scientific courses of Millersville State Normal School, Pennsylvania; have taught for twenty years scarcely anything but mathematics in three of the normal schools in Pennsylvania. Am author of Numbers Symbolized, an elementary algebra, and have in press Numbers Universalized; thirty hours.

207. Twenty hours; Latin.

208. Algebra, three hours; physics, physical geography, rhetoric, Latin, etc.

210. Private study at intervals for four years; twenty-five hours; no other subject.

211. Usual course in academy, normal school, and college.

212. None, except that spent in my regular course in the normal school; fifteen hours; no other branches.

Are students entering your institution thorough in the mathematics required for admission?

Of the forty-five reports received from normal schools three or four give no reply to this; all others answer *no*, excepting the institutions numbered 175, "generally so;" 176, "reasonably so;" 180, "fairly so;" 190 and 194, "yes."

What are the requirements in mathematics for admission?

Number 169 reports, arithmetic and elementary algebra; 190, arithmetic and a little geometry; all others require only arithmetic, generally not the whole of it, but through fractions and the simpler cases of percentage. Number 197 says, "fundamental rules of arithmetic;" 204 says, "fractions and percentage." A few institutions admit all who apply, without examination in mathematics.

Is the metric system taught?

All answered in the affirmative, excepting those bearing the numbers 171, 185, 190, 208.

Which is taught first, algebra or geometry?

All answered "algebra," excepting numbers 181, 183, 190, 210, 211.

Numbers 181, 183, 210 take up geometry first.

Numbers 190, 211 teach both together.

How far do you proceed in the one before taking up the other?

The following carry students first through a full course of elementary algebra: 169, 170, 174, 180, 184, 185, 189, 201, 206, 208, 209.

The following, through quadratics: 173, 178, 197.

The following, to radicals: 193, 196, 212.

The following, to quadratics: 192, 195, 200, 204.

The following, through fractions: 188, 207.

Institution 181 finishes plane geometry before taking up algebra; 183 gives one term of geometry before algebra; 210 observes the following order: (1) A course in form; (2) Rudiments of algebra; (3) Simple geometric theorems and constructions; (4) More difficult algebra; (5) More difficult geometry.

Are percentage and its applications taught before the rudiments of algebra or after?

All who answered said "before," except the following—192, 194, 210, that said "after," though some of the simplest parts of percentage were taught before; 184 and 206 said that both were taught together.

The mathematical course in the normal schools generally embraces a somewhat thorough study of arithmetic, the study of algebra and geometry, and usually a little trigonometry.

(c) ACADEMIES, INSTITUTES, AND HIGH SCHOOLS.

	Name of institution.	Location.	Name of person reporting.	Title or position of person reporting.
214	Toroli's Institute for Boys	Mobile, Ala	A. Toroli	Principal.
215	Public High School	Birmingham, Ala	J. H. Phillips	Superintendent of schools.
216	University High School	Tuscaloosa, Ala	W. H. Verner	Principal.
217	Marianna Institute	Marianna, Ark	F. A. Futrall	
218	Rogers Academy	Rogers, Ark	J. W. Scroggs	Principal.
219	Hopkins Academy	Oakland, Cal	Geo. C. Edwards	Teacher of mathematics.
220	St. Matthew's Hall	San Mateo, Cal	H. D. Robinson	Tutor of mathematics.
221	Boys' High School	San Francisco, Cal	W. N. Bush	Head-teacher of mathematical department.
222	Los Angeles High School.	Los Angeles, Cal	F. A. Dunham	Assistant teacher.
223	Girls' High School	San Francisco, Cal	Fidelia Jewett	Head of department of mathematics.
224	Oakland High School	Oakland, Cal	S. A. Chambers	Teacher of mathematics.
225	High School	Colorado Springs, Colo.	Harriet Winfield	Teacher of mathematics and science.
226	School for Boys	Stamford, Conn	H. M. King	Principal.
227	Connecticut Literary Institution.	Suffield, Conn	M. H. Smith	Do.
228	Public High School	New Britain, Conn	John H. Peck	Do.
229	Sioux Falls High School	Sioux Falls, Dak	Anna Emerson	Assistant high school teacher.
230	Washington High School.	Washington, D. C	Charlotte Smith	Teacher of mathematics.
231	Columbian College Preparatory School.	....do	H. L. Hodgkins	Instructor in mathematics.
232	Sparta Academy	Sparta, Ga	C. E. Little	Principal.
233	Academy of Richmond County.	Augusta, Ga	C. H. Withrow	Do.
234	Allen Academy	Chicago, Ill	I. W. Allen	President.
235	Public High School	Hyde Park, Ill	W. H. Reny	Principal.
236	North Division High School.	Chicago, Ill	O. I. Westcott	Do.
237	West Division High School.	....do	G. P. Welles	Do.
238	Peoria High School	Peoria, Ill	G. E. Knepper	Do.
239	Joliet High School	Joliet, Ill	O. L. Manchester	Do.
240	South Division High School.	Chicago, Ill	J. Slocum	Do.
241	Jennings Seminary	Aurora, Ill	J. E. Adams	Science and higher mathematics.
242	High School	Urbana, Ill	J. W. Hays	Superintendent of schools.
243	Roanoke Classical Seminary.	Roanoke, Ind	D. N. Howe	Principal.
244	Central Grammar High School.	Fort Wayne, Ind	Chester L. Lane	Do.
245	Public High School.	Crawfordsville, Ind	T. H. Dunn	Superintendent of city schools.
246	Indianapolis High School.	Indianapolis, Ind	W. W. Grant	Principal.
247	Union High School	Westfield, Ind	M. E. Cox	Do.
248	Indianapolis Classical School for Boys.	Indianapolis, Ind	T. L. Sewall	Do.
249	Indianapolis Classical School for Girls.	....do	T. L. Sewall, Mary W. Sewall.	Principals.
250	New Hope Female Academy.	Oak Lodge, Choctaw Nation, Ind. T.	A. Griffith	Superintendent.

(c) ACADEMIES, INSTITUTES, AND HIGH SCHOOLS—Continued.

	Name of institution.	Location.	Name of person reporting.	Title or position of person reporting.
251	High School	Davenport, Iowa	T. E. Stratton	Principal.
252	High School	Des Moines, Iowa	J. F. Gowdy	Teacher of mathematics.
253	Iowa City Academy	Iowa City, Iowa	M. R. Tripp	Do.
254	High School	Burlington, Iowa	E. Poppe	Principal.
255	High School	Davenport, Iowa	F. E. Stratton	Do.
256	High School	Topeka, Kans	J. E. Williamson	Do.
257	High School	Ottawa, Kans	G. I. Harvey	Superintendent.
258	High School	Paducah, Ky	A. H. Beals	Do.
259	New Orleans Seminary	New Orleans, La	L. G. Atkinson	President.
260	Girls' High School	....do	M. C. Cusack	Department of mathematics.
261	Madawaska Training School.	Augusta, Me	Vetal Cye	
262	Franklin Female College	Topsham, Me	D. L. Smith	Principal.
263	High School	Saco, Me	L. M. Chadwick	Assistant teacher.
264	High School	Bath, Me	H. E. Cole	Principal.
265	Fryeburg Academy	Fryeburg, Me	M. E. Bussell	Assistant.
266	High School	Portland, Me	A. E. Chase	Principal.
267	McDonogh School	McDonogh, Md	D. C. Lyle	
268	Washington County Male High School.	Hagerstown, Md	G. C. Pearson	Principal.
269	Centreville Academy and High School.	Centreville, Md	A. G. Harley	Principal.
270	Friends' Academy	New Bedford, Mass.	G. B. Dodge	Assistant teacher.
271	Haverhill Training School	Haverhill, Mass	C. A. Newton	Principal.
272	Mount Hermon School	Mount Hermon, Mass.	H. E. Sawyer	Superintendent.
273	Cushing Academy	Ashburnham, Mass.	F. D. Lane	Instructor in mathematics and German.
274	Prospect High School	Greenfield, Mass	Ida F. Foster	Teacher of science and mathematics.
275	High School	New Bedford, Mass.	R. G. Huling	Principal.
276	Eaton School	Middleborough, Mass.	A. A. Eaton	Do.
277	Powder Point School	Duxbury, Mass	F. B. Knapp	Do.
278	Admiral Sir Isaac Coffin's Lancastrian School.	Nantucket, Mass	E. B. Fox	Do.
279	Wheaton Female Seminary.	Norton, Mass	S. L. Dawes	Teacher of mathematics.
280	Lawrence Academy	Falmouth, Mass	S. A. Holton	Principal.
281	Smith Academy	Hatfield, Mass	S. L. Cutler	Do.
282	Partridge Academy	Duxbury, Mass	C. F. Jacobs	Do.
283	High School	Peabody, Mass	C. A. Holbrook	Do.
284	High School	Salem, Mass	A. L. Goodrich	Master.
285	Hanover Academy	Hanover, Mass	A. P. Averill	Principal.
286	Lynn High School	Lynn, Mass	William Fuller	Teacher of mathematics.
287	Bristol Academy	Taunton, Mass	William F. Palmer	Principal.
288	High School	Amherst, Mass	S. A. Sherman	Do.
289	Nichols Academy	Dudley, Mass	E. G. Clark	Do.
290	High School	Haverhill, Mass	Clarence E. Kelley	Do.
291	High School	Fitchburg, Mass	H. W. Kittredge	Do.
292	Sawin Academy and Dawse High School.	Sherborn, Mass	W. F. Gregory	Do.

(c) ACADEMIES, INSTITUTES, AND HIGH SCHOOLS—Continued.

	Name of Institution.	Location.	Name of person reporting.	Title or position of person reporting.
293	Drury High School	North Adams, Mass.	Elizabeth H. Talcott.	First assistant.
294	Charlestown High School.	Boston, Mass	J. O. Morris	Head-master.
295	Girls' High School	do	Adeline L. Sylvester. Emerette O. Patch	Assistant teachers.
296	Public Latin School	do	G. C. Emery	Teacher of mathematics.
297	Cambridge Latin School	Cambridge, Mass	W. F. Bradbury	Head-master.
298	West Roxbury High School	Boston, Mass	G. C. Mann	Principal.
299	English and Classical High School.	Worcester, Mass.	A. S. Roe	Do.
300	High School	Ann Arbor, Mich.	L. D. Wines	Instructor in higher mathematics.
301	Michigan Military Academy.	Orchard Lake, Mich.	W. H. Butts	Principal.
302	High School	Menominee, Mich.	Jesse Hubbard	Superintendent of city schools.
303	High School	Ypsilanti, Mich.	R. W. Putnam	Superintendent of schools.
304	High School	Grand Rapids, Mich	W. A. Greeson	Principal.
305	Michigan Female Seminary.	Kalamazoo, Mich	Isabella G. French	Do.
306	Shattuck School	Faribault, Minn	Wm. W. Champlin.	
307	Public High School	St. Cloud, Minn	C. C. Schmidt	Superintendent.
308	Augsburg Seminary	Minneapolis, Minn.	Wilhelm Pettersen.	Instructor.
309	Minneapolis Academy	do	E. D. Holmes	Principal.
310	Public High School	Vicksburg, Miss.	E. W. Wright	Superintendent of public schools.
311	Smith Academy, Washington University.	St. Louis, Mo	E. R. Offutt	Teacher of mathematics.
312	St. Joseph High School	St. Joseph, Mo	C. S. Thacher	Do.
313	Lincoln High School	Lincoln, Nebr.	S. P. Barrett	Principal.
314	Robinson Female Seminary.	Exeter, N. H	G. N. Cross	Do.
315	Simonds Free High School.	Warner, N. H.	E. P. Barker	Do.
316	Concord High School	Concord, N. H	J. F. Kent	Do.
317	Brewster Free Academy	Wolfborough, N. H.	E. H. Lord	Do.
318	High School	Portsmouth, N. H.	John Pickard	Do.
319	Pennington Seminary	Pennington, N. J.	J. R. Hamlon	Vice-President.
320	Hoboken Academy	Hoboken, N. J	J. Schrenk	Principal.
321	High School	Newark, N. J	H. T. Dawson	Instructor in mathematics.
322	Public High School	Orange, N. J	W. W. Cutts	Principal.
323	Newark Technical School.	Newark, N. J	H. T. Dawson	Instructor in mathematics.
324	Blair Presbyterial Academy.	Blairstown, N. J	J. H. Shumaker	Principal.
325	Stevens High School	Hoboken, N. J	F. L. Sevenoak	Assistant principal and professor of mathematics.
326	Newark Academy	Newark, N. J	S. A. Farrand	Head-master.
327	Dearborn Morgan School.	Orange, N. J	D. A. Kennedy	Principal.
328	New Brunswick High School.	New Brunswick, N. J	C. Jakobs	Superintendent of schools.

(c) ACADEMIES, INSTITUTES, AND HIGH SCHOOLS—Continued.

	Name of institution.	Location.	Name of person reporting.	Title or position of person reporting.
329	Fairfield Seminary.....	Fairfield, N. Y	J. M. Hall.....	Teacher of sciences.
330	College Grammar School.	Brooklyn, N. Y.....	L. W. Hart.....	Principal.
331	Cazenovia Seminary.....	Cazenovia, N. Y	A. White	Teacher of mathematics.
332	Ires Seminary	Antwerp, N. Y.....	E. M. Wheeler	Principal.
333	St. Mary's Academy.....	Troy, N. Y.....	John Hogan	Teacher of classics.
334	Adams Collegiate Institute.	Adams, N. Y.....	L. B. Woodward ..	
335	Grammar School.....	Clinton, N, Y	Isaac O. Best.....	Principal.
336	Gouverneur Seminary	Gouverneur, N. Y...	J. F. Ferthill.....	Superintendent of schools.
337	Union Classical Institute.	Schenectady, N. Y ..	E. E. Veeder	Teacher of mathematics.
338	High School	Troy, N. Y.....	J. P. Worden.....	Professor of mathematics.
339	Oxford Academy	Oxford, N, Y	F. L. Gamage	Principal.
340	Brooklyn Latin School....	Brooklyn, N. Y	C. Harrison	Head-master.
341	The Utica Academy.....	Utica, N. Y	G. C. Sange	Principal.
342	Rome Free Academy	Rome, N. Y	M. T. Scudder.....	Do.
343	High School	Buffalo, N. Y.....	M. T. Karnes	Do.
344	High School	Poughkeepsie, N. Y.	James Winne.....	Do.
345	Buarley School for Girls..	New York, N. Y	Jeannette Fine ...	Teacher of mathematics.
346	Friend's Seminary.....	New York, N. Y	John M. Child	Principal.
347	Central High School.....	Binghamton, N. Y ..	Fannie Webster..	Instructor in mathematics.
348	School for Girls.....	New York, N. Y	A. Brackett.....	Principal.
349	Free Academy.....	Elmira, N. Y	E. T. Wilson	Teacher of mathematics.
350	Delavan Academy.....	Delhi, N. Y	W. D. Graves	Principal.
351	Port Jervis Academy.....	Port Jervis, N. Y ...	John M. Dolph....	Superintendent.
352	Yonkers High School.....	Yonkers, N. Y	E. R. Shaw	Principal.
353	High School	Albany, N. Y	J. H. Gilbert	Professor of mathematics.
354	High School	Syracuse, N. Y.....	O. C. Kinyon	Teacher of physics and mathematics.
355	Fremont Institute.....	Fremont, N. C	W. Wills	Instructor in mathematics and Latin.
356	Bingham School	Bingham School P. O., N. C.	R. Bingham.	Superintendent.
357	High School	Huntersville, N. C ..	W. W. Orr	President.
358	Green Town Academy....	Perrysville, Ohio ...	J. C. Sample	Do.
359	High School	Zanesville, Ohio	W. M. Townsend.	Principal.
360	High School	Dayton, Ohio	C. B. Stivens	Do.
361	Mission House College....	Cleveland, Ohio.....	J.W. Grosshnesch.	Professor.
362	Bishop Scott Academy....	Portland, Oreg	F. E. Patterson ...	Lieutenant Colonel—mathematics.
363	Dickinson Seminary.....	Williamsport, Pa ...	G. G. Brower.....	Teacher of mathematics.
364	Western Pennsylvania Classical and Scientific Institute.	Mt. Pleasant, Pa....	L. Stephens.....	President.
365	Philadelphia Seminary for Young Ladies.	Philadelphia, Pa....	Carrie A. Bitting..	Librarian.
366	Wyoming Seminary	Kingston, Pa.....	R. B. Howland	Teacher of mathematics.
367	Harry Hillman Academy.	Wilkes Barre, Pa ...	E. Scott.....	Principal.
368	William Penn Charter School.	Philadelphia, Pa	A. D. Gray.....	Teacher of mathematics.
369	Central High School.....	Chester, Pa.....	J. F. Reizart	Principal.
370	High School	Titusville, Pa	C. E. Rose	Head of mathematical department.
371	*High School*.....	West Chester, Pa...	J. E. Philips	Teacher of mathematics.
372	*High School*.....	Scranton, Pa	J. C. Lange	Principal.

(c) ACADEMIES, INSTITUTES, AND HIGH SCHOOLS—Continued.

	Name of Institution.	Location.	Name of person reporting.	Title or position of person reporting.
373	High School	Wilkes Barre, Pa	O. W. Potter	Superintendent of schools.
374	High School	York, Pa	A. Wauner	Principal.
375	High School	Carbondale, Pa	H. J. Hoeckenburg	Do.
376	Boys' High School	Harrisburg, Pa	J. H. Wert	Do.
377	High School	Providence, R. I	D. W. Hoyt	Do.
378	Pawtucket High School	Pawtucket, R. I	W. W. Curtis	Head-master.
379	High School	Charleston, S. C	V. C. Dibble	Principal.
380	High School	Chattanooga, Tenn.	J. B. Cash	Do.
381	High School	Austin, Tex	J. B. Bryant	Do.
382	Thetford (Vt.) Academy and Boarding School.	Thetford, Vt	E. F. Morse	Assistant teacher.
383	Brigham Academy	Bakersfield, Vt	F. E. Parlin	Principal.
384	Troy Conference Academy.	Poultney, Vt	C. H. Dunton	Do.
385	High School	Rutland, Vt	L. B. Folsom	Do.
386	Central Female Institute.	Gordonsville, Va	Jas. Dinwiddie	Do.
387	High School	Richmond, Va	W. F. Fox	Do.
388	Thyne Institute	Chase City, Va	J. H. Veazey	Superintendent of schools.
389	High School	Charleston, W. Va	M. R. McGwigan	Principal.
390	West Virginia Academy	Buckhannon, W. Va	W. Johnson	Teacher of mathematics.
391	Male Academy	Charlestown, W. Va.	E. R. Taylor	Principal.
392	Free High School	Sheboygan, Wis	E. G. Haylett	Do.
393	High School	Milwaukee, Wis	G. W. Peckham	Do.
394	High School	Oshkosh, Wis	R. H. Halsey	Do.

What reforms are needed in the teaching of arithmetic?

215. Less adherence to and dependence upon text-books; more thorough primary *drill*.

218. More easy examples.

223. More mental work, more analytical work, greater quickness.

225. Increase in number of problems under each principle, decrease in number of "catch problems"; more mental work.

229. There is too much time put on it in all the lower grades.

232. More attention to rapidity, more every-day sums.

237. Introduction of quick and labor-saving methods in all business methods.

242. Better use of mathematical language; arithmetic as a deductive science.

251. More practice in rapid calculation. Many of the unimportant rules should be scarcely touched. My pupils waste energy by scattering too much.

255. A more judicious selection of subjects that time be not wasted upon non-essentials.

257. More mental arithmetic.

262. Something to make it more practical and the student better able to apply it.

270. Fundamental operations of arithmetic only should be taught before algebra.

274. Text-books are either so childish as to give no inspiration to work after the primary grades, or so abstruse and dependent upon logical reasoning beyond a child's capacity as to discourage.

275. Insistence upon accuracy in fundamental operations, and alertness of mind everywhere.

What reforms are needed in the teaching of arithmetic?—Continued.

276. More thorough work in elementary rules and in common and decimal fractions.

277. Scholars are pushed ahead altogether too fast, allowed to work slowly and incorrectly; should be drilled in quick addition, etc.

281. More attention to accuracy, rapidity, and practical methods.

283. It should be taught as an art rather than as a science.

286. There should be vastly more drill in fundamental processes.

288. Plenty of examples, more oral and "mental" work.

289. More practice. It seems to me that the agitation for reducing time given to arithmetic is a mistake, though greater economy of effort is possible.

294. Fewer subjects, more speed and accuracy in computation.

297. The difficulty (especially with female teachers) is too great subserviency to the text-book—*lack of elasticity in accepting methods.*

300. Hire competent teachers only.

304. More mental arithmetic.

307. More mental work, greater accuracy and rapidity. Scope of the subject reduced.

312. More practical work; judicious omission from ordinary text-book; better development of principles.

314. More mental, less written work.

317. A diminution in the number of subjects and more independent work by the pupil.

322. Particular attention to thoroughness, and abundant practice on fundamental rules and business methods, with the omission of some rules and methods formerly deemed essential.

331. Keep the *keys* out of the way and analyze every problem.

335. To return to the old custom of making the pupil do more thinking. There are too many helps and too much "mince-meat."

338. Many.

340. More philosophy.

341. Return to mental arithmetic, now sadly neglected. More attention to analysis, less to ingenious devices.

344. Do not permit primary teacher to *use* a *figure* in presence of children till they know everything about numbers one to ten.

346. *The use of the Grube method with beginners*, of denominate numbers before abstract, the expansion of the method of analysis in solving problems usually assigned to proportion.

352. A method that will shorten the time, give the pupils the essentials thoroughly. This will come, I believe, only through the experiments in industrial education.

353. More simplicity, less aiming to puzzle, less work that is wholly theoretical.

359. Brief methods of calculation should be insisted on, also independence.

370. *Less of it, in much less time than is now given to it* (*Superintendent of Schools*).

382. More attention to mental arithmetic.

386. The use of such books as Colburn's or Venable's Mental Arithmetic thoroughly at first; and the rejection of such methods as have recently been injected into the new Colburn's Mental Arithmetic. The public schools are teaching for show.

389. Books without answers are needed.

392. We should not go too far in seeking to make all divisions in arithmetic practical. Discipline must be held in mind.

To what extent are models used in teaching geometry?

The following reported that models were not used: 216, 219, 220, 226, 238, 239, 246, 253, 256, 257, 263, 266, 267, 277, 278, 288, 299, 300, 302, 316, 334, 352, 370, 378, 384, 393.

The following reported "occasionally," "not much," "very little": 217, 218, 222, 231, 233, 236, 240, 241, 242, 243, 244, 245, 251, 255, 258, 265, 269, 276, 282, 293, 294, 295, 296, 307, 309, 318, 324, 326, 332, 335, 337, 338, 339, 341, 348, 350, 351, 354, 356, 358, 360, 364, 366, 367, 372, 375, 385, 387, 390, 391.

Nearly all the remaining reports stated that models were used, specifying, in many cases, that they were found particularly serviceable in teaching solid and spherical geometry.

Those reports which stated that the models were made by the pupils themselves were classified with the group "using models." To teach plane geometry to very young students, or solid and spherical geometry to students of any grade, without the aid of models, is a great mistake.

To what extent and with what success original exercises?

All, except about two dozen, reported that original exercises were frequently used, with good success. Some said that one-sixth of the time allotted to geometry was devoted to them, others said one-half of the time; but the large majority of those specifying the relative amount of time given to such work answered one-fourth. Several reporters took occasion to say that the teaching of geometry without introducing original exercises was necessarily more or less of a failure.

Is the metric system taught?

Nearly every report showed that this is taught, though in many schools but little attention is given to it. We observed only one instance in which it was "dropped," after having been taught for some years. How long will it be before this country will wheel in line with the leading European nations and adopt this system to the exclusion of the wretched systems now in use among us?

Which is taught first, algebra or geometry? How far do you proceed in the one before taking up the other?

Excepting a number less than a dozen, all answered that algebra was taught first. The following *complete* a course in elementary algebra, before taking up geometry: 215, 218, 225, 226, 230, 235, 237, 238, 239, 245, 246, 247, 260, 263, 266, 267, 272, 273, 278, 282, 283, 284, 289, 290, 292, 294, 298, 299, 301, 307, 314, 316, 318, 321, 323, 324, 326, 327, 328, 335, 337, 338, 344, 345, 353, 354, 355, 359, 360, 369, 370, 372, 376, 377, 380, 382, 383, 385, 387, 389, 394.

The following take up geometry after having carried the student through quadratics: 214, 223, 236, 244, 252, 255, 256, 258, 263, 274, 275, 280, 286, 291, 295, 297, 300, 303, 305, 306, 311, 317, 334, 336, 339, 343, 350, 351, 356, 363, 378, 381, 384, 391.

The following, after having carried the student to quadratics: 216, 217, 233, 250, 251, 257, 264, 302, 357, 367, 373, 386.

Through radicals: 228, 243, 262, 322.

Through equations: 224, 268, 330.

To simple equations: 219, 231, 288, 374, 379.

Through factoring: 276, 325.

Through L. C. M. and G. C. D.: 220.

To fractions: 232, 248, 249.

To involution: 229.

Of those who take up geometry before algebra, 222 teaches Hill's Geometry for Beginners, 234 teaches the simpler parts of geometry, 242 teaches mathematical drawing, involving about sixty geometric problems (without demonstrations), 315 teaches geometry one year, 293 observes the following order of studies: (1) Beginning geometry; (2) algebra; (3) geometry.

In the two institutions, 269 and 270, algebra and geometry are taught together. Is this scheme *not worthy of* more extended trial?

Are percentage and its applications taught before the rudiments of algebra or after?

Nearly all replied that it was taught "before."

The following answered that in their institution it was taught "after": 217, 224 (arithmetic being reviewed with aid of algebra), 238, 239, 252, 266 (review), 340, 347, 355, 360.

In most, if not all these cases, the *elements* of percentage had been taught to the pupil, before he entered the institution.

In 325 the two subjects are taught "together."

Is it not desirable to introduce the rudiments of algebra earlier than has been the custom in most of our schools?

Are pupils permitted to use "answer-books" in arithmetic and algebra?

"Yes," "yes, but not encouraged": 214, 216, 217, 218, 219, 226 (with younger classes), 228, 230, 235, 238, 239 (in arithmetic), 243, 245, 246, 247, 248, 249, 250, 251, 255, 257, 260, 276, 277, 281, 289, 295, 302, 328, 330, 334, 335, 337, 339, 340, 342, 344, 347, 350, 356, 357, 359, 369 (in algebra, but not in arithmetic), 370, 371, 373, 374, 380, 381, 386, 387, 388, 389, 391, 394.

"No": 215, 220, 221, 222, 224, 226 (with older classes), 230, 237, 239 (in algebra, but not in arithmetic), 244, 256 (in algebra), 264, 278, 286, 287, 288, 290, 291, 293, 299, 304, 305, 310, 314, 315, 317, 318, 321, 323, 327, 331, 338, 343, 348, 349, 351, 360, 367, 372 (in algebra), 375, 385, 393.

"Some of the answers:" 214, 215, 221, 223, 224, 225, 228, 235, 236, 237, 238, 239, 240, 242, 244, 245.

Are students entering your institution thorough in the mathematics required for admission?

Some of the institutions, especially academies and institutes, have no requirements for admission. In the great majority of reports there was a general complaint that students were "not" well prepared or "by no means" well prepared in the requisites for admission.

The following answered "yes," "fairly so:" 214, 220, 221, 223, 224, 227, 228, 235, 237, 238, 245, 254, 266, 268, 293, 311, 322, 337, 352, 359, 390, 394.

What are the requirements in mathematics for admission to the institution?

"Practical arithmetic," "common school arithmetic," was the reply given by one hundred and fourteen institutions.

"Cube root in arithmetic and equations of the second degree in algebra," 217.

Arithmetic and elementary algebra: 222, 230, 273, 288, 303, 357.

Three books in geometry, Brook's Algebra, and arithmetic, 268.

Arithmetic and algebra as far as factoring, 370.

To ratio and proportion in Olney's Practical Arithmetic, 394.

Arithmetic through percentage, 360.

Arithmetic to percentage: 218, 306, 328, 379, 383.

Through fractions in arithmetic: 233, 282, 317.

Fundamental rules in arithmetic: 269, 356, 391.

V.

HISTORICAL ESSAYS.

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*HISTORY OF INFINITE SERIES.**

The primary aim of this paper is to consider the views on infinite series held by American mathematicians. But the historical treatment of this or any similar subject would be meagre indeed were we to confine our discussion to the views held by mathematicians in this country. We might as well contemplate the growth of the English language without considering its history in Great Britain, or study the life-history of a butterfly without tracing its metamorphic development from the chrysalis and caterpillar. A satisfactory discussion of infinite series makes it necessary that the greater part of our space be devoted to the views held by European mathematicians.

Previous to the seventeenth century infinite series hardly ever occurred in mathematics; but about the time of Newton they began to assume a central position in mathematical analysis.

Wallis and Mercator were then employing them in the quadrature of curves. Newton made a most important and far-reaching contribution to this subject by his discovery of the binomial theorem, which is engraved upon his tomb in Westminster Abbey. Newton gave no demonstration of his theorem except the verification by multiplication or actual root extraction. The binomial formula is a finite expression whenever the exponent of $(a+b)$ is a positive whole number; but it is a series with an infinite number of terms whenever the exponent is negative or fractional. Newton appears to have considered his formula to be universally true for any values of the quantities involved, no matter whether the number of terms in the series be finite or infinite.

The binomial theorem was the earliest mathematical discovery of Newton. Further developments on the subject of infinite series were brought forth by him in later works. He made extensive use of them in the quadrature of curves. Infinite series came to be looked upon as a sort of universal mechanism upon which all higher calculations could be made to depend. Special methods of computation, such as contin-

* This article was read before the New Orleans Academy of Sciences in December, 1887, and printed in the "Papers" published by that society, Vol. I, No. 2. Some slight changes have been introduced here.

ued fractions, could easily be reduced, it was said, to the general method of infinite series. It thus appears that series were cultivated by the early analysts with great zeal. They seem to have placed perfect confidence in the universality of the method. The mass of mathematicians never dreamed that the unrestricted use of infinite series was undermining mathematical rigor and opening avenues of doubt and error; they had no idea that in reasoning by means of series it was necessary to consider their *convergency* or *divergency*. To show what implicit confidence was placed in infinite series I shall quote a passage from the large, and in many respects excellent, history of mathematics, written by the celebrated Montucla, who flourished during the latter half of the eighteenth century.

In Volume III, page 272, he expresses the desirability of having a more rigid demonstration of the binomial formula than that given by Newton, so that no rational being might ever entertain the faintest doubt of its truth. Among the early English mathematicians there was one who *did* raise objections to the binomial formula, and of him Montucla says: "Thus we have seen a certain Dr. Green, * * * although professor of physics at the University of Cambridge and a colleague of Cotes, not only doubt it, but pretend that it was false and say he could prove it by examples badly applied; but it does not appear that the English geometers, not even Cotes, his colleague, deigned to reply to him." In the light of modern science, this passage ridiculing Green is very instructive. Time has turned the tables, and the laugh is no longer upon Green, but upon Montucla himself. We now wonder at the recklessness with which infinite series were once used in mathematical reasoning. To be sure, talents of the first order, such as Newton, Leibnitz, Euler, Clairaut, D'Alembert, possessed too much tact and intuitive insight to permit themselves to be dragged to the dangerous extremes and yawning precipices of error, toward which their own imperfect theory of infinite series tended to draw them. And yet, some of them did not escape blunders. The penetrating and teeming mind of Euler, for instance, is said to have fallen into some glaring mistakes by the incautious use of infinite series.

Among the mathematicians who, above all others, made the most unrestricted and reckless use of infinite series, were the Germans. There flourished in Germany during the latter part of the eighteenth century a mathematical school which occupied itself principally with what was termed "combinatorial analysis." This analysis was cultivated in Germany with singular and perfectly national predilection. One of the first problems considered by them was the extension of the binomial formula to polynomials, and the devising of simple rules by which polynomials could be developed into series. The solution of this problem was followed by the problem of "reversion of series." In this connection a quotation from Gerhardt's *Geschichte der Mathematik in Deutschland* (*p.* 205) is instructive.

Says he: "The advocates of the combinatorial analysis were of the opinion that with the complete solution of this problem (of reversion of series) was given also the general solution of equations. But here they overlooked an important point—the convergency or divergency of the series which was obtained for the value of the unknown quantity. Modern analysis justly demanded an investigation of this point, inasmuch as the usefulness of the results is completely dependent upon it." It thus appears that, through the misuse of infinite series, the Germans were temporarily led to believe that they had reached a result which mathematicians had so long but vainly striven to attain, namely, the algebraic solution of equations higher than the fourth degree. It will be observed that their method lacked generality, since it could at best not yield more than one root of an equation. But in the determination of this one root the combinatorial school was deceived. The result was a mere delusion—a mirage produced by the refraction of the rays of reasoning from their true path while passing through the atmosphere of divergent series.

We proceed now to the further consideration of the binomial theorem. After the time of Newton numerous proofs were given of the binomial theorem. James Bernoulli demonstrated the case of whole positive powers by the application of the theory of combinations.

This proof is excellent, and has retained its place in school-books to the present day. But the general demonstration for the case where the exponent may be negative or fractional was still wanting. Maclaurin was among the first to offer a general demonstration. Soon after his followed a host of proofs, each of which met with objections. It is no great exaggeration to say that these early demonstrations seemed to satisfy no one excepting their own authors. Most celebrated is the proof given by Euler. It is still found in some of our algebras. But Euler's proof has one fault which is common to nearly all that have been given of this theorem. It does not consider the convergency of the series. It seems to me that this fault is fatal. Euler claims to prove that the binomial formula is generally true, but if this formula is actually taken as being universally true, then it can be made to lead to all sorts of absurdities. If, for instance, we take, in $(a + b)^n$, $a = 1$, $b = -3$, $n = -2$, then we get from the formula $\frac{1}{4} = \infty$.

One might think that absurdities of this kind would have brought about the immediate rejection of all proofs neglecting the tests of convergency, but this has not been the case.

Another infinite series occupying a central position in analysis is the one known to students of calculus as Taylor's theorem. It was discovered by Brook Taylor and published in London in 1715. One would have thought that the instant it was proposed, this theorem would have been hailed as the best and most useful of generalizations. Instead of this it remained quite unknown for over fifty years, till Lagrange

pointed out its power. In 1772 Lagrange published a memoir in which he proposed to make Taylor's theorem the foundation of the differential calculus. By doing so he hoped to relieve the mind of the difficult conception of a *limit* upon which the calculus has been built by Newton and his disciples. The method of limits was then involved in philosophic difficulties of a serious nature. It was therefore very desirable that an explanation of the fundamental principles should be given which should be so clear and rigorous as to command immediate assent. The illustrious Lagrange attempted to supply such an explanation. He boldly undertook to prove Taylor's theorem by simple algebra, and then to deduce the whole differential calculus from Taylor's theorem. In this way the use of limits or of infinitely small quantities was to be dispensed with entirely. If Taylor's theorem be once absolutely granted, then undoubtedly all the rest may be made to follow by processes which are strictly rigorous. But in proving Taylor's theorem by simple algebra without the use of limits or of infinitesimals, Lagrange avoided the whirlpool of Charybdis only to suffer wreck against the rocks of Scylla. The principles of algebra employed by him in his proof were those which he received from the hands of Euler, Maclaurin, and Clairaut. His proof rested chiefly upon the theory of infinite series. But we have seen that this very theory was at that time wanting in mathematical rigor. Consequently, all conclusions evolved from it possessed the same defect. Though Lagrange's method of treating the calculus was at first greatly applauded, objections were afterward raised against it, because the deductions were drawn from infinite series without first ascertaining that those series were *convergent*. This defect was fatal, and to-day Lagrange's "method of derivatives," as his method was called, has been generally abandoned even in France.

At the beginning of this century the avidity with which the results of modern analysis were sought began so far to subside as to allow mathematicians to examine and discuss the grounds on which the several principles were established. The doctrine of infinite series received its due share of attention. In building up a tenable theory of infinite series, the same course became necessary which was followed some years ago in the erection of the Washington monument in the District of Columbia; after the work had proceeded to a certain height, the old foundation was found to be insecure; it had to be removed and to be replaced by another which was broader and deeper. The engineer to whom more, perhaps, than any other we are indebted for the laying of a new and firm foundation to infinite series and to analysis in general, is Cauchy. In the following few but pregnant sentences, taken from his *Cours d'Analyse* (Paris, 1821, p. 2), he states the object he has striven to attain: "As far as methods are concerned, I have endeavored to give them all the rigor required in geometry, and never to have recourse to the reasons drawn from the generalization of algebra. *Reasons of* that kind, although they are very generally accepted, espe-

cially in passing from converging series to diverging series and from real quantities to imaginary quantities, can be considered, it seems to me, only as inductions, fit to give a glimpse of the truth, but which agree little with the boasted exactness of mathematical science. It is furthermore to be observed that they tend to give to algebraic formulæ an indefinite extent, while in reality most of these formulæ remain true only under certain conditions and for certain values of the quantities which they contain." These weighty words of Cauchy became the parole of a new scientific party. Cauchy himself was eminently successful in his work. To him we owe the first correct proof of Taylor's theorem.

He took very strong and positive grounds against the use of divergent series. All series that were not convergent, he pronounced *fallacious.* Taylor's theorem he considered as being wrong whenever the series became divergent. In his *Cours d'Analyse* no place was given to those troublesome divergent infinite series that had previously been the cause of so much vagueness, uncertainty, and even of error.

But Cauchy was not alone in this protest against the unrestricted use of the time-honored methods of analysis. A youthful mathematician from northern Europe, a worthy descendant of mighty Thor, sided with the French mathematician in the contest. This new combatant was the youthful Abel, who, though he died at the premature age of twenty-seven, left behind him an imperishable name. As in the times of myth and fable, Thor, the thunderer, hurled his huge hammer against the mountain giants, so Abel, with his massive intellectual hammer, dealt powerful blows against some of the mathematical methods of his time. Notice an extract from a letter written by him in 1826, which expresses the convictions to which his profound studies had led him. Says Abel:*

"Divergent series are in general very mischievous affairs, and it is shameful that any one should have founded a demonstration upon them. You can demonstrate anything you please by employing them, and it is they who have caused so much misfortune, and given birth to so many paradoxes. Can anything be more horrible than to declare that

$$0=1-2^n+3^n-4^n+5^n-\text{ etc.},$$

when n is a whole positive number? At last my eyes have been opened in a most striking manner, for, with the exception of the simplest cases, as for example the geometric series, there can scarcely be found in the whole of mathematics a single infinite series whose sum has been rigorously determined; that is to say, the most important part of mathematics is without foundation. The greater part of the results are correct, that is true, but that is a most extraordinary circumstance. I am engaged in discovering the reason of this—a most interesting problem. I do not think that you could propose to me more than a very small number of problems or theorems containing infinite series, without my being able to make well-founded objections to their demonstration. Do

* *Œuvres Complètes de N. H. Abel*, Tome I, Christiania, 1839, p. 264.

so, and I will answer you. Not even the binomial theorem has yet been rigorously demonstrated. I have found that

$$(1+x)^m = 1 + mx + \frac{m(m-1)}{1.2}x^2 + \text{etc.}$$

for all values of x which are less than 1. When $x = +1$, the same formula holds, but only provided that m is > -1; and when $x = -1$, the formula only holds for positive values of m. For all other values of m the series $1 + mx +$ etc., is divergent. Taylor's theorem, the foundation of the whole infinitesimal calculus, has no better foundation. I have only found one single rigorous demonstration of it, and that is the one given by M. Cauchy in his Abstract of Lectures upon the Infinitesimal Calculus, where he has demonstrated that we have

$$p(x+\infty) = p(x) + \infty p'(x) + \frac{\infty^2}{1.2}p''(x) + \text{etc.},$$

as long as the series is convergent; but it is usually employed without ceremony in all cases. * * *

"The theory of infinite series in general rests upon a very bad foundation. All operations are applied to them as if they were finite; but is this permissible? I think not.* Where is it demonstrated that the differential of an infinite series is found by taking the differential of each term? Nothing is easier than to give examples where this rule is not correct. * * * The same remark holds for the multiplication and division of infinite series. I have begun to examine the most important rules which are (at present) esteemed to hold good in this respect, and to show in what cases they are correct and in what not so. This work proceeds tolerably well and interests me infinitely."

Such is the unequivocal language of Abel. His early death prevented him from carrying all his plans into execution. To him we are indebted for the first rigorous proof of the binomial theorem.†

The views on infinite series held by Cauchy and Abel met with hearty acceptance by leading mathematicians on the continent. Thus, Poisson expressed his views in the following language: "It is taught in the elements that a divergent series can not serve to calculate the approximate value of the function from which it results by development, but sometimes it has apparently been thought that such a series can be used in analytical calculations instead of the function; and although this error is far from being general among geometers, nevertheless it is not useless to point it out, for the results which are obtained by means of divergent series are always uncertain and most of the time inexact."

The conditions for convergency and divergency of different series be-

* Dirichlet first pointed out that the most elementary algebraic rule, according to which every sum is independent of the arrangement and grouping of the terms to be added, does not necessarily hold true in infinite series.

† *Œuvres Complètes de N. H. Abel*, Tome I, Christiania, 1839, p. 66.

gan to be carefully investigated. No universal criterion for determining whether a given series is convergent or divergent was then known; nor do we possess such a one even to-day.

A question naturally arising at this point of our inquiry is, whether the views of Cauchy and Abel and their co-workers met at once with general acceptance or not. As might almost be expected, they did not, but encountered firm opposition. The old combinatorial school in Germany would not surrender their orthodox views without a struggle. They obstinately defended every doctrine of their mathematical creed. Even such a man as Dr. Martin Ohm, who was really an enemy of the combinatorial school, and whose achievements in mathematics and physics place him among the coryphæi of science, was not willing to join Cauchy and Abel in calling divergent series fallacious. In an essay written by Ohm, entitled, The Spirit of Mathematical Analysis,* he admits that the great mathematicians of his day, as Gauss, Dirichlet, Jacobi, Bessel, Cauchy, do not employ demonstrations conducted with divergent series, while Poisson speaks decidedly against them. "But," says Ohm, "that the series which are used and from which deductions are drawn ought to be always and necessarily convergent is a circumstance of which the author of this essay has not been able at all to convince himself; on the contrary, it is his opinion that series, as long as they are *general*, so that we can not speak of their convergency or divergency, must always, when properly treated, necessarily and unconditionally produce correct results." By a *general* series Ohm means one in which the letters represent *neither magnitudes nor numbers*, but are considered as perfectly *insignificant* (*inhaltlos*). Whenever the letters are made to represent magnitudes or numbers, then the series is no longer a general series, but is a "*numeric*" series, and in that case Ohm admits that an equality can exist between the function and its series only when the series is convergent. It is very difficult to see exactly what meaning shall be given to letters upon which algebraic operations are to be performed, when the letters represent neither magnitudes nor numbers. Nor is it easy to see in what way formulæ involving these empty, meaningless letters—these "ghosts of departed quantities"—can furnish rigorous methods in mathematical analysis. In fact, this theory of *general* series containing *insignificant* letters is one of the last shifts to which the opponents of the new school resorted; one of the last subterfuges before giving up a contest which had become entirely hopeless.

If we pass from Germany to England we meet there with another mathematician who championed the old cause. I refer to George Peacock, who is well known to mathematicians for his Algebra and his Report, made in 1833 to the British Association, On the Recent Progress and Present State of Certain Branches of Analysis.

Peacock states his views with more clearness than Ohm had stated his. He bases his argument on what he calls the "principle of the per-

*The Spirit of Mathematical Analysis and its Relation to a Logical System, by Dr. *Martin Ohm; translated* by Alexander John Ellis, London, 1843.

manence of equivalent forms," which he considers to be the real foundation of all rules of symbolic algebra. According to this principle, all the rules and operations of arithmetic which have been established by numerical considerations are adopted without reference to relative magnitude; the symbols of algebra are taken to be perfectly general and unlimited in value, and the operations to which they are subject are equally general. To illustrate: In arithmetic we can subtract a smaller number from a larger, but we cannot subtract a larger from a smaller; that is to say, we can subtract 3 from 5, but not 5 from 3. In algebra, on the other hand, no limitation whatever is placed upon the relative values of minuend and subtrahend; there we can subtract 5 from 3 and give the answer a rational interpretation. By the principle of the permanence of equivalent forms every result obtained from mathematical operations must always be a correct result, no matter what the relative values of the quantities be upon which the operations are performed. Peacock applies this principle to the subject of infinite series.

He says (p. 205, Report for 1833) that "the series

$$(1+x)^n = 1^n\left(1 + nx + \frac{n(n-1)}{2}x^2 + \text{etc.}\right)$$

indefinitely continued, in which n is a particular value (a whole number), though general in form, must be true also, in virtue of the principle of the permanence of equivalent forms, when n is general in value as well as in form." Instead of being always a positive whole number, the exponent n may, therefore, be negative or fractional, and the above formula still holds true.

Now, the principle of the permanence of equivalent forms laid down by Peacock is not self-evident, nor did it become known by intuition; on the contrary, it is merely an induction, and can, therefore, hardly be taken as a reliable basis upon which to settle a disputed question; for this very question may be one in which this law established by mere induction might fail. But even granting the principle of the permanence of equivalent forms to be generally applicable, does it really follow from it that infinite series are true, whether they be convergent or divergent? In order to discuss this point let us examine a series resulting from the division of the numerator of an algebraical fraction by its denominator, such as $\frac{1}{1-a}$.

From arithmetic we get the simple but general statement that the numerator of a fraction divided by its denominator is equal to the quotient plus the remainder (if there be any remainder). By the principle of the permanence of equivalent forms this must be true of fractions involving any quantities whatever. Now, if we divide 1 by $1-a$ we get $1+a+a^2+a^3+\frac{a^4}{1-a}$. We oberve there is a remainder, $\frac{a^4}{1-a}$. If we carry the division further there is still a remainder. No

matter how far the division proceeds it will not end, and a remainder will still exist. We may express this fact by writing $\frac{1}{1-a} = 1 + a + a^2 + \ldots\ldots a^n + \frac{a^{n+1}}{1-a}$. Now, if a has a value less than unity the remainder approaches zero, and we may therefore write $\frac{1}{1-a} = 1 + a + a^2 +$ etc., *ad infinitum*. This infinite series is correct whenever $a < 1$. But, according to Peacock, it would follow from the principle of the permanence of equivalent forms that, if this series is correct for $a < 1$, it must be true for all values of a. Hence the series is true when $a > 1$, in which case the series is divergent. Now, this conclusion appears to be inadmissible, because Peacock does not examine the remainder. When $a < 1$, the remainder approaches zero, and can therefore be neglected; but if $a > 1$, then we shall find that the remainder does not approach zero, and therefore cannot be neglected.

To neglect it would be to violate the principle of the permanence of equivalent forms. This principle demands that whenever there is a remainder it shall always be considered and expressed, no matter how far the division be continued. If in the above series we take $a = 2$ and neglect the remainder, then we get.

$$-1 = 1 + 2 + 2^2 + 2^3 + \ldots\ldots \textit{ad infinitum},$$

which is an absurdity. But if the remainder be taken into account, then we have

$$-1 = 1 + 2 + 2^2 + 2^3 + \ldots 2^n + \frac{2^{n+1}}{-1}.$$

This equation is always true, no matter how great n may be; that is to say, no matter how far the division be continued. From similar considerations in other series it would appear that divergent series are false and absurd, except when written with the remainder.

And yet not only Peacock, but even De Morgan was not willing to reject divergent series. Though De Morgan criticised the new school for the unconditional rejection of divergent series, he cannot be pronounced an enthusiastic supporter of the old school. In an article in the Transactions of the Cambridge Philosophical Society, Volume VIII, Part I, he says: "I do not pretend to have that confidence in series which, to judge from elementary writers on algebra, is common among mathematicians, not even convergent series." His views on this subject will be more fully elucidated by the following quotation from his article on "Series" in the Penny Cyclopædia: "A divergent series is, arithmetically speaking, infinite; that is, the quantity acquired by summing its terms may be made greater than any quantity agreed on at the beginning of a process. * * * Nevertheless, as every algebraist knows, such series are frequently used as the *representatives of*

finite quantities. It was usual to admit such series without hesitation; but of late years many of the continental mathematicians have declared against divergent series altogether, and have asserted instances in which the use of them leads to false results. Those of a contrary opinion have replied to the instances, and have argued from general principles in favor of retaining divergent series. Our own opinion is that the instances have arisen from a misunderstanding or misuse of the series employed, though sufficient to show that divergent series should be very carefully handled; but that, on the other hand, no perfectly general and indisputable right to the use of these series has been established *a priori.* They always lead to true results when properly used, but no demonstration has been given that they must always do so."

About the time when Peacock made his report to the British Association, Cauchy was developing new and valuable results on the subject of infinite series. With the aid of the integral calculus he was conducting a careful investigation of the conditions which must be fulfilled in order that a function be capable of being developed into a convergent infinite series. He found that four conditions must be satisfied: (1) The function must *admit of a derivative.* (2) The function must be *uniform*, that is, for any particular value for x the function must have only one value. (3) The function must be *finite.* (4) The function must be *continuous*, that is, it must change gradually as the variable passes from one value to another. These results greatly strengthened the position held by the new school, and notwithstanding the adroit arguments brought forth by various mathematicians of the old school in favor of divergent series, the leading mathematicians of to-day have rejected the old views and adopted those of Cauchy and Abel. In the theory of functions, a branch of mathematics which is now assuming enormous proportions, the convergency of all series employed is carefully and scrupulously tested. In late years more reliable criteria have been invented for determining the convergency. Standard treatises on the subject devote the larger part of their space to the consideration of convergency. Whenever a series is divergent, then either the remainder is inserted or the series is unceremoniously rejected. Indeed, divergent series are now looked upon by our best mathematicians as being nothing more than exploded chimeras.

Having briefly traced the history of infinite series in Europe, we shall consider the views on this subject held by American writers. Previous to the beginning of this century the text-books on algebra used in this country were all imported from abroad. About the only mathematical books published in America before 1800 were arithmetics and some few books on surveying. The earliest imported algebras came from Great Britain. The most important of them were the algebras of Maclaurin, Saunderson, Charles Hutton, John Bonnycastle, and Thomas Simpson. These writers belonged to what we have called the old school. As

might be expected the subject of series was handled by them with the same looseness and recklessness as by the older school of mathematicians on the continent. Thus, in Hutton's Mathematics, which was a standard work in its day, considerable attention was paid to series, but the terms "convergent" and "divergent" were not even mentioned. The earliest American compiler of a course of mathematics for colleges was Samuel Webber. In 1801 he published his "Mathematics." The algebraical part was necessarily elementary in character, and of course contained no formal criteria for convergency. Whatever defects Webber's Algebra may have, it has also its merits. It is pleasing to observe that as far as the author had entered upon the subject of infinite series he was on the right track. Speaking of a certain divergent series he says that "it is false, and the further it is continued the further it will diverge from the truth" (p. 291). This language possesses the true ring; it is free from the discords of error, and we regret that American writers of later date have not imitated it.

In 1814, thirteen years after the publication of Webber's Mathematics, appeared the algebra of Jeremiah Day, of Yale College. All things considered, Day's Algebra is superior to Webber's, but on the particular subject of series it can hardly be said to excel. President Day points out, to be sure, that a certain series must converge in order to come nearer and nearer to the exact value of the fraction from which the series was derived, but he does not even hint at the insecurity or absurdity of divergent series. He gives no demonstration of the binomial theorem, but speaks of it as being universally true.

Four years after the publication of Day's Algebra, John Farrar, professor of mathematics at Harvard, published An Introduction to the Elements of Algebra, * * * Selected from the Algebra of Euler. On the continent of Europe Euler's writings were at that time justly considered as the most profound, and as affording the finest models of analysis. Yet his writings were not faultless. His views on series were those of the old school. The discussion of series as given in Farrar's Euler demands our attention, because subsequent American writers were doubtless greatly influenced by it. On page 76 of this book, the fraction $\frac{1}{1-a}$ is resolved by division into an infinite series. The following comments upon it are then made: "There are sufficient grounds to maintain that the value of this infinite series is the same as that of the fraction $\frac{1}{1-a}$. What we have said may at first seem surprising, but the consideration of some particular cases will make it easily understood. * * * If we suppose $a=2$, our series becomes $=1+2+4+8+16+32+64$, etc., to infinity, and its value must be $\frac{1}{1-2}$ that is to say, $\frac{1}{-1}=-1$, which at first sight will appear absurd. But it must be remarked that if we wish to stop at any term of the above series we can not do so

without joining the fraction which remains." Now this last sentence is certainly a true statement. No fault can be found with it. It simply means that we must consider the *remainder*, the very thing which the new school persistently insists upon. But the next statement made by the author is objectionable. Says he: "Were we to continue the series without intermission, the fraction indeed would be no longer considered, but then the series would still go on." This really amounts to saying that when the series becomes infinite, the remainder shall not be considered.

Now, if the remainder is not taken into account, then we can say in the language of Webber that the further the series is continued, "the further it will diverge from the truth," hence it must be "false."

In addition to this abridgment of Euler's Algebra, Professor Farrar published a translation from the French of Lacroix's Algebra. Lacroix's works are justly celebrated for their purity and simplicity of style. Though more cautious in his statements than the majority of elementary writers, he must still be classed as belonging to the old school. In his algebra * he speaks of divergent series as leading to consequences that are "absurd." The binomial theorem is proved by Lacroix for the case when the exponent is a positive integer, but the proof for the other cases is omitted. In the light of modern mathematics this was a wise omission. A *correct* proof of the general theorem is too difficult for pupils beginning algebra. But the easier proofs are incorrect. Hence it is preferable to give no proof at all than give a wrong one. But Professor Farrar† was not satisfied with this omission. In his translation he adds a foot-note, with the erroneous statement that the binomial formula is "equally applicable to cases in which the exponent is fractional and negative," and he demonstrates this theorem in the last part of his Cambridge Course of Mathematics ("On the Differential and Integral Calculus") without, of course, considering the question of convergency.

Charles Davies, who was appointed professor of mathematics at the United States Military Academy at West Point in 1823, published in 1834 an algebra modeled after the large French treatise of Bourdon. This algebra is familiarly known as "Davies' Bourdon," and, like all other books of Professor Davies, has had a very extensive circulation in all parts of the United States. However excellent this treatise may be in other respects, on the subject of infinite series and the treatment of the binomial theorem it is very defective.

From what has been said it will be seen that the foreign authors whom our American writers took for models in compiling their algebras belonged to the old school. Our early American writers clung faithfully to the orthodox opinions of this school. The only dissenting voice came from Samuel Webber, and it was so feeble that it escaped

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*Elements of Algebra, by S. F. Lacroix, translated by John Farrar. Second edition, Cambridge, N. E., 1825, p. 241. (This second American edition was translated *from the eleventh edition*, printed in Paris in 1815.)

†*Ibid, p. 152.*

all notice. But what, you may ask, were the views held by later American mathematicians?

In answer to this I need not discuss each author individually. If we except a few very recent writers, then we may say that on infinite series the sins of one are quite generally the sins of all. You may consult the large and extensive Treatise on Algebra, of Charles Hackley, or the Elementary Treatise on Algebra, by James Ryan; the Elementary and Higher Algebra, by Theodore Strong; the University Algebra, by Horatio N. Robinson; the Algebra for Colleges and Schools and Private Students, by Joseph Ray; the Elements of Algebra, by Major D. H. Hill; the University Algebra, by Edward Olney; the Binomial Theorem and Logarithms, by William Chauvenet; the Treatise on Algebra, by Elias Loomis. You may consult these and many others, and you will find that they are all swayed more or less by the orthodox ideas of the old school. A few of them give tests for convergency, but none of them treat divergent series with that severity which these mischievous expressions deserve. If divergent series are false, then it ought to be so stated; the student should be informed of the fact that they *are* false. Judging only from American algebras, we might almost conclude that Cauchy, Abel, Poisson, Dirichlet had never lived, or that their ideas had been long since expunged from the creed of true science. Of the algebras which the writer has examined a few of very recent date are the only ones to which this statement is not applicable. But even these give demonstrations of the binomial theorem which are deficient in rigor. The writer has not seen a single proof of this theorem for negative or fractional exponents in any American algebra which is not open to well-founded objections. Our writers often begin the general proof with an equation in which the sign = expresses always a *numerical equality*, and, finally, arrive at an equation (the generalized binomial formula) in which the sign = *does not* express a *numerical equality*, except under certain limiting conditions. The student is not informed in what way such a change in the meaning of the sign = has been brought about, nor is he told by what process of logic this sudden metamorphosis is permissible.

It may be argued that the final equation expresses a *formal truth*. But is this *formal* truth anything more than a perforated shell from which the kernel of *useful* truth has been removed? When the equation expresses merely a *formal* truth, can it be used for numerical calculations? No. Can the series be employed in course of analytical demonstrations in place of the function? No, for it leads to uncertainty, and perhaps even to error. What then is this *formal* truth good for?

There is an American algebraist who says that the formula,

$$(1+x)^n=1+nx+\frac{n(n-1)}{2}x^2+\text{etc.},$$

"is at once true when n is positive or negative, entire or fractional, real or imaginary, rational or irrational." Yet, it was pointed out long

ago by Abel and others that even when the conditions for convergency are satisfied there are still other points to be considered before we are entitled to write the sign of equality between the function and the series. The expression $(1+x)^n$ has in general a multiplicity of different values. In fact, the only case in which it has a single value is when the exponent n is an integer. Whenever n is a rational fraction, the expression has more than one value; whenever n is irrational or imaginary, the expression has an infinite number of values.

The series itself, on the other hand, has always only one value. Now, if we place the function $(1+x)^n$ equal to the series, then the question arises, which one out of the possibly infinite number of functional values is equal to the one value of the series?

A process in which American books are deficient in rigor is the multiplication of one infinite series by another. Some of our books exhibit not the slightest hesitation in multiplying by one another any two series whatever, and placing their product equal to the product of the functions from which the two series were obtained. The same confidence is placed in the process of multiplying *infinite* series as in that of multiplying *finite* expressions. But as a matter of fact, when one or both series are divergent, then their product is an absurd result. It is therefore necessary that both series be convergent. But, strange to say, this necessary condition has not always been found sufficient. There are cases in which the product of two convergent series may actually be a divergent series. For instance, Cauchy has shown that the series

$$1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+\frac{1}{\sqrt{5}}\cdots\cdots$$

is convergent, but that its square

$$1-\frac{2}{\sqrt{2}}+\left(\frac{2}{\sqrt{3}}+\frac{1}{2}\right)-\left(\frac{2}{\sqrt{4}}+\frac{2}{\sqrt{6}}\right)+\cdots\cdots$$

is divergent. The investigation of this difficulty has led to the proof that only the so-called "absolutely convergent" series can be multiplied into each other without liability of error. Thus while our elementary books teach that all infinite series can be multiplied by one another, the most recent and most advanced treaties on the subject teach that only convergent series of a particular kind can be multiplied into each other so as to lead to trustworthy results.

If we had time we could go on and examine the development of functions into series by the method of indeterminate co-efficients, as taught in our elementary books. We should meet with several points which are open to well-founded objections. But we can not enter upon this subject now.

The writing of a good elementary text-book is one of the most difficult undertakings. It is hardly advisable to subject to rigorous proof every rule and every process which ought to go into an elementary *text-book on algebra.*

Many of the proofs would be either exceedingly difficult to the pupil or entirely beyond his comprehension. In consequence of this the problem arises to decide what had best be proved and what might best be assumed without demonstration. It is easy to see how the opinions of able and experienced teachers may differ in making this choice. But there is one point upon which there should be no difference of opinion. Whatever is placed in an elementary text-book ought to be, as near as we can make it, the truth and nothing but the truth. If a subject is so difficult that it can not be stated in an elementary way without *misstating* it, then it had best be left out altogether. Whatever reasoning would be fallacious and wrong when placed in an advanced treatise must be equally fallacious and wrong when placed in an elementary book. If divergent series are unreliable, absurd, or false in advanced articles written by Cayley and Abel, in the *Cours d'Analyse* by Jordan and by Cauchy, or in the *Calcul Différentiel* by Serret, then divergent series must be equally unreliable, absurd, or false in the elementary algebras of Loomis, Davies, or Robinson. Now, if divergent series are actually untrustworthy and fallacious (and the leading mathematicians of to-day consider them so), would it not be best to make a statement to that effect in our elementary algebras and to give at least some of the simplest criteria for determining the convergency. If a *correct* proof of the binomial theorem for negative and fractional exponents is too long and difficult to find a place in an elementary algebra, why should it not be entirely omitted from algebra, and inserted afterward in the differential and integral calculus? There it can be deduced at once as an immediate consequence of Taylor's theorem. But in that case we must be sure that our calculus gives a correct proof of Taylor's theorem.

Unfortunately, many of our American works give what may be called the old proof of this theorem, which proof is pronounced unsatisfactory by all standard writers on the calculus. De Morgan does not consider it a demonstration at all, but treats this old process as "nothing more than rendering it highly probable that

$$\Phi(a+h) \text{ and } \Phi(a)+\Phi'(a)\,h+\Phi''(a)\,\frac{h^2}{2}+\text{etc.},$$

have relations which are worth inquiring into." Todhunter likewise objects to the old proof, and especially to "the use of an infinite series without ascertaining that it is convergent."

We regret to say that many of our American books on calculus are just as reckless and unscrupulous in the treatment of infinite series as our algebras are. But this assertion can not be applied sweepingly to all our works on this subject. Take some of the more recent publications, as, for instance, Byerly's Calculus. On page 118 of Byerly's Differential calculus the following statement is made and emphasized by italics. "*It is very unsafe to make use of divergent series or to base any reasoning upon them.*" This doctrine contradicts the doctrine taught in our algebras. If Byerly's Calculus is correct, then our algebras must

be wrong. Imagine the confusion which will arise in the mind of the student. While he is studying algebra he learns that the binomial theorem is *universally* true. When Byerly's Calculus is placed in his hands, he discovers that this same theorem is not always true, but holds good only when certain conditions are satisfied. The thoughtful student will become disgusted at such glaring contradictions in the presentation and explanation of a science which, in the hands of a careful mathematician, can be made to be the most accurate and consistent of all sciences. In closing, we give the following summary of the views presented in this paper:

1. In calculating with or reasoning by means of infinite series, the question of convergency should always be considered. If a series is divergent, then the sign of equality should not be placed between that series and the function from which it was developed. If the sign of equality *be* used in that way, then it expresses an absurdity, which is no less an absurdity when found in an elementary text-book than when found in a more advanced treatise.

2. Those parts of the subject which are too difficult for correct treatment in algebra, may be assumed temporarily without demonstration, and may afterward be proved in the differential and integral calculus. This suggestion applies particularly to the binomial formula for all cases in which its exponent is not a positive integer.

ON PARALLEL LINES AND ALLIED SUBJECTS.

There are few subjects in mathematics which have been discussed to greater extent than that of parallel lines. The various attempts at improving the theory of this subject may be classified under four heads: I. In which a new definition of parallel lines is suggested. II. In which a new axiom, different from Euclid's, is proposed. III. In which efforts have been made to deduce the theory of parallels from the nature of the straight line and plane angle. IV. In which, during the present century, the whole subject of geometrical axioms has been re-investigated and searched to the very bottom, and in which the novel and startling conclusion has been reached that the space defined by Euclid's axioms is not the only possible non-contradictory space. This gave birth to what is now termed non-Euclidian geometry.

It is our intention to take up the discussion under the above four heads, with a view of presenting the ideas advanced by American mathematicians or given in text-books used in this country.

I.--NEW DEFINITIONS.

Euclid's definition of parallel lines is as follows: "*Parallel straight lines are such as are in the same plane, and which being produced ever so far both ways do not meet.*" This definition has been retained by the larger

number of American writers,[1] and seems indeed the most desirable one to use in elementary geometry.

Parallel lines are lines everywhere equally distant. This definition has been adopted by Hutton,[2] Webber,[3] T. Walker,[4] A. Schuyler,[5] and probably by other authors whose books have not been examined by the writer. This definition has never been popular here since the time of Webber. Chief among older and foreign authors who used it are Wolf, Dürer, Boscovich, T. Simpson (in the first edition of his Elements), and Bonnycastle. Clavius assumed that a line which is everywhere equidistant from a straight line is itself straight. This axiom or postulate, which, by the way, does not hold true in pseudo-spherical space (according to the ordinary methods of measurement), lies hidden in disguise in the above definition. The objections to that definition are that it is an advanced *theorem*, rather than a definition; that it involves a number of considerations of great subtlety; and that it has to be abandoned as a fundamental definition in the more generalized view which is taken of this science in what is called non-Euclidian geometry. To be rejected for similar reasons is the following definition.

Two lines that make equal angles with a third line, all being in the same plane, are parallel. This is given by H. N. Robinson.[6] It was used in France by Varignon and Bezout, and in England by Cooley.

Parallel lines are straight lines which have the same direction. This definition has been growing in favor in this country. The reason of its popularity lies in the fact that it appears to contribute to the brevity and simplicity of demonstrations. Its validity will be considered further on. One of the first, perhaps the first, to use it in this country was James Hayward, teacher of mathematics at Harvard College.[7] It was used by Benjamin Peirce,[8] N. Tillinghast,[9] Charles W. Hackley,[10] Davies and Peck,[11] Eli T. Tappan,[12] William T. Bradbury,[13] and G. A. Wentworth.[14]

In England the concept of direction was made the basis of a work

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[1] Of the books examined by the writer, the following employ this definition: Farrar, F. H. Smith, and Davies, in their respective editions of Legendrie; also, Chauvenet, Newcomb, Venable, Halsted, Loomis, Grund, Olney, Hassler, Hunter, Whitlock, and Wentworth (in the revised edition of his geometry, 1888).

[2] Hutton's Mathematics, edited by Robert Adrain, New York, 1831, Vol. I., p. 275.

[3] Webber's Mathematics, Cambridge, N. E., 1808, p. 340.

[4] Walker's Elements of Geometry, Boston, 1831, p. 30.

[5] Schuyler's Elements of Geometry, 1876, p. 33.

[6] Elements of Geometry, Plane and Spherical Trigonometry, Cincinnati, 1852, p. 11.

[7] Geometry, Cambridge, 1829, p. 7.

[8] Elementary Treatise on Plane and Solid Geometry, Boston, 1837.

[9] Plane Geometry for the use of schools, Concord and Boston, 1841.

[10] Elementary Course of Geometry for the use of schools and colleges, New York, 1847.

[11] Mathematical Dictionary, Article, "Parallel Lines."

[12] Treatise on Plane and Solid Geometry, "Ray's Series," Cincinnati, 1864.

[13] Elementary Geometry and Trigonometry, Boston, 1872.

[14] Elements of Plane and Solid Geometry, Boston, 1878. All editions of this most popular book, except the *revised* edition of June, 1888, contain the above definition of *parallels*.

on geometry by J. M. Wilson, 1868, but in his new book of 1878 the whole theory of direction is ignored.

II.—NEW "AXIOM."

Euclid proves in his Elements (I, 27) that "If a straight line falling on two other straight lines make the alternate angles equal to one another, the two straight lines shall be parallel to one another." But before any other step can be made, it is necessary either to prove or assume that in every other case the two lines are not parallel.

Being unable to prove this, Euclid assumed it. His assumption consists in what is generally called the twelfth, by some the eleventh "axiom:" "If a straight line meet two straight lines so as to make the two interior angles on the same side of it taken together less than two right-angles, these straight lines, being continually produced, shall at length meet on that side on which are the angles which are less than two right-angles." It has been validly urged against Euclid that this statement is far from being axiomatic. But H. Hankel* has shown that Euclid himself placed this among the postulates (where it more properly belongs) and not among the axioms. The mistake of calling it an axiom was due to later editors. Euclid thus placed the whole difficulty of parallel lines in an assumption.

It has been objected that this assumption is not sufficiently simple and obvious. Accordingly, Playfair proposed the following "axiom": "Two straight lines which intersect one another can not be both parallel to the same straight line." This is merely Euclid's "axiom" in a better and more obvious form. It has been adopted by the best American works on geometry.†

A large number of our geometries give neither Euclid's nor Playfair's "axiom," but pretend to *prove* some "theorem" which states, in substance, what is equivalent to Euclid's "axiom." This leads us to the next heading.

III.—"PROOFS."

Since neither Euclid's nor Playfair's "axiom" is axiomatic, innumerable attempts have been made to *prove* one or the other. Until within twenty years it was believed by many leading mathematicians that valid proofs could be deduced from reasonings on the nature of the straight line. But the researches which led to the development of non-Euclidian geometry have, at last, made it clear that all such attempts must necessarily remain fruitless.

We shall call attention to a few so-called proofs found in text-books used in this country. Hutton‡ proves the "theorem" that "when a line

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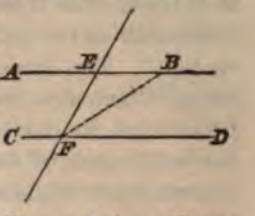
* *Vorlesungen über Complexe Zahlen und ihre Funktionen*, p. 52.

† The writer has seen it in the geometries of Davies, F. H. Smith, Venable, Loomis, Chauvenet, Hunter, Brooks, Grund, Newcomb, Halsted, and Wentworth (in his *revised edition*, 1888).

‡ *Hutton's Mathematics*, edited by Robert Adrain, New York, 1831, Vol. I, p. 288.

intersects two parallel lines, it makes the alternate angles equal to each other" (which is the equivalent of Euclid's "axiom")—in the following manner: If angles *AEF* and *EFD* are not equal, "one of them must be greater than the other; let it be *EFD*, for instance, which is the greater, if possible, and conceive the line *FB* to be drawn, cutting off the part or angle *EFB* equal to the angle *AEF, and meeting the line AB in the point B.* Then, since the outward angle *AEF*, of the triangle *BEF*, is greater than the inward opposite angle *EFB* (th. 8); and since these two angles also are equal (by the construction), it follows that those angles are both equal and unequal at the same time, which is impossible. Therefore, the angle *EFD* is not unequal to the alternate angle *AEF;* that is, they are equal to each other." The error of this proof lies in the (implied) statement that the line *FB* must always intersect the line *AB*, which is, virtually, an assumption of the thing to be proved. We know that in pseudo-spherical geometry one of the angles (say *EFD*) is greater than the other, and that the line *FB* does not cut *AB*.



This same proof is given in Davies' Elementary Geometry, p. 26. Attempts at proving the "parallel-axiom" were made also by Hassler, by a writer (James Wallace) in the Southern Review (Vol. I, 1828), and by A. C. Twining, in Silliman's Journal (1846, pp. 47 and 89).

Olney* proves Playfair's axiom in this way: "Let *AB* be the given line, and *G* the given point, there can be one and only one perpendicular through *G* to *AB* (127). Let this be *FE*. Now through *G* one and only one perpendicular can be drawn to *FE*. Let this be *CD*. Then is *CD* parallel to *AB* by the proposition (just proved in the book), and it is the only such parallel, since it is the only perpendicular to *FE* at *G*." The fallacy of this lies in the assumption that every line in the plane, drawn through the point *G* and not cutting the line *AB must necessarily* be perpendicular to *EF*.

An interesting attempt to prove Euclid's axiom is given anonymously in Crelle's Journal (1834), and translated and published by W. W. Johnson in the Analyst (Vol. III, 1876, p. 103). According to De Morgan, this proof is due to Bertrand. Professor Johnson says that "this demonstration seems to have been generally overlooked by writers of geometrical text-books, though apparently exactly what was needed to put the theory upon a perfectly sound basis." The error in the proof seems to lie in the statement that, if lines *AB* and *CD* in a plane lie on the same side of the line *AC* and are equally inclined to it, then the infinite space *BACD* must always be less than the infinite space *BAE*, provided only that angle *BAE* be not taken less than angle *BAC*. That this is not true in spherical geometry, is seen very readily; nor is it true in pseudo-spherical geometry.

* Treatise on Special or Elementary Geometry, University edition, New York, 1872, p. 70. In later editions this proof is omitted and Playfair's axiom assumed.

Those authors who adopt the idea of "direction," and define parallels as lines having the same direction, dispose of the whole subject in a trice. To them the theory of parallels gives no trouble. The difficulties of the subject are all hidden from sight by the notion of "direction." The following is the proof given by Hayward* of a "theorem" which says, in substance, the same thing as the "parallel-axiom." "The straight line has the same direction in every part. An angle is the inclination of one straight line to another; that is, the inclination to each other of these two directions. Two parallel straight lines have the same direction. Therefore, a straight line (which has but one direction in every part), meeting two straight lines which have but one direction in all their parts, must have the same inclination to both. That is, *when a straight line meets two parallel straight lines, the angles which it makes with the one are equal to those which it makes with the other.* Clearer evidence of the truth of this proposition can not be desired." A little further on we shall consider the question, Is the directional method scientific?

A mathematician whose attempts to prove the "parallel-axiom" were awarded with the most fruitful results, was M. Legendre. In the earliest editions of his celebrated Elements, he makes a direct appeal to the senses. In the seventh edition he assumes that a magnitude increases without limit when perpetual increase is all that is demonstrable. But his early proofs of the "parallel-axiom" did not satisfy even him, and he temporarily returned to Euclid's mode of treating parallels. Farrar's second edition of Legendre, brought out in 1825, contains this last presentation of the subject. Further investigations led Legendre to the beautiful result that the theory of parallels can be strictly deduced, if it can previously be shown that the three angles of a triangle are equal to two right angles. In Farrar's Legendre of 1831 and 1833 is given Legendre's attempt to prove this theorem without previously assuming the "parallel-axiom." The attempted proof is somewhat long, and introduces an infinite series of triangles. In Volume XII of the Memoirs of the Institute is a paper by Legendre, containing his last attempt at a solution of the problem. Assuming space to be infinite, he proved satisfactorily that it is impossible for the sum of the three angles of a triangle to exceed two right angles; and that if there be any triangle the sum of whose angles is two right angles, then the same must be true of all triangles. But in the next step, to show that this sum can not be less than two right angles, his demonstration failed.

IV.—RECENT RESULTS.

Some years before Legendre completed the above investigations, Lobatchewsky of Russia adopted the bold plan of constructing a geometry without assuming the parallel-axiom. He succeeded in this, and

* Elements of Geometry, p. ix.

it opened up the subject of non-Euclidian geometry. His discoveries were first made public in a discourse at Kasan, February 12, 1826. We can not here discuss the investigations on this subject that were made by Lobatchewsky, Gauss, Bolyai, Beltrami, Riemann, Helmholtz, Klein, and our own Newcomb.*

We shall only state that the possibility of constructing geometries upon different assumptions than those made by Euclid has become evident. We know now that, assuming space to be of uniform curvature, there are really three sorts of geometries possible—those of spherical space, of Euclid's space, and of pseudo-spherical space. Each of these is consistent in itself. These three do not contradict each other, but form rather one great system of which each is only a special case.

Much light has been thrown by the above generalizations upon the vexed subject of geometric "axioms." Do our more recent text-books profit by these researches? Some of them do. Take, for instance, Newcomb's Elementary Geometry. On page 14 we read, "We are to think of the geometric figures as made of perfectly stiff lines which can be picked up from the paper and moved about without bending or undergoing any change of form or magnitude." This statement embraces a property that is a common characteristic of all three geometries mentioned above, and distinguishes that group from any other which might be conceived, namely, the property that a figure can be moved about without undergoing either stretching or tearing.† "Through a given point one straight line can be drawn, and only one, which shall be parallel to a given straight line." The assumption, "one straight line can be drawn," divides the Euclidian and pseudo-spherical geometries from the spherical geometry; for in the last there are no (real) straight lines that are parallel to each other. The assumption contained in the words "and only one," separates Euclidian geometry from the pseudo-spherical. In the latter more than one line can be drawn through the same point, none of which intersect a given straight line. The assumptions thus made completely define the geometry of Euclid from the other two. A good statement of the assumptions about Euclid's space is found also in Halsted's Geometry.

This may be a convenient place to inquire into the scientific value of the term "direction" as a fundamental geometric concept. Professor Halsted says, in the preface to his geometry: "In America the geometries most in vogue at present are vitiated by the immediate assumption and misuse of that subtle term, 'direction;' and teachers who know something of the non-Euclidian, or even the modern synthetic gometries, are seeing the evils of this superficial 'directional' method. * * *

* For a bibliography of hyper-space and non-Euclidian geometry, see a paper by George Bruce Halsted in the American Journal of Mathematics, Vol. I, pp. 261–276, and pp. 384, 385; Vol. II, pp. 65–70.

† The property that figures can be moved about "without bending" distinguishes the geometry on an ordinary plane or on a sphere from that on a surface like the cone.

The present work, composed with special reference to use in teaching, yet strives to present the elements of geometry in a way so absolutely logical and compact, that they may be ready as rock-foundation for more advanced study." We quote on this subject also from a prominent German work of Dr. Wilhelm Killing.* "The attempts to establish a natural basis for geometery have, thus far, not been accompanied with desired success. The reason for that lies, in my opinion, in this, that even as geometry has been compelled to abandon the concept of direction (*Begriff der Richtung*) in the senserequired by the parallel-axiom, so it will not be able to hold on to the concept of distance (*Begriff des Abstandes*) as a fundamental concept, and must, therefore, pass far beyond the non-Euclidian forms of space (*Raumformen*) in the narrower sense." Thus, according to Dr. Killing, geometry has discarded the term direction as a fundamental concept.

There are several objections which can be urged against the term "direction." When we think of two straight lines as having different directions, we imagine ourselves placed on the point of intersection and looking along one of the lines, then the other. The term seems clear as long as we apply it to lines which intersect each other, or which coincide with one another. In the latter case we say that the two lines have the same direction. But we, as yet, have no geometric meaning of the phrase "the same direction," except when used of lines having a common point. Simply because lines which intersect each other have *different* directions, we can not logically conclude that lines which do not meet each other have the *same* direction. This objection was urged by De Morgan twenty years ago in his review of J. M. Wilson's geometry.† Says he, "There is a covert notion of direction, which, though only defined with reference to lines which meet, is straightway transferred to lines which do not. According to the definition, direction is a relation of lines which *do* meet, and yet lines which have the same direction can be lines which *never* meet. * * * How do you know, we ask, that lines which have the same direction never meet? Answer—lines which meet have *different* directions. We know they have; but how do we know that, under the definition given, the relation called *direction* has any application at all to lines which never meet? The notion of limits may give an answer; but what is a system of geometry which introduces continuity and limits to the mind as yet untaught to think of space and of magnitude?"

Benjamin Peirce says, in the preface to his geometry, "The term *direction* is introduced into this treatise without being defined; but it is regarded as a simple idea, and to be as incapable of definition as *length*, *breadth*, and *thickness*." But in case of length we have clear and rigorous means of testing by the method of superposition whether two lengths are equal or unequal. The same is true of breadth and thick-

* *Die Nicht-Euclidischen Raumformen in Analytischer Behandlung*, Leipzig, 1885, p. iv.
† *Athenæum, July 18*, 1868.

ness. In case of *direction*, on the other hand, comparisons cannot always be instituted, at least not without becoming involved in logical difficulties. We have no satisfactory means of telling whether two non-intersecting lines in a plane have the same or different directions. We are not even sure that the relation of direction can be applied to them. On a pseudo-spherical surface a whole pencil of lines can be drawn through a given point which do not intersect a given line. The lines in this pencil do not have the same directions with respect to one another. The question then arises, which one, if any, of these lines in the pencil has the same direction as the given line? If we can not distinguish between the presence and absence of a quality, then that quality is useless.

But suppose that, for the sake of argument, we waive the above objection, and say that parallel lines have the same direction. After defining straight line, angle, parallel lines, in accordance with the concept of "direction," we can reason in the same way as Hayward does in the quotation given under the third heading. But that mode of treating parallels excludes the possibility of the existence of pseudo-spherical geometry, inasmuch as it renders absurd the statement that two or more lines intersecting one another may exist, none of which intersect a third line, for lines in a plane which have different directions with respect to one another cannot all have the same direction with respect to a third line. The above use of the term direction involves assumptions as to the character of space which are too narrow to admit the use of that term as a fundamental concept. As far as possible, our Euclidian geometry should be made to rest upon concepts which need not be abandoned when we take a generalized view of the science. Our treatment of the elements should be a "rock-foundation for more advanced study."

One of the many objections to all attempts to found the elements of geometry on the word "direction" is stated by Professor Halsted in the following manner:* "Direction is a common English word, and in Webster's Dictionary, our standard, the only definition of it in a sense at all mathematical is the fourth: 'The line or course upon which anything is moving * * * ; as, the ship sailed in a southeasterly *direction*.' Direction, to be understood in any strict sense whatever, posits and presupposes three fundamental geometric ideas, namely, straight line, angle, parallels. After the theory of parallels founded upon an explicit assumption has been carefully established, a strict definition of direction may be based upon these three more simple concepts, and we may use it as Rowan Hamilton does in his Quaternions. But in American geometries, for example Wentworth's, the fallacy *petitio principii* is three times repeated by defining the three component parts of direction, each by direction itself."

Professor Halsted objects also to the word "distance" as a funda-

* Letter to the writer, November 17, 1888.

mental idea. He says, in the preface to his elements, that the attempt, "to take away by definition from the familiar word 'distance' its abstract character and connection with length-units, only confuses the ordinary student. A reference to the article 'Measurement,' in the Encyclopædia Britannica, will show that around the word 'distance' centers the most abstruse advance in pure science and philosophy. An elementary geometry has no need of the words 'direction' and 'distance.'" This view receives support from Dr. Killing in the above quotation. Professor Halsted has introduced the new word *sect*, meaning "part of a line between two definite points," and corresponding to the German word *Strecke*. The objections to the word distance are stated by him in the following words:* "Distance is also a common English word, and Webster as its first definition gives, 'An interval or space between two objects; the length of the shortest line which intervenes between two things that are separate. Every particle attracts every other with a force * * * inversely proportioned to the square of the *distance*. Newton.' Thus, distance posits shortest line and *length*, therefore measurement, therefore *ratio*, never treated before the fifth book in the Euclidian geometry, and never adequately treated at all in any other geometry without the use of the whole theory of limits. Yet American geometries, for example Wentworth's, give in place of the well-known simple proof of the theorem that any two sides of a triangle are together greater than the third, the abstruse assumption 'a straight line is the shortest distance between any two points,' and that after having explicitly said that there is only *one distance* between two points."

Before concluding this essay we should like to express our belief that detailed discussions of the fundamental geometric concepts should be avoided with students beginning geometry. Such discussions can be carried on with more profit when reviewing the subject near the end of the course, or when beginning the study of non-Euclidian geometry. In this connection I can not forbear quoting from a letter of Dr. E. W. Davis, of the University of South Carolina. Says he, "This getting down to the ultimate basis of our assumptions is a long and painful process, and should not be insisted upon in *elementary* instruction. The first beginning in mathematical reasoning should be reasoning that shows the student facts that are new to him. It disgusts him to have continually *proved* to him what he has always *known*, or to *begin* by asking him to *doubt* what he can not help but deem *true* in spite of all our fine logic. Confidence in logic should be gained by long experience in *predicting* by it the *unforeseen*, before we proceed by it to invalidate deeply-rooted and universally cherished conceptions." While we fully indorse these views, we at the same time insist upon a scientific treatment of geometry in our text-books, for the two following reasons: (1) When the student advances to a more generalized view of the subject, he will find that his first studies in this line rested upon a rock-foundation, and that the old edifice can be enlarged without being first de-

* Letter to the writer, November 17, 1888.

molished; (2) A teacher, like an honest preacher, prefers to teach doctrine which is, to the best of his knowledge, logically and philosophically true.

ON THE FOUNDATION OF ALGEBRA.

From Peacock's Report to the British Association, in 1833, on the Recent Progress and Present State of Certain Branches of Analysis (p. 188) we quote the following words: "Algebra was denominated in the time of Newton *specious* or *universal arithmetic*, and the view of its principles which gave rise to its synonym has more or less prevailed in almost every treatise upon this subject which has appeared since his time. In a similar manner, algebra has been said to be a science which arises from that generalization of the processes of arithmetic which results from the use of symbolical language; but though in the exposition of the principles of algebra arithmetic has always been taken for its foundation, and the names of the fundamental operations in one science have been transferred to the other without any immediate change in their meaning, yet it has generally been found necessary subsequently to enlarge this very narrow basis of so very general a science, though the reason of the necessity of doing so, and the precise point at which, or extent to which, it was done, has usually been passed over without notice."

From the same Report (p. 284) we quote the following: "In the early part of the last century the algebra of Maclaurin was almost exclusively used in the public education of this country. It is unduly compressed in many of its most essential elementary parts, and is unduly expanded in others which have reference to his own discoveries. * * * It was, subsequently, in a great measure susperseded, in the English universities at least, by the large work of Saunderson. It was swelled out to a very unwieldy size by a vast number of examples worked out at great length; and it labored under the very serious defect of teaching almost exclusively arithmetical algebra, being far behind the work of Maclaurin in the exposition of general views of the science."

There was indeed, in those days, some opposition at Cambridge (England) to the use of negative quantities in algebra. Among Cambridge algebraists who argued against the use of such quantities were Baron Francis Maseres (fellow of Clare), author of a dissertation on the negative sign in algebra (1758), and W. Frend, author of Principles of Algebra (1796–99). Both of these persons set themselves against Saunderson, Maclaurin, and the rest of the world; for they rejected negative quantities no less than imaginaries; and, like Robert Simson, "made war of extermination on all that distinguished algebra from arithmetic."*

The algebras studied by the early teachers and pupils in this country were all English works. Maclaurin, Saunderson, Thomas Simpson,

* *Scholæ Academicæ:* Some Account of the Studies at English Universities in the Eighteenth Century, by C. Wordsworth, 1877, p. 68.

Hutton, Bonnycastle, and Bridge were authors that could be found in the libraries of our American professors of mathematics As pointed out by Peacock, these authors began their treatises with arithmetical algebra, but gradually and disguisedly introduced negative quantities.

It is to be expected that our early compilers of algebra and writers on mathematics should imitate the English. The first publication in this country of a mathematical work which can, perhaps, lay some little claim to originality, was the work by Jared Mansfield, entitled, Essays, Mathematical and Physical, Containing New Theories and Illustrations of some very Inportant and Difficult Subjects of the Sciences.*

In the first essay, Mansfield says that "affirmative quantities are to be added, negative ones to be subtracted." Negative quantities "can never exist alone or independently; * * * for to suppose a compound where the elements have been all exhausted by the diminishing quantities, and something still left, would be very absurd. This, however, may be the case apparently, and in reality no absurdity follow. Thus the case above mentioned, 8—12, is absurd in itself, when pure numbers are considered; but an algebraist who knows how often the signs are changed in order to develop the unknown quantity, and that the quantities are often assumed without knowing on which side the difference lies, views this expression as nothing else than the difference of 12 and 8, or as 12—8; for those terms which have the sign + prefixed to them, have precisely the same effect on those to which the sign — is prefixed, as those which have the sign—on those which have the sign +. The signs are totally indifferent, excepting as to the operations, and where no operation is to be performed they are to be neglected."

These views suggest an algebra purely arithmetical, which finds it as impossible to give a *clear* explanation of negative quantities as it would of the imaginary $\sqrt{-1}$. In fact, negative quantities are the true "imaginaries" of such an algebra.

Day's Algebra contains a detailed discussion of positive and negative quantities. "A negative quantity is one which is required to be subtracted. When several quantities enter into a calculation, it is frequently necessary that some of them should be added together, while others are subtracted. The former are called affirmative or positive, and are marked with the sign +; the latter are termed negative, and distinguished by the sign —." But when a negative quantity is greater than a positive, how can the former be subtracted from the latter? "The answer to this is, that the greater may be supposed first to *exhaust* the less, and then to leave a remainder equal to the difference between the two." The interpretation of positive and negative quantities is then given by employing the ideas of *gain* and *loss*, *ascent* and *descent*, *north* and *south* latitude, etc.

* The work contains eight essays. Their titles are as follows: (1) On the Use of the Negative Sign in Algebra, (2) Goniometrical Properties, (3) Nautical Astronomy, (4) Orbicular Motion, (5) Investigation of the Loci, (6) Fluxionary Analysis, (7) *Theory of Gunnery*, (8) Theory of the Moon.

The treatment of this subject in Day's Algebra is essentially the same as that given by all American books, excepting those of recent date. It is only within the last ten or fifteen years that our writers on algebra (such as Olney, Wentworth, Wells, Thomson and Quimby, Bowser, Newcomb, Oliver, Wait, and Jones, Van Velzer and Slichter), have explicitly *assumed* the existence of negative as well as positive quantities at the very beginning of their text-books, and have clearly explained that the series of algebraic numbers is assumed as going out from 0 indefinitely in both directions, and that the signs + and − are used not only as signs of *operation* to indicate addition and subtraction, but also as signs of *quality* to indicate the nature of the quantities as positive or negative.

The algebra that is usually found in our school-books, wherein quantities are considered as being one or the other of two diametrically opposite kinds, has been called *single algebra*. It differs from *pure arithmetic* in assuming the existence not only of positive, but also of negative quantities. Neither pure arithmetic nor single algebra is perfect in itself, since each leads to expressions that are meaningless. In pure arithmetic, $a-b$, whenever $a<b$, expresses an impossibility and is an "imaginary value." In single algebra this ceases to be impossible, but there we are led to another impossibility, another "imaginary," namely, $\sqrt{-1}$.

By proceeding one step further in our generalization we come to *double algebra*, in which the existence of complex quantities (of the form $a \pm \sqrt{-1}\, b$) is assumed. Geometrically, such a quantity represents a line of definite length in some one definite direction in a plane. This algebra is capable of giving meaning to all the expressions to which it leads and is, therefore, perfect in itself. Some of our recent text-books (as Wentworth's, Bowser's, Newcomb's, Van Velzer and Slichter's, and especially Oliver, Wait, and Jones's) give a more or less complete account of this kind of algebra in their chapters on imaginaries. Triple, quadruple (quaternions), and other multiple algebras have been invented.

It will be seen that the true foundations of algebra have not been understood before the present century. The theory of imaginaries in double algebra has been developed chiefly by Argand, Gauss, and Cauchy. The philosophy of the first principles of algebra has been studied by Peacock, De Morgan, Hankel, and others. They established the three great laws of operations, *i. e.*, the distributive, associative, and commutative laws. A flood of additional light on this subject was thrown by the epoch-making researches of Hamilton, Grassman, our own Peirce, and their followers. They conceived new algebras, whose laws differ from the laws of ordinary algebra.*

* An excellent historical sketch of Multiple Algebra, by J. W. Gibbs, of Yale, will be found in the Proceedings of the American Association for the Advancement of Science, *Vol. XXXV, 1886.*

*DIFFERENCE BETWEEN NAPIER'S AND NATURAL LOGARITHMS.**

The term "Napierian logarithms" has been used in three different senses: (1) as meaning Napier's logarithms, or the ones invented by him and published in 1614 in his *Mirifici Logarithmorum Canonis Descriptio;* (2) as a synonym for "natural logarithms;" (3) as conveying the first and second meanings combined, and, thereby, implying that the natural logarithms are the ones invented by Napier. Though this last use of the term is inadmissible, because the logarithms invented and published by Napier are really different from the natural logarithms, it has, nevertheless, been the most prevalent; especially has it been prevalent in this country.

An examination of the algebras which have been in use in our schools will at once convince us that this error has been very general. We may consult the algebras of Ray, Greenleaf, Ficklin, Schuyler, Loomis, Robinson, F. H. Smith, Hackley, Davies, Bowser, Stoddard and Henkle, Thomson and Quimby, and many others, and we find it stated either that Lord Napier selected for the base of his system $e = 2.718\ldots$, or that he assumed the modulus equal to unity. Either of these two statements is equivalent to saying that the logarithms invented by Napier are identical with the natural logarithms. Some authors make statements like the following one, taken from the revised edition of Wells's University Algebra (p. 363): "The system of logarithms, which has e for its base, is called the Napierian system, from Napier, the inventor of logarithms."

The objection to statements like this is that they almost invariably mislead the student. What inference is more natural than that *Napierian* logarithms were invented by *Napier?* Some explanation ought therefore to be made guarding against this error.

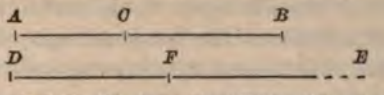
But I have seen only two American books doing this, namely, J. M. Peirce's Mathematical Tables, and Van Velzer and Slichter's Course in Algebra (of which a preliminary edition has just appeared). In these two books the truth is conveyed in plain words that Napier's logarithms differ from the natural. It is the object of this article to explain that difference.

It is important to note that, in Napier's time, our exponential notation in algebra had not yet come into use. To be sure, Stifel in Germany and Stevin in Belgium had, previous to this, made some attempts at denoting powers by indices; but this notation was not immediately appreciated, nor was it generally known to mathematicians, not even to the celebrated Harriot, whose algebra appeared long after Napier's death. It is one of the greatest curiosities in the history of mathe-

* This article has been published in the Mathematical Magazine, Vol. II, No. 1, and *is here reproduced* with some very slight changes.

matics that logarithms should have been constructed before exponents were used. We know how naturally logarithms flow from the exponential symbol, but to Napier this symbol was entirely unknown.

The interesting inquiry then arises, What was Napier's treatment of logarithms? It may be briefly stated as follows:



Let AB be a line of definite length, DE a line extending from D indefinitely. Imagine two points set in motion at the same time, and with the same initial velocity; the one point moving from D toward E with uniform velocity; the other from A to B with a velocity decreasing in such a way that when it arrives at any position, C, its velocity is proportional to the remaining distance, BC. While the latter point travels a distance, AC, suppose the former to move over the space DF. Napier called DF the logarithm of BC. He first applied this idea to the calculation of a table of logarithms for the natural sines in trigonometry. In the above figure, AB would represent the sine of 90° or the radius, which was taken by him equal to 10,000,000 or 10^7. BC would be the sine of an arc, and DF its logarithm.

"The *logarithm*, therefore, of any sine is a number very nearly expressing the line which increased *equally* in the meantime, while the line of the whole sine decreased *proportionally* into that sine, both motions being equal-timed, and the beginning equally swift."*

This treatment of the subject is certainly very unique. Let us now establish the relation between these Napierian logarithms and our natural logarithms. Let $m=AB$, $x=DF$, $y=BC$, then $AC=m-y$. The velocity of the point C is $\frac{d(m-y)}{dt}=ry$, r being a constant. Integrating, we have

$$-\text{Nat log } y=rt+c.$$

When $t=0$, then $y=m$, and $c=-\text{Nat log } m$. The velocity of the point C is rm, when $t=0$. Since the two points start with the same velcity, we have $\frac{dx}{dt}=rm$ as the uniform velocity of the point F. Hence $x=rmt$. Substituting for t and c their values, and remembering that, by definition, $x=\text{Nap log } y$, we get

$$\text{Nap log } y=m \text{ Nat log } \frac{m}{y}$$

The constant m was taken equal to 10^7. Substituting we get

$$\text{Nap log } y=10^7 \text{ Nat log } \frac{10^7}{y}$$

as the equation expressing the relation between Napierian and natural logarithms.

* Definition 6, p. 3, of Napier's *Mirifici Logarithmorum Canonis Descriptio*, etc., 1614.

That there is a difference between the two is evident at once. We easily observe the characteristic property of Napierian logarithms, that they *decrease* as the number itself *increases*. This property alone should have been a sufficient guard against declaring the two systems identical. The Napierian logarithm of 10^7 is equal to zero. The Napierian logarithms of numbers smaller than it are positive; those of numbers larger than it are negative, or, in the language of Napier, "less than nothing." In further illustration we give the following:

Nap. log. $1 = 161\ 180\ 956.509$; Nat. log. $1 = 0$
Nap. log. $2 = 154\ 249\ 484.703$; Nat. log. $2 = 0.6\ 931.472$
Nap. log. $10 = 138\ 155\ 105.578$; Nat. log. $10 = 2.3\ 025\ 851$

The question may be asked what *base* Napier selected for his system. We answer that he did not calculate his logarithms to a base at all. He never thought nor ever had any idea whatever of a *base* in connection with logarithms. The notion of a base suggested itself to mathematicians later, after the algorithm of powers and exponents, both integral and fractional, had come to be better understood.

If we inquire what the base to the logarithms in Napier's tables would have been had he used one, then it will be found that it does not coincide with the natural base e, but is very nearly equal to its reciprocal. In theory, that base is exactly equal to the reciprocal of e, as will be seen from the following relation,* which is merely another form of the one given above,

$$\frac{y}{10^7} = \left(\frac{1}{e}\right)^{\frac{\text{Nap log } y}{10^7}}$$

The base $\frac{1}{e}$ would not lead accurately to Napier's logarithmic figures, because the inventor's method of calculation was necessarily somewhat rude and inexact. The modulus of his logarithms is not equal to 1, but nearly equal to —1. If the base were exactly $\frac{1}{e}$, then the modulus would be exactly — 1; for the modulus of any system of logarithms is the logarithm, in that system, of the Napierian base e.

The first calculation of logarithms to the base of the natural system was made by John Speidell in his New Logarithms, published in London, in 1616, or five years after the first appearance of Napier's logarithmic tables.†

* To make the theory of exponents applicable to Napier's logarithms, it becomes necessary to divide the number y by 10^7, otherwise the base raised to the zero power would not be equal to unity. This division really amounts to making the length of the line AB equal to 1 instead of 10^7. If this be done, then *Nap. log. y* must also be divided by 10^7, so as to retain the inventor's conception that the two points on the lines AB and DE, respectively, move with equal initial velocities.

† The error of calling the Napierian and natural logarithms one and the same system has been wide-spread. We may pardon the celebrated Montucla, the eldest *prominent writer on the history of mathematics*, for making this mistake (Montucla,

CIRCLE SQUARERS.

It would be strange if America had not produced her crop of "circle-squarers," just as other countries have done. Our history of them will be quite incomplete. We have not gone out of our way to seek the acquaintance of this singular race of "mathematicians," nor have we avoided them. A few individuals have come across our path, and we proceed to tell about them for the benefit and edification not so much of mathematicians as of psychologists. The mathematician contemplates the products of only sound intellects; the psychologist studies also the utterings of minds that are or seem to be diseased.

The history of the quadrature of the circle is not without its sober lessons to mathematicians. It extends back through centuries almost to the beginning of geometry as a science.

The student of the history of mathematics is impressed by the fact that this science, more than any other, has always been a progressive one. He does not find a period in authentic history during which mathematics was not cultivated quite successfully by some nation or other The earliest contributions were made by the Babylonians and Egyptians, then came the Greeks, then the Hindoos, then the Arabs, and finally the Europeans. Like metaphysics, mathematics has encountered fundamental problems apparently of insurmountable difficulty. But it has generally had the good fortune to perceive that fortifications can be taken in other ways than by direct attack with open force; that, when repulsed from a direct assault, it is well to reconnoitre

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Histoire les Mathématiques, Tome II, Paris, 1758, p. 21), but there is hardly any excuse for a modern writer, such as Hoefer (*Histoire des Mathématiques depuis leurs Origines jusqu'au Commencement du Dix-neuvième Siècle*, Paris, 1874, p. 378), for stumbling over the same stone. The difference between the two systems was pointed out in Germany by Karsten in 1768, Kaestner in 1774, and Mollweide in 1808, but no attention was paid to their writings on this subject. A lucid proof of the non-identity of the two systems was given by Wackerbarth ("*Logarithmes Hyperboliques et Logarithmes Népériens*," Les Mondes, Tome XXVI, p. 626). The French mathematician Biot wrote likewise on this subject (*Journal des Savants*, 1835, p. 259), as did also De Morgan in England (English Cyclopædia, Article "Tables"). Still more recently attention has been called to this matter by J. W. L. Glaisher (Encyclopædia Britannica, 9th ed., Article "Logarithms"), and by Siegmund Guenther (*Untersuchungen zur Geschichte der mathematischen Wissenschaften*, Leipzig, 1876, p. 271). The writings of these scientists do not seem to have received the attention they deserve, and the erroneous notion of the identity of Napierian and natural logarithms still continues to be almost universal.

Napier's original works on logarithms are very scarce. The *Mirifici Logarithmorum Canonis Descriptio*, etc., *Edinburgi*, 1614, can be found in the Congressional Library in Washington and in the Ridgway Library in Philadelphia. The latter library has also the English edition of the above work, translated by Edward Wright in 1616. "So rare are these original editions that, of the two greatest historians of logarithms, Delambre never saw the Latin edition and Montucla never heard of the English." (Mark Napier's Biography of Lord Napier, p. 379).

and occupy the surrounding country and discover the secret paths by which the apparently unconquerable position can be taken.*

From this we can draw the valuable lesson that it is not always best to "take the bull by the horns."

The value of this precept may be seen by giving an instance in which it has been violated. The history of the quadrature of the circle is in point. An untold amount of intellectual energy has been expended upon this problem, yet no conquest has been made by direct assault. The circle-squarers existed in crowds even before the time of Archimedes and in all succeeding ages in which geometry was cultivated, down even to our own. After the invention of the differential calculus abundant means were introduced to complete the quadrature, if such a thing were possible. Persons versed in mathematics became convinced that the problem could not be solved, and dropped it. But those who still continued to make attempts upon this "enchanted castle," as it was supposed to be, were completely ignorant of the history of the subject, and generally misunderstood the conditions of the problem. "Our problem," says De Morgan,† "is to square the circle with the *old allowance of means:* Euclid's postulates and nothing more. We can not remember an instance in which a question to be solved by a *definite method* was tried by the best heads and answered at last *by that method*, after thousands of complete failures."

But great advance has been made on this problem by approaching it from a different direction and by newly discovered paths. Lanbert, an Alsacian mathematician, proved in 1761 that the ratio of the circumference of a circle to its diameter is incommensurable. Only nine years ago Lindemann, a German mathematician, demonstrated that this ratio is also transcendental, and that the quadrature of the circle by means of the ruler and compass only, or by means of any algebraic curve, is *impossible.*‡ He has thus shown by actual proof that which keen-minded mathematicians had long suspected, namely, that the great army of circle-squarers have, for more than two thousand years, been assaulting a fortification which is as impossible to be torn down as the firmament of heaven is by the hand of man.

Now-a-days, a person claiming to have solved this problem is ranked by mathematicians in the same class with inventors of "perpetual motion," and discoverers of the "fountain of perpetual youth." A very peculiar characteristic of circle-squarers, or quadrators, as Montucla calls them, is that they cannot be convinced of their errors. The first American quadrator we shall mention is William David ClarkMurdock, who, in a pamphlet of eight pages, bearing no date, gives a Demonstration of the Quadrature of the Circle.

The next man on our list is John A. Parker, whose work on the Quad-

* H. Hankel, *Entwickelung der Mathematik in den letzten Jahrhunderte*, p. 16.
† English Cyclopædia; article, "Quadrature of the Circle."
‡ *Mathematische Annalen*, Band XX, p. 213.

rature of the Circle (1851) was reviewed in the New Englander of February, 1852. The most prominent characteristics of this work, says the reviewer, are, a contempt for "algebra," and a grudge against "professors." The author proves that all geometers from Euclid to his (Parker's) great forerunner, Seba Smith, have been but blockheads in the very A B C's of their science. He solves in a twinkling the vexed problem of the "three bodies." He seems ashamed of his usher, Seba Smith, and takes him to task for "stealing his thunder." Over twenty years of experience seem to have made him no wiser. In 1874 he republished his book of over three hundred pages in almost exactly its original form.

The next publication is the following: The Regulated Area of the Circle and the Area of the Surface of the Sphere, by Charles P. Bouché, Citizen of the United States of North America, Cincinnati, 1854. It covers sixty pages. The author says: "Notwithstanding the perfection at which mathematics may have arrived in rectilineal geometry, planimetry, and stereometry, yet with regard to the *curve line*, as well as the spherical surface, we have remained in great darkness till it pleased the author of Spheres to afford us some light in this respect, and from a source little expected, *i. e.*, through the medium of a plain, but a moderately cultivated, seeker after truth. By the assiduous application of mind and the blessing of God, I have ultimately succeeded in correcting some great errors respecting *curvilineal geometry*."

The next circle-squarer on our list is Lawrence Sluter Benson, the author of a geometry. In 1879 he published in New York a work called Mathematics in a Dilemma, in which he also gives an extremely interesting history of his efforts on this subject. He says that after completing a course of studies at college in 1858, and while residing on his former place in South Carolina, his mind drifted into geometrical abstractions. He published new modes of demonstrating the quadrature in 1860 and in 1862. He says that he offered "one thousand dollars to any one who could refute the result which I gave for the circle, namely, that the perimeter of its equivalent square *is exactly equidistant between the squares circumscribed and inscribed about the circle; the sides of all the squares being respectively parallel.* This offer and demonstration drew me in many discussions, for mathematicians claim themselves able to prove that this intermediate square is just equal to an inscribed decagon in the circle; whence they argue that I make the circle *too small.* Committees of expert mathematicians—professors in colleges were selected to decide this issue; but no decision was made. Therefore, in 1864, while the Civil War was raging, I ran the blockade and visited Europe, and laid my demonstration before scientific societies and distinguished mathematicians there." He then says that he returned to New York and published a simplification of the Elements of Euclid, with the repudiation of the *reductio ad absurdum.* He says that these changes met the approval of Professor Docharty of the Col-

lege of the City of New York, and that, in 1873, Charles Davies published a book where he also repudiated the *absurdum* reasoning. "For nearly twenty years mathematicians and myself have been at loggerheads on the issue made by me about the circle. I now propose to set at rest all doubts against the demonstration published by me in 1860 and 1862." In more recent years Mr. Benson's efforts to revolutionize mathematics have been unabated.

Dr. A. Martin tells us of a quadrator who deposited with him a manuscript, in 1885, proving that the long sought for ratio is exactly $3\frac{1}{8}\frac{1}{1}$. Mr. Faber, the writer of it, distinguished himself also in other branches of mathematical inquiry. In a phamphlet of thirty-four pages, in 1872, "Theodore Faber, a citizen of the United States, New York," makes the "extraordinary and most significant discovery of a *lacking link* in the demonstration of the world-renowned Pythagorean problem, utterly disproving its absolute truth, although demonstrated as such for twenty-three centuries." In justice to Mr. Benson, it should be remarked that he, too, is waging war against Euclid, I, 47.*

*Since writing the above, we have received from Dr. Artemas Martin a copy of the Notes and Queries, Vol. V, Nos. 6 and 7, June and July, 1888, giving a Bibliography of Cyclometry and Quadratures. From this article we see that Theodore Faber has appeared in print also on the subject of the quadature of the circle. The article gives over twenty publications, besides the ones mentioned above, by American writers who believe that they have found the true and exact ratio.

APPENDIX.

BIBLIOGRAPHY OF FLUXIONS AND THE CALCULUS.

TEXT-BOOKS PRINTED IN THE UNITED STATES.

HUTTON, CHARLES. *Course of Mathematics*, in two volumes.

American editions, revised by Robert Adrain, appeared in 1812, 1816 (†), and 1822. Evert Duyckinck brought out an edition in New York in 1828. Another edition appeared in 1831. The second volume contains a short account of the doctrine of fluxions, using the Newtonian notation.

VINCE, REV. S. *The Principles of Fluxions*, first American edition, corrected and enlarged. Philadelphia: Kimber & Conrad. 1812. Pp. 256.

Employs the Newtonian notation.

BEZOUT. *First Principles of the Differential and Integral Calculus, or the Doctrine of Fluxions*, from the Mathematics of Bezout, and translated from the French for use of students of the University at Cambridge, New England. Boston, 1824. Pp. 195.

This book forms a part of Farrar's Cambridge Mathematics. It is the first work published in this country employing the notation of Leibnitz and the infinitesimal method. "In the Introduction, taken from Carnot's *Réflexions sur la Métaphysique du Calcul Infinitésimal*, a few examples are given to show the truth of the infinitesimal method, independently of its technical form." This is done by explaining that there is a "compensation of errors."

RYAN, JAMES. *The Differential and Integral Calculus*. New York, 1828.* Pp. 328.

"The works which I have principally used in preparing this treatise are Lacroix, Lardner, Boucharlat, Garnier, and Du Bourgnet's Differential and Integral Calculus; Lagrange's *Calcul des Functions*, Simpson's Fluxion's, Peacock's Examples on the Differential and Integral Calculus, and Hirsch's Integral Tables" (advertisement). The first section of the book is given to "preliminary principles," in which the three methods of Newton, D'Alembert, and Lagrange are explained. The method adopted by the author is that of limits, but no formal definition of the term "limit" is given. The symbol (0), indicating the absence of quantity, is everywhere treated with the same courtesy and implicit confidence as though it were actually a quantity. The inquiry as to whether the laws of analysis are really applicable to this foreign intruder into the society of actual magnitudes, or whether it has to be governed by laws of its own, is nowhere deemed necessary. These remarks apply with equal force to other works on calculus and to works on algebra.

YOUNG, J. R. *Elements of the Differential Calculus*, comprehending the general theory of surfaces, and of curves of double curvature. Revised and corrected by Michael O'Shannessy. Philadelphia, 1833. Pp. 255.

In the preparation of this American edition, the editor was assisted by Professor Dod, of Princeton College.

In the explanation of the process of differentiation, he makes h absolutely zero, in an expression like this:

$$\frac{y'-y}{h}=3x^2+3xh+h^2.$$

* Ryan's Calculus is now a rare book. The copy we have before us was kindly lent to us by Prof. W. Rutherford, of the University of Georgia.

"In both these cases (of which that given here is one), as indeed in every other the respective differential co-efficients are only so many particular values of the general symbol $\frac{0}{0}$, to which $\frac{y'-y}{h}$ always reduces, when $h=0$." In the above example $\frac{0}{0}=3x^2$. "The expressions $\frac{dy}{dx}$ and $\frac{dz}{dy}$ have, we see, the advantage over the symbol $\frac{0}{0}$, of particularizing the function and the independent variable under consideration and this, it must be remembered, is all that distinguishes $\frac{dy}{dx}$ or $\frac{dz}{dy}$ from $\frac{0}{0}$, for dy, dz, dx, are each absolutely 0." "These differentials, although each = 0, have, nevertheless, as we have already seen a determinate relation to each other (!); thus, in the last example, this relation is such that $dy=2b\,(a+bx)\,dx$, and, although this is the same as saying that $0=2b\,(a+bx)\,0$; yet, as we can always immediately obtain from this form the true value of $\frac{0}{0}$ or $\frac{dy}{dx}$, we do not hesitate occasionally to make use of it." It will thus be seen that the author has no hesitation whatever in breaking up the differential co-efficient.

YOUNG, J. R. *The Elements of the Integral Calculus*, with its applications to geometry and to the summation of infinite series. Revised and corrected by Michael O'Shannessy. Philadelphia, 1833. Pp. 292.

DAVIES, CHARLES. *Analytical Geometry and Differential and Integral Calculus.* 18—.

——— *Elements of the Differential and Integral Calculus.* 1836.

Several editions of Davies' calculus appeared. In the improved edition of 1843 (pp. 17 and 18) the author says that $2ax$ is the *limit* toward which the ratio $\frac{u'-u}{h}=2ax+ah$ approaches in proportion as h diminishes, and hence "expresses that particular ratio which is independent of the value of h." Bledsoe objects to this, saying, "Shall they (teachers) continue to seek and find what no rational beings have ever found, namely, that particular value of $\frac{u'-u}{h}$ which does not depend on the value of h? That is to say, that particular value of a fraction which does not depend on its denominator!" Davies represents by dx "the last value of h, which can not be diminished, according to the law of change to which h or x is subjected, without becoming 0." "It may be difficult," says the author, "to understand why the value which h assumes in passing to the limiting ratio is represented by dx in the first member and made equal to 0 in the second." To this Bledsoe says: "Truly this is a most difficult point to understand, and needs explanation. For if h be made absolutely zero, or nothing on one side of the equation, why should it not also be made zero on the other side?" "Why should 'a trace of the letter x' be preserved in the first member of the equation and not in the second? The reason is, just because dx is needed in the first member and not in the second to enable the operator to proceed with his work."

As regards the conception of the term "limit," Davies believed that a variable actually reached its limit. "The limit of a variable quantity is a quantity toward which it may be made to approach nearer than any given quantity, and which it reaches under a particular supposition."*

Davies believed that by the definition of M. Duhamel, according to which a variable never reaches its limit, there seemed to be placed an "impassable barrier" between a variable quantity and its limit. "If these two quantities are thus to be forever separated," says he, "how can they be brought under the dominion of a common law, and enter together in the same equation?"†

* Nature and Utility of Mathematics, by Charles Davies, New York, 1873, p. 291.

† *Ibid.*, p. 326.

PEIRCE, BENJAMIN. *An Elementary Treatise on Curves, Functions, and Forces.* Volume I, containing analytic geometry and the differential calculus. Boston and Cambridge, 1841. Pp. 301. Volume II, containing calculus of imaginary quantities, residual calculus, and integral calculus. Boston, 1846. Pp. 290.

The method followed in these volumes is the infinitesimal, of which the author was a great admirer. The differential co-efficients are here denoted by D, D', etc. The second volume treats of many rather advanced subjects, such as imaginary infinitesimals, imaginary logarithms, imaginary angles, the imaginary angle whose sine exceeds unity, potential functions, residuals, definite integrals, elliptic integrals, method of variations, linear differential equations, Riccati's equation, and particular solutions of differential equations.

CHURCH, ALBERT E. *Elements of the Differential and Integral Calculus.* New York, 1842.

This is in many respects a good work, but the explanation of fundamental principles therein contained is too brief, and fails to convey a philosophic knowledge of them. The difficulties which a student is likely to encounter in a treatise like this have been well stated by a writer in the Nation of October 18, 1888: "What vexes and perplexes him (the student) is that he seems to himself to comprehend very clearly what he is doing, and to be doing what all his previous training had taught him he must not do. It all seems very easy, very simple, and very absurd. He is told to 'take the limit' of one side of his equation by striking out a quantity because it 'is approaching zero,' while on the other side the same quantity must be carefully preserved, because it is one of the terms of the ratio which is the very essence of the whole process."

MCCARTNEY, WASHINGTON. *Principles of the Differential and Integral Calculus,* and their application to Geometry. Philadelphia, 1844. Pp. 340.

The author makes use of the doctrine of limits, but retains the language of infinitesimals. "$\frac{dy}{dx}$ is used as a mere symbol to denote the ultimate ratio, $\frac{dy}{dx}$ being in reality $\frac{0}{0}$. But inasmuch as the rules for differentiating and the geometrical application of ultimate ratios are more readily understood by regarding the increments of the ordinate and abscissa as indefinitely small, we will call these increments in their ultimate state, *indefinitely small quantities.*" "For the sake of convenience," the student is asked to call dy and dx what he has just been told that they really are not. Such an exposition of a *fundamental* principle is quite apt to fail to give satisfaction to beginners.

McCartney's Calculus is a book possessing several good features.

LOOMIS, ELIAS. *Analytical Geometry and Calculus.* 1851.

Later the Calculus was published in a separate volume and much enlarged. The unfolding of fundamental principles, as given in the improved edition of 1874, is less objectionable than that in the preceding works which adopt the method of limits. The term "limit of a variable" is here subjected to *definition,* but the student is not informed whether or not the variable ever reaches its limit. The symbol $\frac{dy}{dx}$ is made to represent the limiting value of $\frac{incr.\ y}{incr.\ x}$. Confusion is apt to arise in the mind of the student from the fact that dx is "put for the *incr.* x in the limiting value" (which value is zero), and is afterward said to be "indeterminate" in value, "either finite or indefinitely small."

SMYTH, WILLIAM. *Elements of the Differential and Integral Calculus.* 1854.

The author uses the infinitesimal method, but says (p. 229) that "as a logical basis of the calculus, the method of Newton and especially that of Lagrange have some advantage. In other respects the superiority is immeasurably on the side of the method of Leibnitz."

COURTENAY, EDWARD H. *Treatise on the Differential and Integral Calculus and on the Calculus of Variations.* New York, 1855. Pp. 501.

The exposition of the method of limits, as given in this in many respects admirable work, is likewise open to objection. dx is pronounced to be "indefinitely small" and equal to h, but when $h=0$ at the limit, dx continues to remain indefinitely small.

ROBINSON, HORATIO N. *Differential and Integral Calculus,* 1861.

Some of Robinson's elementary works on mathematics became popular, but not so his advanced works. His calculus and astronomy met with able but severe criticism in the Mathematical Monthly. Robinson's work did not appear in a second edition, but the work of Quinby was added to "Robinson's Series" in place of it.

DOCHARTY, GERARDUS BEEKMAN. *Elements of Analytical Geometry and of the Differential and Integral Calculus.* New York, 1865. Pp. 306.

The part on the calculus covers 204 pages.

The method of limits is employed and treated in the manner customary with us at the time the book was written.

SPARE, JOHN. *The Differential Calculus: with Unusual and Particular Analysis of its Elementary Principles, and Copious Illustrations of its Practical Application.* Boston, 1865. Pp. 244.

This work I have never seen. Dr. Artemas Martin, who kindly sends me its title, calls it a unique work, as may be seen from the following, which he quotes from its preface: "The calculus being algebra, a strictly numerical science, the present treatise claims to have labored successfully in putting on the true character as such. No insinuation is allowed to prevail that it is any part whatever of analytical geometry or that it is other than the natural sequel and supplement of common algebra; useful, indeed, as an appliance, to borrow, in investigation of the few kinds of geometrical quantity."

QUINBY, I. F. *A New Treatise on the Elements of the Differential and Integral Calculus.* New York, 1868. Pp. 472.

Here, as in other works based on the method of limits, the student encounters at the outset the perplexing statement that $\frac{0}{0}$, where 0 denotes "absolute zero," is equal to some particular quantity.

STRONG, THEODORE. *A Treatise on the Differential and Integral Calculus.* New York. 1869. Pp. 617.

This work was printed, but, we understand, never published. The author died while the work was in press. Theodore Strong was professor at Rutgers College from 1827 to 1863, and enjoyed the reputation of being one of the very deepest and most erudite mathematicians in America. He was a very frequent contributor to our mathematical periodicals. To students who possessed taste for mathematical investigation he was a good teacher, but to those who had no taste he was unintelligible. He had an unconscious tendency to diverge into regions where the ordinary student could not follow him. This same tendency is exhibited in his Calculus, and also in his Elementary and Higher Algebra, published in 1859. Both works possess many original features, but the novelties contained in them are not always improvements. These books are defective in arrangement, and not at all suited for use in the class-room. In his general view of the calculus Strong follows Lagrange, but his mode of presentation is quite new. He believed that his treatment divested the calculus of all its old metaphysical encumbrances. He attempted to show how the foundations of this science could be established without the intervention of any of the antiquated hypotheses. "It is hence clear," says he, "that the differential and integral calculus are deducible from what has been done, without using infinitesimals or limiting ratios" (p. 271).

PECK, WILLIAM G. *Practical Treatise on the Differential and Integral Calculus*, with some of its applications to mechanics and astronomy. New York and Chicago, 1870. Pp. 208.

Employs the infinitesimal method.

SESTINI, B. *Manual of Geometrical and Infinitesimal Analysis.* Baltimore, 1871. Pp. 131.*

The infinitesimal method is used.

OLNEY, EDWARD. *General Geometry and Calculus.* New York, 1871.

The part on the infinitesimal calculus covers 152 pages. The infinitesimal method is used. It is the experience of the large majority of teachers in this country that the infinitesimal method, taken by itself, unaided by any other method, *does not seem rigorous* to a student *beginning* the study of the calculus, and *does not fully satisfy his mind.*

RICE AND JOHNSON. *The Elements of the Differential Calculus*, founded on the method of rates or fluxions. (Printed for the use of the cadets of the U. S. Naval Academy.) New York, 1874.

Without abandoning the ordinary notation, the writers return, in this work, to the method of Newton. Newton's method of rates or fluxions is employed in subsequent treatises written by the same authors, and also in the works of Buckingham and Taylor. By these writers much-longed-for improvements in the philosophical exposition of the fundamental principles of the transcendental analysis have been introduced.

JOHNSON, W. WOOLSEY. *Integral Calculus.*

RICE AND JOHNSON. *An Elementary Treatise on the Differential Calculus*, founded on the method of rates or fluxions. New York, 1877. Pp. 469.

RICE AND JOHNSON. *Differential Calculus* (abridged).

RICE AND JOHNSON. *Differential and Integral Calculus* (abridged).

CLARK, JAMES G. *Elements of the Infinitesimal Calculus* (in "Ray's Series"). New York and Cincinnati, 1875. Pp. 441.

The doctrine of limits is made the basis of this work. The author follows mainly the excellent philosophical treatise of M. Duhamel.

BUCKINGHAM, C. P. *Elements of the Differential and Integral Calculus.* By a new method, founded on the true system of Sir Isaac Newton, without the use of infinitesimals or limits. Chicago, 1875.

BYERLY, W. E. *Elements of the Differential Calculus*, with examples and applications. Boston, 1880.

The doctrine of limits is used as a foundation of the subject and preliminary to the adoption of the more convenient infinitesimal method. The notation $D_x y$ is employed.

BYERLY, W. E. *Elements of the Integral Calculus*, with a key to the solution of differential equations. Boston, 1882.

BOWSER, EDWARD A. *An Elementary Treatise on the Differential and Integral Calculus.* New York, 1880.

Adopts infinitesimal method.

TAYLOR, JAMES M. *Elements of the Differential and Integral Calculus.* Boston, 1884.

The author employs the conception of rates.

*A copy of this work was lent to us by Prof. J. F. Dawson, S. J., of Georgetown College, West Washington.

NEWCOMB, SIMON. *Elements of the Differential and Integral Calculus.* New York, 1887.

The author uses the method of infinitesimals, based on the doctrine of limits. An infinitesimal quantity is here defined as one "in the act of approaching zero as a limit." This definition of an infinitesimal has now been very generally adopted.

It has been said that years ago a cadet at West Point, extremely fond of mathematics, thus estimated the calculus: "The inventors of the differential and integral calculus have claimed that this branch of so-called science belongs to the department of mathematics, and, laboring under that delusion, have introduced it into the course of academical instruction for the torture of students. Such classification is obviously incorrect, because the principles of mathematics fall within the scope of the reasoning faculty. The calculus, on the contrary, lies without the boundaries of reason."* That such should have been the impression received by the student of the early works on calculus is not at all strange. Our recent publications on the subject have, however, made decided progress in the philosophical exposition of the fundamental principles. With a modern book and a competent teacher there is no reason why the ordinary student should not get a *rational* understanding of the calculus.

*Life of General Nathaniel Lyon, p. 30. The passage is quoted in the Analyst, Vol. I, 1874, "Educational Testimony Concerning the Calculus."

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